Distributed Estimation Based on Observations Prediction in Wireless Sensor Networks

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***Abstract*—We consider wireless sensor networks (WSNs) used for distributed estimation of unknown parameters. Due to the limited bandwidth, sensor nodes quantize their noisy observations before transmission to a fusion center (FC) for the estimation process. In this letter, the correlation between observations is exploited to reduce the mean-square error (MSE) of the distributed estimation. Specifically, sensor nodes generate local predictions of their observations and then transmit the quantized prediction errors (innovations) to the FC rather than the quantized observations. The analytic and numerical results show that transmitting the innovations rather than the observations mitigates the effect of quantization noise and hence reduces the MSE.**

***Index Terms*—Correlation, mean square error, prediction, quantization, wireless sensor networks.**

I. Introduction

ONE typical application of wireless sensor networks (WSNs) is the distributed estimation of scalar parameters using a fusion center (FC) (see, e.g., [1], and references therein). In such applications, sensor nodes are deployed in the area of interest in order to observe physical or environmental conditions such as temperature, pressure, or humidity, to name a few. The sensor node observations are transmitted over multiple access channel (MAC) to the FC for estimating the observed parameter. One challenge in implementing the estimation at the FC is the limited communication bandwidth between the sensor nodes and the FC. Thus, the observations have to be quantized first before transmission. Different distributed estimation schemes with discrete transmit signals have been studied in the literature. For example, a one-bit quantizer is proposed in [2] where each sensor node performs probabilistic local quantization for its observation. The authors in [3] consider the quantization bits and power scheduling problem for distributed estimation with quantized observations to minimize the total transmit power while ensuring a given mean-square error (MSE) performance.

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For typical WSNs observing a physical phenomenon, sensor node observations are highly correlated due to the dense deployment of sensor nodes and the nature of the observed phenomenon. The correlation between sensor observations is exploited in [4], [5] to achieve better routing and medium access control.

In addition, the correlation has also been utilized to improve the performance of the distributed estimation with quantized observations [6], [7]. The effect of the correlation between the sensor node observations on the MSE performance of the distributed estimation is studied in [8].

In this paper, we utilize the following two constraints on the sensor node observations to compress the transmitted information from the sensor nodes to the FC:

- The sensor node observations are correlated.
- Due to the orthogonal multiple access communication nature with the FC, sensor nodes overhear each other.

Specifically, each sensor node will use the transmissions overheard from other sensor nodes to compress its transmission and only transmit the innovative part of its measurement. This compression will result in improved power efficiency and better utilization of the bandwidth. To further improve the power efficiency and resilience to additive noise, each sensor will transmit a quantized and digitalized version of its innovation. The effect of using the innovations for the distributed estimation on the performance of the linear minimum mean-square error (LMMSE) estimator is studied in this letter. Numerical results confirm the performance improvement due to the proposed scheme.

The notations used in the following are stated here. Small letters, bold small letters, and bold capital letters designate scalars, vectors, and matrices, respectively. If $\mathbf{A}$ is a matrix, then $\mathbf{A}^T$ and $\mathbf{A}^{-1}$ denote the transpose and the inverse of $\mathbf{A}$, respectively. The matrix $\mathbf{I}$ is the identity matrix of an appropriate size. $\mathrm{diag}(\mathbf{a})$ denotes a diagonal matrix formed from the vector $\mathbf{a}$ and $\mathbf{a}_{(k)}$ is the first k elements in the vector $\mathbf{a}$. Finally, the statistical expectation is denoted as $\mathrm{E}\{\cdot\}$.

II. System Model

Consider a collection of K sensor nodes deployed in an area of interest to observe a parameter θ and communicate their observations to a FC according to the system model shown in Fig. 1. The observed parameter θ is modeled as a scalar Gaussian random variable with zero mean and variance σ_θ^2. Let x_k be the k^{th} noisy observation of the parameter θ, i.e.

$$x_k = s_k + n_k, \tag{1}$$

where s_k is the value of the k^{th} sensor measurement and n_k is a spatially uncorrelated Gaussian observation noise with zero

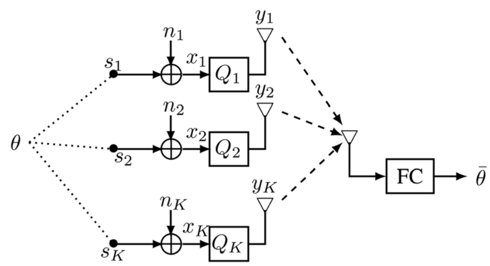

Fig. 1. System model of WSN consists of K sensor nodes and a FC for distributed estimation of a scalar parameter θ.

mean and variance $\sigma^2{}_n$. The correlation coefficient between the kth sensor observation x_k and the observed parameter θ (denoted by ρ_k and referred to as the *source-node* correlation) and the correlation coefficient between the two observations x_k and x_l (denoted by ρ_{kl} and referred to as the *inter-node* correlation) are respectively defined as

$$\rho_k = \mathrm{E}\{\theta x_k\} = \mathrm{E}\{\theta s_k\}, \quad \rho_{kl} = \mathrm{E}\{x_k x_l\} = \mathrm{E}\{s_k s_l\}. \quad (2)$$

We can write things more compactly using the source-node correlation vector $\mathbf{r} \triangleq [\rho_1, \rho_2, \ldots, \rho_K]^T$ and the inter-node correlation matrix $\mathbf{R_{ss}}$ with $\rho_{k,l}$ as its (k, l) entry. Consequently, the observation correlation matrix is given by $\mathbf{R_{xx}} = \mathbf{R_{ss}} + \mathbf{R_{nn}}$ where $\mathbf{R_{nn}} = \sigma^2{}_n\mathbf{I}$ is the observation noise correlation matrix.

Each observation is converted into a digitally modulated signal for transmission. Thus, the observation x_k, corresponding to the kth sensor node, is quantized first into an L bits message, i.e. the message assumes 2^L discrete values which can be represented by L bits. Let $q(x_k)$ be the quantized version of x_k with a range limited to $[-W, W]$. The quantization range is divided into intervals with step size $\Delta = 2W/2^L$ and the observation x_k is rounded to the closest mid-point of these small intervals[1]. The quantization noise in this case is $\tilde{x}_k = x_k - q(x_k)$ with variance $\sigma^2{}_q = \Delta^2/12$. For fine quantization, i.e. larger L, the quantization error $\tilde{x}_k$ has negligible correlation with the observation x_k[9].

Sensor nodes use time division multiple access (TDMA) to transmit their messages to the FC using some convenient digital modulation scheme. The received message y_k at the FC can thus be written as

$$y_k = q(x_k) + v_k = x_k + \tilde{x}_k + v_k, \quad (3)$$

where v_k is the demodulation error at the FC with variance $\sigma^2{}_v$. The value of $\sigma^2{}_v$ depends on the probability of error of the digital modulation scheme used for transmission, which is function of the communication signal-to-noise ratio (SNR).

Having the messages y_k at the FC, the LMMSE estimator can now be used to estimate θ as

$$\bar{\theta} = \mathbf{R}_{\theta\mathbf{y}}^T \mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{y}, \quad (4)$$

where $\mathbf{y} \triangleq [y_1, y_2, \ldots, y_K]^T$ is the message vector, $\mathbf{R}_{\theta\mathbf{y}}$ is the correlation matrix between the received messages and the observed parameter and $\mathbf{R_{yy}}$ is the message correlation matrix.

[1]For Gaussian parameters, the quantizer captures almost all observation values when we set $W = 3\sigma_\theta$, for small σ_n, and the step size Δ in this case would be the same for all sensor nodes.

Note that $\mathrm{E}\{y_k\theta\} = \mathrm{E}\{s_k\theta\} = \rho_k$ and thus $\mathbf{R}_{\theta\mathbf{y}} = \mathbf{r}$. Moreover, $\mathbf{R_{yy}} = \mathbf{R_{xx}} + \mathbf{R_{qq}} + \mathbf{R_{vv}}$, where $\mathbf{R_{qq}} = \sigma^2{}_q\mathbf{I}$ and $\mathbf{R_{vv}} = \sigma^2{}_v\mathbf{I}$ represent the quantization error correlation matrix and the demodulation error correlation matrix, respectively. The matrix $\mathbf{R_{xx}}$ is symmetric and diagonalizable and therefore the inverse of $\mathbf{R_{yy}}$ exists. Finally, the LMMSE estimator can be written as

$$\bar{\theta} = \mathbf{r}^T(\mathbf{R_{xx}} + \mathbf{R_{qq}} + \mathbf{R_{vv}})^{-1}\mathbf{y}, \quad (5)$$

and the corresponding MSE is

$$\mathrm{MSE} = \mathrm{E}\{(\bar{\theta} - \theta)^2\} = \sigma^2{}_\theta - \mathbf{r}^T(\mathbf{R_{xx}} + \mathbf{R_{qq}} + \mathbf{R_{vv}})^{-1}\mathbf{r}. \quad (6)$$

Note that all the noise and demodulation error correlation matrices are diagonal. Thus, the matrix $\mathbf{R_{yy}}$ would be a diagonal matrix when the message correlation matrix $\mathbf{R_{xx}}$ is diagonalized which allows more convenient analysis. In the following, the information about correlation between sensor node observations is exploited to generate orthogonal messages that have diagonal correlation matrix and thus adapt the quantization process to reduce the MSE.

III. Exploiting the Correlation Between the Observations to Reduce the Estimation MSE

The aforementioned scheme implements the LMMSE in a conventional way where all observations are treated equally. Intuitively, this is not an optimal scheme since some observations may carry more information than others. Power and bit allocation are typically used to improve the MSE performance [3]. Here, we use a different method where the correlation between observations is utilized to remove the redundant transmitted information from the different sensor nodes. Recall the fact that the sensor nodes transmit their observations sequentially in a TDMA arrangement. Hence, sensor nodes overhear each other and each sensor node can process its local observation given the received transmissions from other sensor nodes and the correlation knowledge. Consequently, only the "new" part of the information or the innovation should be transmitted. This has the potential to reduce the MSE of the distributed estimation even with equal power allocation among sensor nodes. With this in mind, the observation x_k can be expressed as

$$x_k = \bar{x}_{k|k-1} + e_k, \quad (7)$$

where $\bar{x}_{k|k-1}$ is the part of x_k that can be predicted from the previous $k-1$ sensor node observations and e_k is the innovation in x_k, i.e. the new part of x_k which is uncorrelated with the previous observations. Therefore, at the kth sensor node, instead of quantizing and transmitting the observation x_k, only its innovation, e_k, is calculated and quantized, which results in $q(e_k) = e_k + \tilde{e}_k$, then transmitted.

Thus, the received signal at the FC from the k^{th} sensor node can be alternatively expressed as

$$\hat{e}_k = e_k + \tilde{e}_k + v_k. \quad (8)$$

Now, the prediction should be based on the signals received from the previous $k-1$ sensor nodes, i.e. $\hat{\mathbf{e}}_{(k-1)} = [\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_{k-1}]^T$, which are the quantized innovations corrupted by demodulation errors. The prediction $\bar{x}_{k|k-1}$ should be generated in a way so that the elements of the innovation vector $\mathbf{e} = [e_1, e_2, \ldots, e_K]^T$ are uncorrelated with each other. The innovations are generated using Gram-Schmidt procedure which produces an orthonormal set of signals out of a given set. At the kth sensor node, the innovation e_k is given by [10]

$$
\begin{aligned}
e_k &= x_k - \bar{x}_{k|k-1} \\
&= x_k - \frac{\mathrm{E}\{x_k \hat{e}_{k-1}\}}{\mathrm{E}\{\hat{e}_{k-1}^2\}}\hat{e}_{k-1} \ldots - \frac{\mathrm{E}\{x_k \hat{e}_2\}}{\mathrm{E}\{\hat{e}_2^2\}}\hat{e}_2 - \frac{\mathrm{E}\{x_k \hat{e}_1\}}{\mathrm{E}\{\hat{e}_1^2\}}\hat{e}_1 \\
&= x_k - \mathrm{E}\left\{x_k \hat{\mathbf{e}}_{(k-1)}^T\right\}\left(\mathrm{E}\left\{\hat{\mathbf{e}}_{(k-1)}\hat{\mathbf{e}}_{(k-1)}^T\right\}\right)^{-1}\hat{\mathbf{e}}_{(k-1)} \\
&= x_k - \mathbf{b}_{k-1}^T \mathbf{R}_{\hat{\mathbf{e}}_{(k-1)}\hat{\mathbf{e}}_{(k-1)}}^{-1} \hat{\mathbf{e}}_{(k-1)} = \mathbf{w}_k^T \mathbf{z}_k,
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{x}_{1|0} &= 0, \\
\mathbf{z}_k &\triangleq [\hat{\mathbf{e}}_{(k-1)}^T, x_k]^T = [\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_{k-1}, x_k]^T, \\
\mathbf{b}_{k-1} &\triangleq \mathrm{E}\{x_k \hat{\mathbf{e}}_{(k-1)}^T\}, \\
&= [\mathrm{E}\{x_k \hat{e}_1\}, \mathrm{E}\{x_k \hat{e}_2\}, \ldots, \mathrm{E}\{x_k \hat{e}_{k-1}\}]^T, \\
&= [\mathrm{E}\{x_k e_1\}, \mathrm{E}\{x_k e_2\}, \ldots, \mathrm{E}\{x_k e_{k-1}\}]^T, \\
&= \mathrm{E}\{x_k \mathbf{e}_{(k-1)}^T\}, \qquad (9) \\
\mathbf{R}_{\hat{\mathbf{e}}_{(k-1)}\hat{\mathbf{e}}_{(k-1)}} &\triangleq \mathrm{diag}\left\{\mathrm{E}\{\hat{e}_1^2\}, \mathrm{E}\{\hat{e}_2^2\}, \ldots, \mathrm{E}\{\hat{e}_{k-1}^2\}\right\}, \\
\mathbf{w}_k &\triangleq \left[-\mathbf{b}_{k-1}^T \mathbf{R}_{\hat{\mathbf{e}}_{(k-1)}\hat{\mathbf{e}}_{(k-1)}}^{-1}, 1\right]^T. \qquad (10)
\end{aligned}
$$

We refer to $\mathbf{R}_{\hat{\mathbf{e}}_{(k-1)}\hat{\mathbf{e}}_{(k-1)}}$ as the innovation covariance matrix and $\mathbf{w}_k$ as the prediction weights vector. Equation (10) is obtained by exploiting the equality $\mathrm{E}\{x_k \hat{e}_j\} = \mathrm{E}\{x_k(e_j + \tilde{e}_j + v_k)\} = \mathrm{E}\{x_k e_j\}, \forall j < k$. Note that the innovation for the first sensor node is the sensed signal itself, i.e. $e_1 = x_1$.

Similar to the LMMSE in (4), the observed parameter can be estimated based on the received innovations $\hat{\mathbf{e}}$ as

$$\bar{\theta} = \mathbf{r}_{\hat{\mathbf{e}}}^T \mathbf{R}_{\hat{\mathbf{e}}\hat{\mathbf{e}}}^{-1} \hat{\mathbf{e}}, \tag{11}$$

and the corresponding MSE in this case is

$$\mathrm{MSE}' = \mathrm{E}\{(\bar{\theta} - \theta)^2\} = \sigma^2{}_\theta - \mathbf{r}_{\hat{\mathbf{e}}}{}^T \mathbf{R}_{\hat{\mathbf{e}}\hat{\mathbf{e}}}^{-1} \mathbf{r}_{\hat{\mathbf{e}}}, \tag{12}$$

where $\mathbf{R}_{\hat{\mathbf{e}}\hat{\mathbf{e}}} = \mathbf{R}_{\hat{\mathbf{e}}_{(K)}\hat{\mathbf{e}}_{(K)}}$. The kth diagonal element of $\mathbf{R}_{\hat{\mathbf{e}}\hat{\mathbf{e}}}$ can be written as

$$
\begin{aligned}
\mathrm{E}\{\hat{e}_k^2\} &= \mathrm{E}\{(\mathbf{w}_k^T \mathbf{z}_k)^2\} + \sigma_q^2 = \mathbf{w}_k^T \mathbf{z}_k \mathbf{z}_k^T \mathbf{w}_k + \sigma_q^2 \\
&= \mathbf{w}_k^T \begin{bmatrix} \mathbf{R}_{\hat{\mathbf{e}}_{(k-1)}\hat{\mathbf{e}}_{(k-1)}} & \mathbf{b}_{k-1} \\ \mathbf{b}_{k-1}^T & \sigma_s^2 + \sigma_n^2 \end{bmatrix} \mathbf{w}_k + \sigma_q^2 + \sigma_v^2, \qquad (13)
\end{aligned}
$$

and the correlation between the observed parameter θ and the innovation e_k is given by

$$
\begin{aligned}
\mathbf{r}_{\hat{e}_k} &= \mathrm{E}\{\theta \hat{e}_k\} = \mathrm{E}\{\theta e_k\} \\
&= \mathrm{E}\{\theta \mathbf{w}_k^T \mathbf{z}_k\} = \mathbf{w}_k^T \mathrm{E}\{\theta \mathbf{z}_k\} \\
&= \mathbf{w}_k^T [\mathrm{E}\{\theta \hat{e}_{(k-1)}\}, \quad \rho_k]^T = \mathbf{w}_k^T [\mathbf{r}_{\hat{e}_{(k-1)}}, \quad \rho_k]^T \qquad (14)
\end{aligned}
$$

Note that the innovations e_k do not have the same variance for all sensor nodes. This difference in variances allows having a different step size for the quantizer at each sensor node, i.e., $W_k = 3\sqrt{\mathrm{E}\{\hat{e}_k^2\}}$ and $\Delta_k = 2W_k/2^L$ which results in performance improvement as shown in the following section.

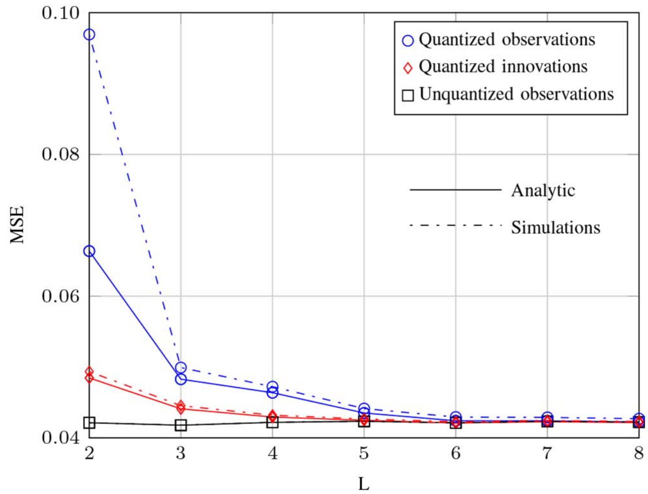

Fig. 2. Achieved MSE as function of the number of quantization bits ($K = 10$, $d_0 = 1000, \alpha = 1, \sigma_\theta^2/\sigma^2{}_n = 15$ dB, $(\sigma_s^2+\sigma_n^2)/\sigma^2{}_v = 15$ dB, and $\sigma_s^2 = 1$).

IV. Numerical Results

In this section, we confirm the performance analysis of the proposed scheme by numerical simulations. Consider a WSN that consists of $K = 10$ sensor nodes randomly located in a square sensing area with side length of 100 m. Here, both the observation SNR, $\sigma_\theta^2/\sigma^2{}_n$, and the communication SNR, $(\sigma_s^2 + \sigma_n^2)/\sigma^2{}_v$, are set to 15 dB for all sensor nodes, where $\sigma_\theta^2 = 1$. The correlation coefficients ρ_k and ρ_{kl} are non-negative and monotonically decreasing with distance. We assume that ρ_k and ρ_{kl} follow exponential functions of the distance d_k between the source and the kth sensor node and the distance d_{kl} between the kth and lth sensor nodes, respectively, as [8],

$$\rho(d) = e^{-\left(\frac{d}{d_0}\right)^\alpha}, \quad d_0 > 0, 0 < \alpha \le 2, d \in \{d_k, d_{kl}\}, \tag{15}$$

where d_0 is the distance normalization and α is the correlation decay rate, which are set to $d_0 = 1000$ and $\alpha = 1$. The quantizer at the sensor nodes uses $L = 3$ bits. The aforementioned settings are assumed for all the following simulations unless otherwise stated. All simulation results are averaged over 5000 independent Monte Carlo runs with the sensor nodes and the observed parameter locations are independently generated for each run according to a uniform distribution. For each of the following figures, the MSE performance of the LMMSE estimator with both quantized observations and innovations are shown. Also, the LMMSE estimator with unquantized observations is included as a lower bound of the MSE performance in the presence of observation and demodulation noise only. The dashed and solid lines refer to simulation and analytic results, respectively. The quantized data are transmitted using PAM modulation with a constellation of size 2^L.

Fig. 2 shows the MSE as a function of the number of quantization bits L. As expected, the LMMSE estimator with unquantized observations has the lowest MSE where only the observation and demodulation noise corrupt the observations. At lower number of quantization bits (L = 2,3, and 4), the LMMSE estimator with quantized innovations outperforms the same estimator with quantized observations. There is a deviation between the simulation results and theoretical analysis at $L = 2$ for the quantized observations case because the step size of the quantizer at this value is large. Consequently, the quantization errors

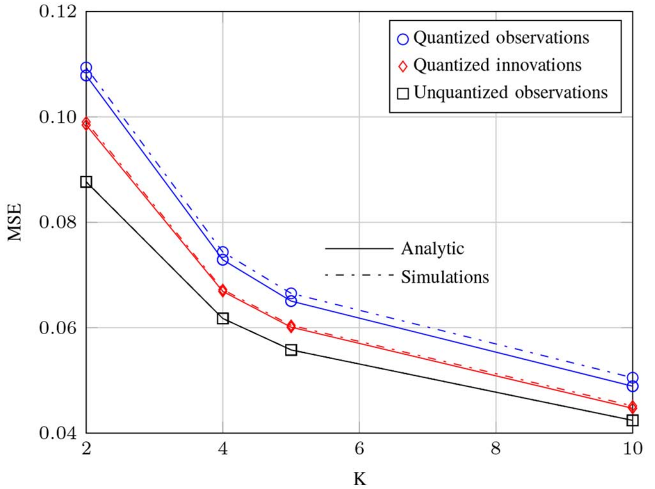

Fig. 3. Achieved MSE as function of the number of sensor nodes ($d_0 = 1000$, $\alpha = 1$, $\sigma_\theta^2/\sigma^2{}_n = 15$ dB, $\sigma_s^2 = 1$, and total communication SNR budget = 25 dB).

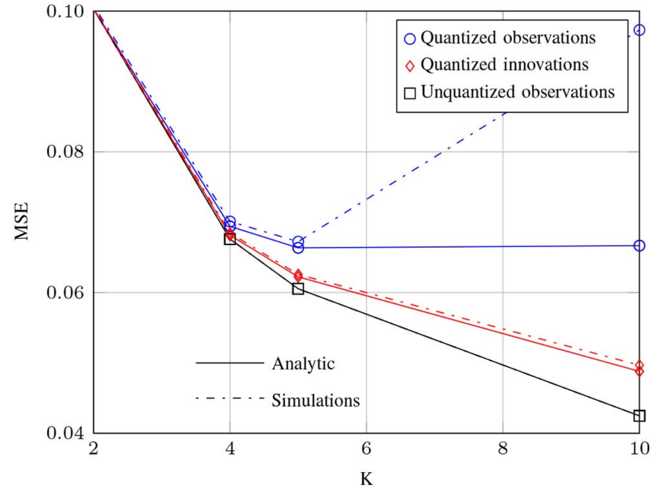

Fig. 4. Achieved MSE as function of the sensor nodes ($d_0 = 1000$, $\alpha = 1$, $\sigma_\theta^2/\sigma^2{}_n = 15$ dB, $(\sigma_s^2 + \sigma_n^2)/\sigma^2{}_v = 15$ dB, $\sigma_s^2 = 1$, and network bits budget = 20 bits).

retain some correlation with the observations and the quantization error correlation matrix has some non-zero off-diagonal elements. The MSE gap of the quantized observations and quantized innovations based estimators almost vanishes at $L = 5$ where the quantization error becomes negligible in both cases. Extra quantization bits in this case do not result in significant improvement on the MSE performance since most of the distortion comes from the observation and demodulation noise.

Fig 3 and Fig. 4 show the effect of the number of sensor nodes K on the MSE performance for two different scenarios. In Fig 3, we assume total communication power budget of 25 dB (for all sensor nodes) and fixed individual bit budget of 3 bits (per sensor node). In this scenario, increasing the number of sensor nodes reduces the MSE even that lower communication power is assigned for each node in this case (and consequently the demodulation errors increase). Note that the quantization error variance is the same for all sensor nodes since they use the same number of quantization bits $L = 3$.

Fig. 4 shows the opposite scenario. Specifically, it shows the MSE as function of the number of sensor nodes K with total bit budget of 20 bits (for all sensor nodes) and fixed individual communication power budget of 15 dB (per sensor node). Using the quantized innovations for the estimation achieves lower MSE than using the quantized observations. While increasing the number of sensor nodes reduces the MSE for the three cases in Fig. 4, larger number of sensor nodes results in lower number of quantization bits per sensor node which results in increasing MSE gap between the quantized observations and quantized innovations estimators. A difference between the analytic and simulation results at $K = 10$ (i.e., $L = 2$) is noticed here for the quantized observations case as expected.

The previous results show the effect of having fixed budget for the quantization bits and communication SNR on the MSE performance. It can be concluded that as long as the number of quantization bits per sensor node is more than about 4 bits, it is better to distribute the power budget over larger number of sensor nodes.

V. Conclusions

In this letter, we introduced a distributed estimation scheme utilizing the knowledge of the correlation between sensor node observations. The innovations in sensor node observations are locally predicted and transmitted to the FC. The innovations have smaller signal range which reduces the quantization error for the same number of bits as compared to the original observations. An improvement in the MSE performance was found in this case for low bit. It is also shown that observation noise and quantization errors degrade the MSE performance more than demodulation errors when the number of sensor nodes increases.

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