A Low-Complexity Near-ML Differential Spatial Modulation Detector

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***Abstract*—Differential spatial modulation (DSM) is a newly-emerging differential scheme tailored to the spatial modulation technique, which selects only one among a group of antennas for transmission at any time instant. DSM, however, gives rise to prohibitive search complexity when the number of transmit antennas is large. In this letter, a low-complexity suboptimal detector is proposed for DSM. It is designed based on the maximum-likelihood criterion but takes more candidates for the antenna activation orders into account. The detection is performed in two steps: the first step is to confine the number of candidates for the modulated symbols to a small portion by exploiting the symmetry of the signal constellation; the second step is to select the most likely modulated symbols from the output of the first step according to the determined antenna activation order via a Viterbi-like algorithm. Analyses and simulations show that the proposed detector achieves near-optimal performance yet largely reduces the search complexity.**

***Index Terms*—Differential modulation, search complexity, spatial modulation (SM).**

I. Introduction

SPATIAL MODULATION (SM) activates a single transmit antenna for transmission at any particular time instant [1]–[4]. However, this feature significantly challenges the acquisition of channel state information (CSI), which is necessary for coherent SM decoding [2]. To overcome this challenge, recently, we proposed a differential (D-)SM scheme, which dispenses with the CSI acquisition [5]–[7]. It is shown that DSM not only preserves most advantages of SM but also has the potential to pay less than 3 dB signal-to-noise ratio (SNR) penalty with respect to SM for a target bit error rate (BER). In contrast to SM, which uses the index of the active antenna to implicitly convey information, DSM relies on a unique antenna activation order in which each antenna is activated once and only once during N_T time instants, with N_T denoting the number of transmit antennas. Consequently, unlike SM, which has to search through N_T transmit antennas to identify the most probable active one from the received signal, DSM has to identify the most probable order out of nearly $\log_2 N_T!$ antenna activation orders from the received signals within N_T time instants. This implies that the development of a low-complexity detector for DSM is as crucial as that for SM [8].

In this letter, we propose a low-complexity suboptimal DSM detector, which is designed based on the maximum-likelihood (ML) criterion but takes more candidates for the antenna activation orders into account. The proposed detection is comprised of two steps. For the first step, the symmetric property of the phase-shift keying (PSK) constellation is taken advantage of and only about one-eighth of all constellation points are involved in the calculation. The output of the first-step detection is N_T^2 modulated symbol candidates that are to be fed to the input of the second-step detection. For the second step, the space of the search through all possible permutations of the indices of transmit antennas is reduced to a very small portion. This is carried out in a manner similar to the Viterbi decoder [9] by resorting to the property that each permutation, similar to convolutional coding, has memory. The output of the second-step detection gives the final estimated antenna activation order and modulated symbols. The search complexity of the proposed detector is shown to be much lower than that of the optimal DSM detector, and Monte-Carlo simulations are conducted to validate its near-optimal performance. The rest of the letter is organized as follows. Section II reviews the ML-based optimal DSM detector. The implementation, the search complexity, and the performance of the proposed detector are discussed in Section III.

Notation: Upper and lower case boldface letters denote matrices and column vectors, respectively. The (i,j)-th entry of a matrix $\mathbf{X}$ is denoted by $X(i,j)$. The i-th entry of a vector $\mathbf{x}$ is denoted by $x(i)$. The i-th element of a set κ is denoted by $\kappa\{i\}$, e.g., if $\kappa = \{\{1,2\},3\}$, then $\kappa\{1\} = \{1,2\}$ and $\kappa\{2\} = 3$. $(\cdot)^H$ and $|\cdot|$ denote the Hermitian operation and the absolute value, respectively. $\Re\{\cdot\}$ and $\Im\{\cdot\}$ represent the real and the imaginary components of their arguments, respectively. $\mathrm{sign}\{\cdot\}$ and $\lfloor\cdot\rfloor$ return the polarity of and the largest integer less than or equal to the argument, respectively. $\kappa \oplus \nu$ returns a new set which is formed by appending ν to κ, e.g., if $\kappa = \{1,2\}$ and $\nu = \{3\}$, $\kappa \oplus \nu = \{1,2,3\}$. $\kappa\backslash\nu$ returns a new set which is formed by removing ν from κ, e.g., if $\kappa = \{1,2,3\}$ and $\nu = \{3\}$, $\kappa\backslash\nu = \{1,2\}$. $\mathrm{Tr}_{\{}\cdot\}$ denotes the trace operation. $\mathbf{I}_P$ denotes the $P \times P$ identity matrix. $\mathrm{C}(n,r) = n!/(n-r)!/r!$.

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II. Optimal DSM Detector

In DSM, the communication is carried out block-wise, where each transmitted block occupies N_T time slots. In each transmitted block, the incoming bits are partitioned into two parts: the first part is comprised of $\lfloor \log_2 N_T! \rfloor$ bits and determines the activation order of all transmit antennas, while the second part is comprised of $\sum_{i=1}^{N_T} \log_2 M_i$ bits and generates the modulated symbols, $\{s_i\}_{i=1}^{N_T}$, with s_i being drawn from the M_i-PSK constellation of normalized power, χ_i. Consider the t-th transmitted block. The time slots from $(t-1)N_T + 1$ to tN_T are taken to transmit the $N_T \times N_T$ matrix $\mathbf{X}^{(t)}$, which is generated according to

$$\mathbf{X}^{(t)} = \mathbf{X}^{(t-1)} \mathbf{S}^{(t)}, \tag{1}$$

with $\mathbf{X}^{(0)} = \mathbf{I}_{N_T}$, where $\mathbf{S}^{(t)}$ is defined as the $N_T \times N_T$ information matrix. Here, only the $(A(m,i),i)$-th entry of $\mathbf{S}^{(t)}$ is nonzero and is given by $S^{(t)}(A(m,i),i) = s_i$, where $i \in \xi (= \{1, \ldots, N_T\})$ and $\{A(m,i)\}_{i=1}^{N_T}$ indicate the activation order of all transmit antennas corresponding to the integer parameter $m \in \omega (= \{1, \ldots, 2^{\lfloor \log_2 N_T! \rfloor}\})$, which has a one-to-one mapping relationship with the first part of the incoming bits [7]. Note that we have omitted the notation t in both $A(m,i)$ and s_i for notational simplicity, and will do so hereafter. An example for a one-to-one mapping with $N_T = 3$ is given by

$$\begin{aligned}
00 &\Rightarrow m = 1 \Rightarrow \{A(1,1), A(1,2), A(1,3)\} = \{1,2,3\}, \\
01 &\Rightarrow m = 2 \Rightarrow \{A(2,1), A(2,2), A(2,3)\} = \{1,3,2\}, \\
10 &\Rightarrow m = 3 \Rightarrow \{A(3,1), A(3,2), A(3,3)\} = \{2,1,3\}, \\
11 &\Rightarrow m = 4 \Rightarrow \{A(4,1), A(4,2), A(4,3)\} = \{2,3,1\}.
\end{aligned}$$

From the above explanation, it is clear that the data rate of the DSM system is $\frac{1}{N_T}(\lfloor \log_2 N_T! \rfloor + \sum_{i=1}^{N_T} \log_2 M_i)$(bps/Hz).

Let $\mathbf{Y}^{(t)}$, $\mathbf{H}^{(t)}$, and $\mathbf{N}^{(t)}$, all of dimension $N_R \times N_T$, represent the received signal matrix, the channel matrix with zero mean and covariance $\mathbf{I}_{N_R}$, and the complex Gaussian noise matrix with zero mean and covariance $1/\gamma \mathbf{I}_{N_R}$, corresponding to the t-th transmitted block, respectively, where N_R denotes the number of receive antennas and γ represents the receive SNR. The received signal in the matrix representation is given by

$$\mathbf{Y}^{(t)} = \mathbf{H}^{(t)} \mathbf{X}^{(t)} + \mathbf{N}^{(t)}. \tag{2}$$

Assuming quasi-static fading, in which case $\mathbf{H}^{(t-1)} = \mathbf{H}^{(t)}$, Eq. (2) can be rewritten from Eq. (1) as

$$\mathbf{Y}^{(t)} = \mathbf{Y}^{(t-1)} \mathbf{S}^{(t)} - \mathbf{N}^{(t-1)} \mathbf{S}^{(t)} + \mathbf{N}^{(t)}, \tag{3}$$

and thus the optimal ML detector can be derived as [7]

$$\begin{aligned}
&\{\hat{m}, \hat{s}_1, \ldots, \hat{s}_{N_T}\} \\
&= \underset{\tilde{m} \in \omega \& \{\tilde{s}_i \in \chi_i\}_{i=1}^{N_T}}{\arg\max} \operatorname{Tr}\left\{ \Re\left\{ \mathbf{Y}^{(t)^H} \mathbf{Y}^{(t-1)} \tilde{\mathbf{S}}^{(t)} \right\} \right\}.
\end{aligned} \tag{4}$$

In light of Eq. (4), it is clear that the optimal DSM detector necessitates an exhaustive search through all joint candidates $\{\tilde{m}\}$ and $\{\tilde{s}_i\}_{i=1}^{N_T}$, which leads to a search complexity of $O(2^{\lfloor \log_2 N_T! \rfloor} \prod_{i=1}^{N_T} M_i)$. Here, we do not take into account the computational cost of the involved matrix multiplication but only the comparisons among all candidates.

III. Proposed Detector

In this section, we present a low-complexity near-ML detector for DSM. To begin with, let $\mathbf{W}^{(t)} = \mathbf{Y}^{(t)^H} \mathbf{Y}^{(t-1)}$, which is of dimension $N_T \times N_T$. Due to the independence among the modulated symbols transmitted in different time slots, Eq. (4) can be simplified as

$$\hat{L}^{(t)}(\tilde{m}, i) = \underset{\tilde{s}_i \in \chi_i}{\arg\max} \Re\left\{ W^{(t)}(i, A(\tilde{m}, i)) \tilde{s}_i \right\}, \tag{5}$$

and

$$\begin{cases} \hat{m} = \underset{\tilde{m} \in \omega}{\arg\max} \sum\limits_{i=1}^{N_T} \Re\left\{ W^{(t)}(i, A(\tilde{m}, i)) \hat{L}^{(t)}(\tilde{m}, i) \right\} \\ \hat{s}_n = \hat{L}^{(t)}(\hat{m}, n), \; n = 1, \ldots, N_T \end{cases}, \tag{6}$$

where $\hat{\mathbf{L}}^{(t)}$ is of dimension $2^{\lfloor \log_2 N_T! \rfloor} \times N_T$. Since given $\tilde{m}_1 \in \omega$, there always exists at least an $\tilde{m}_2 \in \omega$ with $\tilde{m}_2 \neq \tilde{m}_1$ that satisfies $A(\tilde{m}_2, i) = A(\tilde{m}_1, i)$ for some $i \in \xi$ and large N_T, one will discover from Eq. (5) that $L^{(t)}(\tilde{m}_2, i) = L^{(t)}(\tilde{m}_1, i)$ in those cases. To avoid duplicate computation, we can further simplify Eqs. (5) and (6) as

$$\hat{G}^{(t)}(i, j) = \underset{\tilde{s}_i \in \chi_i}{\arg\max} \Re\left\{ W^{(t)}(i, j) \tilde{s}_i \right\}, \tag{7}$$

and

$$\begin{cases} \hat{m} = \underset{\tilde{m} \in \omega}{\arg\max} \sum\limits_{i=1}^{N_T} D^{(t)}(i, A(\tilde{m}, i)) \\ \hat{s}_n = \hat{G}^{(t)}(n, A(\hat{m}, n)), \; n = 1, \ldots, N_T \end{cases}, \tag{8}$$

where both $\hat{\mathbf{G}}^{(t)}$ and $\mathbf{D}^{(t)}$ are of dimension $N_T \times N_T$ and $D^{(t)}(i,j) = \Re\{W^{(t)}(i,j) \hat{G}^{(t)}(i,j)\}$ with $i, j \in \xi$. The proposed detector is based on Eqs. (7) and (8), which is carried out in the following two steps. The first step is to solve Eq. (7), and the second step is to solve Eq. (8) but with $\tilde{m} \in \omega$ relaxed to $\tilde{m} \in \{1, \ldots, (N_T - 1)(N_T - 1)!\}$ so as to facilitate low-complexity detection, which will be discussed later. Note that the above-mentioned relaxation only applies to $N_T \geq 3$ cases, which are of particular interest as we focus on the reduction of the computational complexity for a large N_T. With such relaxation, however, the detector may decide on $\hat{m}$ that is not included in ω, thus resulting in performance loss. Nevertheless, we will see from the simulation results that this performance loss is negligible. In the sequel, details on how each step works will be presented, and we will omit the notation t for brevity.

A. First Step: Signal Domain Pre-detection

The first step aims at low-complexity calculation of the matrix $\hat{\mathbf{G}}$ defined in Eq. (7). This step invokes an initial detection of modulated symbols and thus is termed signal domain pre-detection in this letter. The idea is clarified in the following.

Let us reformulate Eq. (7) as

$$\hat{G}(i,j) = \underset{\tilde{s}_i \in \chi_i}{\arg\max} \left\{ \Re\{W(i,j)\} \Re\{\tilde{s}_i\} - \Im\{W(i,j)\} \Im\{\tilde{s}_i\} \right\}. \tag{9}$$

Eq. (9) indicates that $\hat{G}(i,j)$ is a modulated symbol drawn from χ_i, for which the maximum of the expression is achieved. From Eq. (9), it can be deduced that $\Re\{\hat{G}(i,j)\}$ must have the same polarity as $\Re\{W(i,j)\}$ and $\Im\{\hat{G}(i,j)\}$ must have the opposite polarity to $\Im\{W(i,j)\}$. Therefore, we can solve Eq. (9) by first searching for the constellation point, $\breve{s}_i$, belonging to the first quadrant, which satisfies

$$D(i,j) = \Re\{W(i,j)\}\Re\{\hat{G}(i,j)\} - \Im\{W(i,j)\}\Im\{\hat{G}(i,j)\} = |\Re\{W(i,j)\}|\Re\{\breve{s}_i\} + |\Im\{W(i,j)\}|\Im\{\breve{s}_i\}, \quad (10)$$

based on the symmetry of the PSK constellation and then mapping $\breve{s}_i$ to $\hat{G}(i,j)$ based on the above-mentioned polarity property in analogy with [8]. However, we find that a further reduction of search complexity, which accounts for a half fewer constellation points than [8], is possible, thanks to the symmetry of the real and imaginary parts of the constellation points belonging to the first quadrant. To show this, define $\Delta(i,j) = 0$ if $|\Re\{W(i,j)\}| > |\Im\{W(i,j)\}|$, otherwise, $\Delta(i,j) = 1$. Further, let $\breve{s}_i = e^{\sqrt{-1}\frac{2\pi}{M_i}(K'(i,j)+\frac{M_i}{8}\Delta(i,j))}$ and $\hat{G}(i,j) = e^{\sqrt{-1}\frac{2\pi}{M_i}K(i,j)}$. By definition, the solutions of $\breve{s}_i$ and $\hat{G}(i,j)$ are now translated into the solutions of $K'(i,j)$ and $K(i,j)$, respectively. Since given $a_1 > a_2 > 0$ and $b_1 > b_2 > 0$ it follows that $a_1b_1 + a_2b_2 > a_1b_2 + a_2b_1$, from Eq. (10) the search for $\breve{s}_i$ can be completed by

$$K'(i,j) = \underset{k\in\left\{0,\ldots,\frac{M_i}{8}\right\}}{\arg\max} \left\{ |\Re\{W(i,j)\}| \cos\left(\frac{2\pi k}{M_i} + \frac{\pi\Delta(i,j)}{4}\right) + |\Im\{W(i,j)\}| \sin\left(\frac{2\pi k}{M_i} + \frac{\pi\Delta(i,j)}{4}\right) \right\}. \quad (11)$$

Then, according to $\hat{G}(i,j) = \text{sign}\{\Re\{W(i,j)\}\} \cdot \Re\{\breve{s}_i\} - \sqrt{-1}\,\text{sign}\{\Im\{W(i,j)\}\} \cdot \Im\{\breve{s}_i\}$, the relationship between $K'(i,j)$ and $K(i,j)$ can be readily established as

$$K(i,j) = \frac{M_i}{4}\left(2 + \text{sign}\{\Im\{W(i,j)\}\} + \frac{\text{sign}\{\Re\{W(i,j)\}\}}{\text{sign}\{\Im\{W(i,j)\}\}} - \frac{\text{sign}\{\Re\{W(i,j)\}\}}{\text{sign}\{\Im\{W(i,j)\}\}}\left(K'(i,j) + \frac{M_i}{8}\Delta(i,j)\right)\right). \quad (12)$$

Note that Eq. (12) is not unique. For example, one could have used $\text{sign}\{\Re\{W(i,j)\}\} \cdot \text{sign}\{\Im\{W(i,j)\}\}$ in place of $\text{sign}\{\Re\{W(i,j)\}\}/\text{sign}\{\Im\{W(i,j)\}\}$.

Example: Assume 16-PSK modulation for the i-th time slot, i.e., $M_i = 16$ and $\chi_i = \{1, e^{2\sqrt{-1}\pi/16}, \ldots, e^{2\sqrt{-1}\pi 15/16}\}$. Suppose that $W(i,j) = -2 - 3\sqrt{-1}$. Thus, we have $\Delta(i,j) = 1$, $\text{sign}\{\Re\{W(i,j)\}\} = -1$, and $\text{sign}\{\Im\{W(i,j)\}\} = -1$. Searching through about one-eighth of the constellation points according to Eq. (11) gives $K'(i,j) = 1$ and accordingly $\breve{s}_i = e^{2\sqrt{-1}\pi 3/16}$. Then, from Eq. (12) we have $K(i,j) = 5$ and accordingly $\hat{G}(i,j) = e^{2\sqrt{-1}\pi 5/16}$, which agrees with that solved by Eq. (9) via searching through the whole constellation.

When $\hat{\mathbf{G}}$ is ready, it is straightforward to obtain the metric matrix $\mathbf{D}$ from Eq. (10).

B. Second Step: Joint Detection

As discussed earlier, the second step is to search for an optimal order of indices of transmit antennas, parametrized by $\hat{m}$, via

$$\hat{m} = \underset{\tilde{m}\in\{1,\ldots,(N_T-1)(N_T-1)!\}}{\arg\max} \sum_{i=1}^{N_T} D(i, A(\tilde{m}, i)), \quad (13)$$

and to search for the most-likely transmitted symbols through the candidates included in $\hat{\mathbf{G}}$. This step invokes both signal domain and spatial domain detections and thus is termed joint detection in this letter. As the signal domain detection is simply $\hat{s}_n = \hat{G}(n, A(\hat{m}, n))$ with $n = 1, \ldots, N_T$, we will mainly focus on the spatial domain detection in the following.

TABLE I
DEFINITION OF PARAMETERS IN ALGORITHM 1

ψ^{pre}	a set that stores all most-likely antenna activation orders up to the last time slot
ψ^{now}	a set that stores all most-likely antenna activation orders up to the current time slot
$\mathbf{d}^{pre}$	a vector that stores accumulated metrics corresponding to those antenna activation orders in ψ^{pre}
$\mathbf{d}^{now}$	a vector that stores accumulated metrics corresponding to those antenna activation orders in ψ^{now}
ζ	a set that stores the indices of transmit antennas that are probably chosen at the next time slot

As indicated in Eq. (13), to perform the spatial domain detection, we have to first initialize the $(N_T - 1)(N_T - 1)! \times N_T$ matrix $\mathbf{A}$, which becomes impractical for a larger N_T. Noticing that the metric accumulation in Eq. (13) is dictated by the sequence $\{A(\tilde{m}, i)\}_{i=1}^{N_T}$, which has memory, we are motivated by the Viterbi algorithm [9] to reduce the search complexity. The idea can be clarified through the following example. Assume that $N_T = 4$ and there exist two candidates $\tilde{m}_1$ and $\tilde{m}_2$, giving $\{A(\tilde{m}_1, i)\}_{i=1}^4 = \{1,2,3,4\}$ and $\{A(\tilde{m}_2, i)\}_{i=1}^4 = \{2,1,3,4\}$. Then, from Eq. (13), it can be inferred that $\tilde{m}_1$ is preferred over $\tilde{m}_2$ when $D(1,1) + D(2,2) + D(3,3) + D(4,4) > D(1,2) + D(2,1) + D(3,3) + D(4,4)$, and vice versa. In fact, the above decision can be made merely via comparing the following two accumulated metrics $D(1,1) + D(2,2)$ and $D(1,2) + D(2,1)$. Moreover, to extend, provided that $D(1,1) + D(2,2)$ is larger (smaller) than $D(1,2) + D(2,1)$, the antenna activation orders with the first two indices being $\{2,1\}$ ($\{1,2\}$) can be all removed from the search space.

To realize this idea, we find it useful to predefine some parameters, as listed in Table I, and introduce the following enumeration method. The adopted enumeration method is based on the combinational number system (CNS) [10], which enables us to map a natural number $r \in \{1, \ldots, \mathrm{C}(N_T, n)\}$ to a unique permutation $\{c_n, \ldots, c_1\}$ with $c_1, \ldots, c_n \in \{1, \ldots, N_T\}$ and $c_n > \cdots > c_1$, via

$$r = \mathrm{C}(c_n - 1, n) + \cdots + \mathrm{C}(c_1 - 1, 1) + 1. \quad (14)$$

The enumeration procedure starts by choosing the maximal c_n that satisfies $\mathrm{C}(c_n - 1, n) \le r - 1$, and proceeds by choosing the maximal c_{n-1} that satisfies $\mathrm{C}(c_{n-1} - 1, n - 1) \le r - \mathrm{C}(c_n - 1, n) - 1$ and so on until Eq. (14) is satisfied. As an example, for $N_T = 5$, $n = 3$, and $r = 6$, the permutation $\{c_3, c_2, c_1\}$ can be calculated as $\{5, 3, 1\}$. With the above-mentioned preliminaries, we summarize the entire process of the proposed spatial domain detection in Algorithm 1. In the algorithm, the initialization corresponds to the 1-st time slot case, where $d^{pre}(N_T) := -\infty$ guarantees that $\tilde{m} \in \{1, \ldots, (N_T - 1)(N_T - 1)!\}$[1]. For the $(i \ge 2)$-th time slot case, in total $(N_T - i + 1)\mathrm{C}(N_T, i - 1)$ permutations of i out of N_T transmit antenna indices are to be grouped into $\mathrm{C}(N_T, i)$ classes. In each class, i permutations

[1]This implies that all antenna activation orders starting with index N_T are excluded in the search [7]. Note that it is also feasible to enable $\tilde{m} \in \omega$ in order to achieve the optimal performance, which, however, will significantly complicate the search as we have to judge the legality of each possible antenna activation order up to the current time slot.

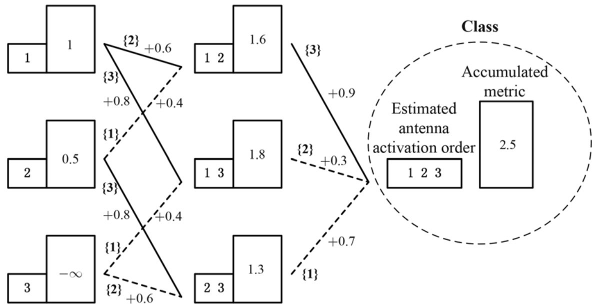

Fig. 1. Illustration of the example, where the solid line represents the survivor path whereas the dotted line represents the discarded path.

TABLE II
PARAMETERS EVOLUTION IN THE EXAMPLE

	$i=1$	$i=2$	$i=3$
ψ^{pre}	$\{1,\ldots,N_T\}$	$\{1,\ldots,N_T\}$	$\{\{1,2\},\{1,3\},\{2,3\}\}$
ψ^{now}	–	$\{\{1,2\},\{1,3\},\{2,3\}\}$	$\{1,2,3\}$
$\mathbf{d}^{pre}$	$[1, 0.5, -\infty]$	$[1, 0.5, -\infty]$	$[1.6, 1.8, 1.3]$
$\mathbf{d}^{now}$	–	$[1.6, 1.8, 1.3]$	2.5

of the same i indices will compete with each other and only the one with the largest accumulated metric will be saved and fed to the $(i+1)$-th time slot. The process continues until $i = N_T$, i.e., all transmit antennas have been considered in the comparison.

Algorithm 1 Viterbi decoding based spatial domain detector

1: Initialization: $\psi^{pre} := \xi, \mathbf{d}^{pre} := D(1,:), d^{pre}(N_T) := -\infty$
2: **for** $i = 2$: N_T % *loop 1 starts*
3: $\mathbf{d}^{now} := [0,\ldots,0]$ % $\mathrm{C}(N_T, i)$ zeros
4: **for** $z = 1$: $\mathrm{C}(N_T, i-1)$ % *loop 2 starts*
5: $\zeta := \xi \backslash \psi^{pre}\{z\}$
6: **for** $k = 1$: $N_T - i + 1$ % *loop 3 starts*
7: Sort $\psi^{pre}\{z\} \oplus \zeta\{k\}$ in descending order and get its address via (14) as r
8: **if** $d^{pre}(z) + D(i, \zeta\{k\}) > d^{now}(r)$
9: $\psi^{now}\{r\} := \psi^{pre}\{z\} \oplus \zeta\{k\}$
10: $d^{now}(r) := d^{pre}(z) + D(i, \zeta\{k\})$
11: **end if**
12: **end for** % *loop 3 ends*
13: **end for** % *loop 2 ends*
14: $\mathbf{d}^{pre} := \mathbf{d}^{now}$, $\psi^{pre} := \psi^{now}$
15: **end for** % *loop 1 ends*
16: Transfer ψ^{now} to the corresponding parameter $\hat{m}$

Example: Suppose that $N_T = 3$ and $\mathbf{D} = [1, 0.5, 0.2;. .0.4, 0.6, 0.8; 0.7, 0.3, 0.9]$. According to Algorithm 1, the evolution of some important parameters for different time slots (before line 14) is presented in Table II. A graphical illustration is also provided in Fig. 1.

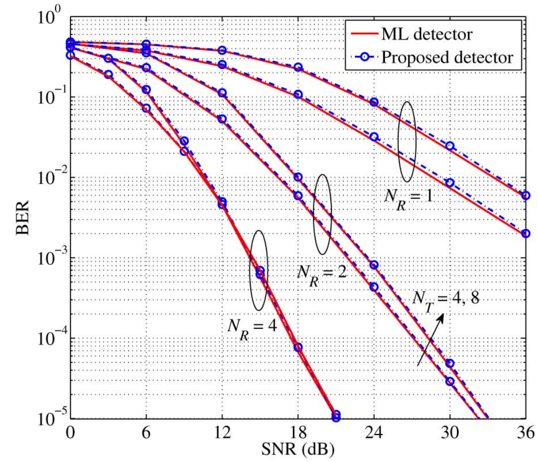

Fig. 2. Performance comparison between the ML detector and the proposed detector under the assumption of 8-PSK modulation, slow Rayleigh fading channels, $N_T = 4, 8$, and $N_R = 1, 2, 4$.

C. Complexity Analysis

From Section III-A, it is clear that the task of signal domain pre-detection is mainly focused on the calculation of the $N_T \times N_T$ matrix $\hat{\mathbf{G}}$, whose i-th row contains $N_T M_i$-PSK symbols, where $i = 1,\ldots,N_T$. Since each M_i-PSK symbol is obtained via searching through about one-eighth of the constellation points from Eq. (11), the overall search complexity of the signal domain pre-detection is of order $O(\frac{N_T}{8}\sum_{i=1}^{N_T} M_i)$. On the other hand, from Section III-B one can see that the task of joint detection is mainly focused on the spatial domain detection, which is detailed in Algorithm 1. The search starts from the $(i \geq 2)$-th time slot and ends at the N_T-th time slot. At the $(i \geq 2)$-th time slot, $\mathrm{C}(N_T, i)$ antenna activation orders will be saved, each of which is chosen by comparing $(N_T - i + 1)\mathrm{C}(N_T, i-1)$ ones. This means that a search through $(N_T - i + 1)\mathrm{C}(N_T, i-1)$ antenna activation orders is involved at the $(i \geq 2)$-th time slot. Therefore, the overall search complexity of the spatial domain detection can be obtained as $\sum_{i=2}^{N_T}(N_T - i + 1)\mathrm{C}(N_T, i-1) = N_T(2^{N_T-1} - 1)$ by virtue of [11, Eq. (0.154.1)]. Consequently, the overall search complexity of the proposed detector is about $O(\frac{N_T}{8}\sum_{i=1}^{N_T} M_i + N_T 2^{N_T-1})$.

D. Simulation Results

We perform simulations to examine the BER performance of the proposed detector. In the simulations, 8-PSK modulation and slow Rayleigh fading are assumed; the number of transmit antennas is chosen as $N_T = 4$ and 8, indicating that the data rates of the system are 4 bps/Hz and 4.875 bps/Hz, respectively; the number of the receive antennas is chosen as $N_R = 1, 2$, and 4, respectively. From the complexity analysis, one can see that for $N_T = 4$ and 8, the search complexities of the proposed detector amount to 48 and 1088, whereas those of the optimal detector are 65536 and 5.50×10^{11}, respectively. The simulation results are illustrated in Fig. 2. Obviously, the proposed detector achieves nearly the same BER performance as the optimal detector.

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