Optimal Bartlett Detector Based SPRT for Spectrum Sensing in Multi-antenna Cognitive Radio Systems

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***Abstract*—This letter presents the optimal Bartlett detector based sequential probability ratio test (SPRT) for spectrum sensing in multi-antenna array cognitive radio networks. The optimal Wald test based sequential Bartlett spectral detector (S-BSD) is derived for a single/multiple primary user scenario, considering a fading wireless channel. Further, this is also extended to a scenario with multiple-input multiple-output (MIMO) wireless systems. Closed form expressions are derived for the average number of blocks required for the sequential detector in terms of the desired probability of false alarm and mis-detection. The S-BSD framework is also subsequently extended to an AWGN channel scenario. Simulation results are presented to illustrate the performance of the proposed detection schemes and verify the derived analytical results. The proposed S-BSD scheme is seen to significantly reduce the average number of symbols/blocks required to achieve the desired detection performance in comparison to the fixed block size BSD (F-BSD).**

***Index Terms*—Bartlett Detector, cognitive radio, MIMO, sequential probability ratio test, spectrum sensing.**

I. Introduction

COGNITIVE radio [1] [2], enables unlicensed secondary users to dynamically access the vacant spectral bands [3], thereby increasing the efficiency of spectrum utilization. Naturally, a key requirement for dynamic spectrum access is to reliably identify unutilized spectrum blocks, also known as spectral holes. This procedure is termed as spectrum sensing [4]. Several spectrum sensing techniques such as energy detection (ED) [5], matched filter detection (MFD) [6], cyclostationary feature detection (CFD) [7] etc, exist in literature. However, a majority of them employ the fixed block size likelihood ratio test (LRT). In this context, the sequential probability ratio test (SPRT) [8] is better suited which requires a lower number of blocks/symbols in comparison to the fixed block size tests, thereby improving the overall efficiency of the system.

Very few works in existing literature have characterized the performance of the SPRT for spectrum sensing. Pilot based spectrum sensing algorithms, such as in [9] can potentially be employed in cognitive radio (CR) scenarios. However, they require the transmission of pilot symbols and knowledge of the fading channel coefficients, which renders them unsuitable for a general dynamic spectrum access scenario due to the large overheads and loss of spectral efficiency. The scheme proposed in [10] is based on CFD and is therefore relatively more complex to implement, while also requiring the existence and prior knowledge of the cycle frequencies of the primary user signals. Further, it does not consider the effect of the fading wireless channel, which makes the analysis unrealistic for wireless scenarios. On the other hand, time domain ED for sequential spectrum sensing proposed in [11], [12], [13] are simple to implement and do not need elaborate prior information about the primary user signal. However, these are based on artificial truncation which leads to suboptimal performance. It has been shown in papers [10], [14], [15], [16] that the average number of blocks needed for sequential detection schemes is significantly lower in comparison to the fixed block size based detectors. Therefore, the suboptimal truncated sequential test is rarely discussed.

Although there are many works in current literature on time domain ED. Recently, power spectral density (PSD) estimation based frequency domain ED such as the Periodogram, Bartlett and Welch detectors have been proposed and studied in [17], [18], [19]. However, they consider only fixed block size detectors. Another drawback of these works is that they consider a Gaussian transmit signal covariance, which is not well suited to model digital modulation based finite constellation communication signals. Further, none of the existing sequential detectors for spectrum sensing consider a multiple antenna array [20], especially in a multi-user scenario. Also, multiple-input multiple-output (MIMO) wireless systems, which are increasingly being employed in current 3G/ 4G wireless cellular networks, have not been addressed in the works mentioned above in the context of sequential detection. Therefore, to address these shortcomings in the existing schemes, we derive a novel frequency domain Bartlett spectral detector (BSD) considering digital modulation based narrowband communication signals. Subsequently, the SPRT or Wald test based sequential BSD (S-BSD) is developed for spectrum sensing in multi-antenna cognitive radio scenarios. Closed form expressions have been derived for the average number of blocks required for the S-BSD in terms of the desired probabilities of false alarm and detection. The scope of the proposed detection framework and allied performance results is also extended to include multiple primary users. Further, the proposed S-BSD detection framework is also extended to MIMO wireless and conventional AWGN channel based cognitive radio scenarios. Simulation results demonstrate a significant performance improvement in comparison to the conventional fixed block size schemes, while also verifying the analytical results derived.

II. System Model

We now consider the single/multiple primary user spectrum sensing scenario with multiple antennas at the cognitive radio receiver. Let $s_k(n)$ denote the primary user signal, given as,

$$s_k\left(n\right) = \sum_{j=1}^{J} a_{k,j} g_j\left(nT_s - kT_b\right) e^{j2\pi f_j nT_s},$$

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where J denotes the number of primary users, $a_{k,j}, f_j$ denote the symbols and the carrier frequency respectively of the j th user and $g_j(t)$ denotes the pulse shaping waveform. The symbol $a_{k,j} = 0$ if user j is inactive. The quantity T_s denotes the sampling time, N_b represents the total number of independent symbols, and $T_b = LT_s$ denotes the symbol duration where L represents the number of samples per block. The symbol vector $\mathbf{s}_k$ for the multiple primary user scenario above is defined as $\mathbf{s}_k = \sum_{j=1}^{J} a_{k,j}\boldsymbol{\beta}_k(f_j)$, with $\boldsymbol{\beta}_k(f_j) = \mathbf{G}_{k,j}\mathbf{u}(f_j)$, where $\mathbf{G}_{k,j} \in \mathbb{R}^{L\times L}$ is the diagonal pulse shaping filter matrix with $g_j(iT_s - kT_b), 0 \le i \le L-1$ denoting its ith diagonal element and the column vector $\mathbf{u}(f_j) \in \mathbb{C}^{L\times 1}$ defined as,

$$\mathbf{u}(f_j) = \left[1, e^{j\frac{2\pi}{L}\frac{f_j}{f_b}}, \cdots, e^{j\frac{2\pi(L-1)}{L}\frac{f_j}{f_b}}\right]^T.$$

Thus each element of $\mathbf{u}(f_j)$ is a primitive Lth root of unity provided f_j is an integer multiple of f_b. Without loss of generality, consider the pulse power normalized to unity as $\sum_{n=0}^{L-1} g^2(nT_s - kT_b) = 1$, which implies $\|\boldsymbol{\beta}_k(f_1)\|^2 = 1$. The hypothesis testing problem for spectrum sensing can now be formulated as follows. Let the hypotheses $\mathcal{H}_0$, $\mathcal{H}_1$ denote the absence and presence of the primary user respectively and $\mathbf{x}_k \in \mathbb{C}^{LM\times 1}$ denote the kth symbol vector received across the M dimensional antenna array. The spectrum sensing problem for the single primary user ($J=1$) scenario is given as,

$$\begin{aligned}&\mathcal{H}_0 : \mathbf{x}_k = \mathbf{w}_k, \\ &\mathcal{H}_1 : \mathbf{x}_k = h_k a_{k,1}\underbrace{\boldsymbol{\beta}_k(f_1)\otimes\mathbf{d}(\theta_1)}_{\mathbf{v}_k(f_1,\theta_1)} + \mathbf{w}_k, \quad 1\le k\le N_b,\end{aligned} \tag{1}$$

where $\mathbf{d}(\theta_1) = [1, e^{j2\pi\delta\cos\theta_1}, \cdots, e^{j2\pi\delta(R-1)\cos\theta_1}]^T$ denotes the M dimensional steering vector for the primary user to be detected at a direction of arrival θ_1, δ denotes the ratio of the inter-antenna spacing to the wavelength [20], $\otimes$ denotes the Kronecker product and vector $\mathbf{v}_k(f_1,\theta_1) \in \mathbb{C}^{LM\times 1}$. It can also be observed that $\|\mathbf{d}(\theta_1)\|^2 = M$. The vector $\mathbf{w}_k = [w_k(0), w_k(1), \ldots, w_k(LM-1)]^T$ is the concatenated white Gaussian noise vector with the variance of the zero mean noise samples $w_k(n)$ given as $\mathrm{E}\{|w_k(n)|^2\} = \sigma^2$ and h_k denotes the standard Rayleigh fading wireless channel coefficient distributed as the zero-mean symmetric complex normal random variable $\mathcal{CN}(0,\rho)$. We now derive the optimal BSD for a single user cognitive radio scenario, based on the observation matrix $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{N_b}]$, which is subsequently generalized for a multiple user scenario.

Theorem 1: The optimal BSD statistic for the binary hypothesis testing based spectrum sensing problem in (1), for a single user scenario i.e. $J=1$ is given as,

$$S(\mathbf{X}) = \sum_{k=1}^{N_b} \underbrace{\left|\sum_{n=0}^{L-1} \tilde{\mathbf{d}}_k^H(n)\,\mathbf{x}_k((0:M-1)+nM)\, e^{\frac{-j2\pi n}{L}\frac{f_1}{f_b}}\right|^2}_{y_k}, \tag{2}$$

where $\tilde{\mathbf{d}}_k(n) = g(nT_s - kT_b)\mathbf{d}(\theta_1)$.

Proof: It can be seen from (1) for $J=1$ that the observation vector $\mathbf{x}_k$ is distributed as $\mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_{LM})$, $\mathcal{CN}(\mathbf{0}, \mathbf{C}+\sigma^2\mathbf{I}_{LM})$ under hypotheses $\mathcal{H}_0$, $\mathcal{H}_1$ respectively, where $\mathbf{C} = P\rho\mathbf{v}_k(f_1,\theta_1\mathbf{v}_k^H(f_1,\theta_1)$ is the rank one received signal covariance matrix and P represents the transmit power of the primary user signal. Therefore, from the optimal likelihood ratio test (LRT) [6], the detector decides $\mathcal{H}_1$ if,

$$\begin{aligned} L(\mathbf{X}) &= \frac{\prod_{k=1}^{N_b} f(\mathbf{x}_k|\mathcal{H}_1)}{\prod_{k=1}^{N_b} f(\mathbf{x}_k|\mathcal{H}_0)} \ge \gamma, \\ &= \frac{\prod_{k=1}^{N_b} \frac{1}{(\pi)^{LM}\left|\mathbf{C}+\sigma^2\mathbf{I}_{LM}\right|} e^{-\mathbf{x}_k{}^H\left(\mathbf{C}+\sigma^2\mathbf{I}_{LM}\right)^{-1}\mathbf{x}_k}}{\prod_{k=1}^{N_b}\left(\frac{1}{\pi\sigma^2}\right)^{LM} e^{-\frac{1}{\sigma^2}\mathbf{x}_k{}^H\mathbf{x}_k}} \ge \gamma. \end{aligned}$$

Taking the logarithm on both sides and using the Woodbury matrix inversion identity [6] for $(\mathbf{C}+\sigma^2\mathbf{I}_{LM})^{-1}$ as,

$$\left(\mathbf{C}+\sigma^2\mathbf{I}_{LM}\right)^{-1} = \frac{\mathbf{I}_{LM}}{\sigma^2} - \frac{\mathbf{v}_k(f_1,\theta_1)\mathbf{v}_k^H(f_1,\theta_1)}{\sigma^4\left(\frac{M}{\sigma^2}+\frac{1}{P\rho}\right)},$$

the LRT can be simplified to yield the BSD test statistic $S(\mathbf{X})$ given in equation (2) where we use the fact $\|\mathbf{v}_k(f_1,\theta_1)\|^2 = M$ in the above simplification. The BSD statistic in (2) can be equivalently expressed as,

$$S(\mathbf{X}) = \sum_{k=1}^{N_b} \mathbf{x}_k^H\mathbf{D}(\theta_1)\mathbf{G}_{k,1}\mathbf{A}(f_1)\mathbf{G}_{k,1}\mathbf{D}^H(\theta_1)\mathbf{x}_k,$$

where the matrices $\mathbf{A}(f_1) \in \mathbb{C}^{L\times L}$, $\mathbf{D}(\theta_1) \in \mathbb{C}^{LM\times L}$ are defined as $\mathbf{A}(f_1) = \mathbf{u}(f_1)\mathbf{u}^H(f_1)$, $\mathbf{D}(\theta_1) = \mathbf{I}_L \otimes \mathbf{d}(\theta_1)$ respectively. This yields the analogous Hermitian quadratic form for the optimal BSD. ■

The BSD derived above can be extended to a multi-user scenario with $J > 1$ in a straightforward fashion as,

$$S'_j(\mathbf{X}) = \sum_{k=1}^{N_b} \underbrace{\left|\mathbf{b}_k^H(f_j,\theta_j)\mathbf{x}_k\right|^2}_{y_{k,j}}, \tag{3}$$

where the column vector $\mathbf{b}_k(f_j,\theta_j) \in \mathbb{C}^{LM\times 1}$ for the multi-user scenario is the projection of the array manifold vector $\mathbf{v}_k(f_j,\theta_j)$ along the null space of the beamformer matrix $\mathbf{B}_k \in \mathbb{C}^{LM\times J-1} = [\mathbf{v}_k(f_1,\theta_1), \ldots, \mathbf{v}_k(f_{j-1},\theta_{j-1}), \mathbf{v}_k(f_{j+1},\theta_{j+1}), \ldots, \mathbf{v}_k(f_J,\theta_J)]$ i.e. $\mathbf{b}_k = \mathbf{V}_k^{\perp}(\mathbf{V}_k^{\perp})^H\mathbf{v}_k(f_j,\theta_j)$, where $\mathbf{V}_k^{\perp}$ is a basis for the null space of $\mathbf{B}_k$. Next we present the optimal S-BSD and the associated expressions for the average number of blocks for a multi-antenna cognitive radio scenario in Rayleigh fading, MIMO and AWGN channels.

III. Optimal S-BSD for Rayleigh Flat Fading Cognitive Radio Scenarios

Spectrum sensing is a challenging task in a fading channel scenario due to the random fluctuations of the wireless channel gains. This section considers the effect of a Rayleigh flat fading channel on the proposed sequential spectrum sensing scheme. The result below derives the optimal S-BSD for a spectrum sensing scenario defined in equation (1) employing the double threshold SPRT based on the aggregate Wald test statistic computed from the K test statistics $y_k, 1 \le k \le K$.

Theorem 2: The optimal SPRT decision rule based S-BSD for spectrum sensing in a multi-antenna Rayleigh flat fading channel scenario is given as,

$$\begin{aligned} \Lambda_K(\mathbf{y}) &\le \gamma_0 : decide\ \mathcal{H}_0 \\ \gamma_0 < \Lambda_K(\mathbf{y}) &< \gamma_1 : Wait\ for\ y_{K+1} \\ \Lambda_K(\mathbf{y}) &\ge \gamma_1 : decide\ \mathcal{H}_1, \end{aligned} \tag{4}$$

where the cumulative test statistic $\Lambda_K(\mathbf{y}) = \sum_{k=1}^{K} y_k$, and the threshold γ_j is defined as,

$$\gamma_j = \frac{\sigma^2\left(\sigma^2+MP\rho\right)}{P\rho}\left(\ln\eta_j - K\ln\frac{\sigma^2}{\sigma^2+MP\rho}\right), j\in\{0,1\}.$$

Proof: We begin by considering the case $J=1$. It follows from Theorem 1 that y_k is distributed as a central χ_2^2 random variable with 2 degrees of freedom and each component of variance $\frac{1}{2}M\sigma^2$ and $\frac{1}{2}(M\sigma^2+M^2P\rho)$ corresponding to the hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ respectively. The log-likelihood ratio (LLR) for the kth test statistic y_k denoted by $L(y_k) = \ln\frac{f(y_k|\mathcal{H}_1)}{f(y_k|\mathcal{H}_0)}$ is,

$$L\left(y_k\right) = y_k\left[\frac{1}{M\sigma^2} - \frac{1}{M\sigma^2+M^2P\rho}\right] + \ln\frac{M\sigma^2}{M\sigma^2+M^2P\rho}.$$

Therefore, employing the SPRT framework [8], the aggregate S-BSD test statistic for the K observed blocks is given as $\tilde{\Lambda}_K(\mathbf{y}) = \sum_{k=1}^{K} L(y_k)$, which can be simplified as,

$$\tilde{\Lambda}_K\left(\mathbf{y}\right) = K\ln\frac{\sigma^2}{\sigma^2+MP\rho} + \frac{P\rho}{\sigma^2\left(\sigma^2+MP\rho\right)}\sum_{k=1}^{K} y_k,$$

where $\mathbf{y}\in\mathbb{R}^{K\times 1}$ is defined as $\mathbf{y} = [y_1, y_1, \ldots, y_K]^T$. From [8], the SPRT decision rule can now be formulated as,

$$\begin{aligned} \tilde{\Lambda}_K\left(\mathbf{y}\right) &\le \ln\eta_0 : \textit{decide } \mathcal{H}_0 \\ \ln\eta_0 < \tilde{\Lambda}_K\left(\mathbf{y}\right) &< \ln\eta_1 : \textit{Wait for } y_{K+1} \\ \tilde{\Lambda}_K\left(\mathbf{y}\right) &\ge \ln\eta_1 : \textit{decide } \mathcal{H}_1, \end{aligned} \tag{5}$$

where the quantities η_0 and η_1 above are defined in terms of the target false alarm and mis-detection probabilities α, δ respectively as $\eta_0 = \frac{\delta}{1-\alpha}, \eta_1 = \frac{1-\delta}{\alpha}$. Simplifying the detector in (5) by appropriately rearranging the constant factors yields the sequential detector in (4). ■

Result below analytically characterizes the average number of blocks required by the S-BSD for given values of the probability of detection (P_D) and false alarm (P_{FA}).

Lemma 1: The average number of blocks $\bar{K}$ required for the S-BSD detector in Theorem 1 with prior probabilities $\Pr(\mathcal{H}_0)$ and $\Pr(\mathcal{H}_1)$ corresponding to hypotheses $\mathcal{H}_0, \mathcal{H}_1$ respectively, is given as,

$$\begin{aligned} \bar{K} = E\left\{K\right\} &= \Pr\left(\mathcal{H}_0\right)\frac{\alpha\ln\eta_1 + (1-\alpha)\ln\eta_0}{\frac{MP\rho}{\sigma^2+MP\rho} + \ln\frac{\sigma^2}{\sigma^2+MP\rho}} \\ &+ \Pr\left(\mathcal{H}_1\right)\frac{(1-\delta)\ln\eta_1 + \delta\ln\eta_0}{\frac{MP\rho}{\sigma^2} + \ln\frac{\sigma^2}{\sigma^2+MP\rho}}. \end{aligned} \tag{6}$$

Proof: The quantity $\mathrm{E}\{L(y_k)|\mathcal{H}_0\}$ i.e. the expected value of the likelihood ratio of the observation y_k corresponding to the hypothesis $\mathcal{H}_0$, can be evaluated as,

$$\begin{aligned} \mathrm{E}\left\{L\left(y_k\right)|\mathcal{H}_0\right\} &= M\sigma^2\left[\frac{P\rho}{\sigma^2\left(\sigma^2+MP\rho\right)}\right] + \ln\frac{\sigma^2}{\sigma^2+MP\rho}, \\ &= \frac{MP\rho}{\sigma^2+MP\rho} + \ln\frac{\sigma^2}{\sigma^2+MP\rho}. \end{aligned} \tag{7}$$

Similarly, the expression for $\mathrm{E}\{L(y_k)|\mathcal{H}_1\}$ can be simplified as,

$$\mathrm{E}\left\{L\left(y_k\right)|\mathcal{H}_1\right\} = \frac{MP\rho}{\sigma^2} + \ln\frac{\sigma^2}{\sigma^2+MP\rho}. \tag{8}$$

Further, from [8], the quantities $\mathrm{E}\{K|\mathcal{H}_0\}$, $\mathrm{E}\{K|\mathcal{H}_1\}$ corresponding to the hypotheses $\mathcal{H}_0, \mathcal{H}_1$ respectively, are given as,

$$\begin{aligned} \mathrm{E}\left\{K|\mathcal{H}_0\right\} &= \frac{\alpha\ln\eta_1 + (1-\alpha)\ln\eta_0}{\mathrm{E}\left\{L\left(y_k\right)|\mathcal{H}_0\right\}}, \\ \mathrm{E}\left\{K|\mathcal{H}_1\right\} &= \frac{(1-\delta)\ln\eta_1 + \delta\ln\eta_0}{\mathrm{E}\left\{L\left(y_k\right)|\mathcal{H}_1\right\}}. \end{aligned} \tag{9}$$

Substituting the expressions derived in (7), (8) in the equations above yields the average number of blocks for each hypothesis. Therefore, the average number of blocks $\bar{K} = \mathrm{E}\{K\}$ for the overall sequential detection process is given as,

$$\bar{K} = \Pr\left(\mathcal{H}_0\right)\mathrm{E}\left\{K|\mathcal{H}_0\right\} + \Pr\left(\mathcal{H}_1\right)\mathrm{E}\left\{K|\mathcal{H}_1\right\}, \tag{10}$$

which can be simplified as (6) by employing the expressions for $\mathrm{E}\{K|\mathcal{H}_0\}$, $\mathrm{E}\{K|\mathcal{H}_1\}$ derived in (9) above. ■

The equivalent result for a multi-user spectrum sensing scenario can now be readily derived by considering the test statistic $y_{k,j}$ defined in (3). Following an approach similar to the one above, it can be shown that the equivalent sequential detector and the average number of blocks for a multi-user scenario can be derived by replacing the power P with $\tilde{P} = P|\mathbf{b}_k^H(f_j,\theta_j)\mathbf{v}_k(f_j,\theta_j)|^2$ in equations (4), (6) respectively.

IV. Optimal S-BSD for MIMO Cognitive Radio Scenarios

MIMO technology can be effectively used to increase the spectral efficiency of modern cognitive radio systems. In this context, we now extend the proposed optimal S-BSD to a MIMO spectrum sensing scenario. Consider a scenario with multiple transmit (T) and receive (M) antennas at the primary user transmitter and secondary user receiver respectively. The binary hypothesis testing problem for this MIMO single primary user cognitive radio scenario can be formulated as,

$$\begin{aligned} \mathcal{H}_0: &\quad \mathbf{x}_{k,r} = \mathbf{w}_{k,r}, \\ \mathcal{H}_1: &\quad \mathbf{x}_{k,r} = \sum_{t=1}^{T} h_k\left(r,t\right) a_{k,t}\boldsymbol{\beta}_k\left(f_1\right) + \mathbf{w}_{k,r}, \end{aligned} \tag{11}$$

where $\boldsymbol{\beta}_k(f_1)$ is as defined in Section II, $a_{k,t}$ is the k th symbol transmitted from the t th antenna, $h_k(r,t)$ is the Rayleigh fading channel coefficient of average power ρ across the (t,r)th transmit, receive antenna pair and $\mathrm{E}\{\mathbf{w}_{k,r}\mathbf{w}_{k,r}^H\} = \sigma^2\mathbf{I}_L$. The result below formulates the sequential detector for the MIMO scenario above.

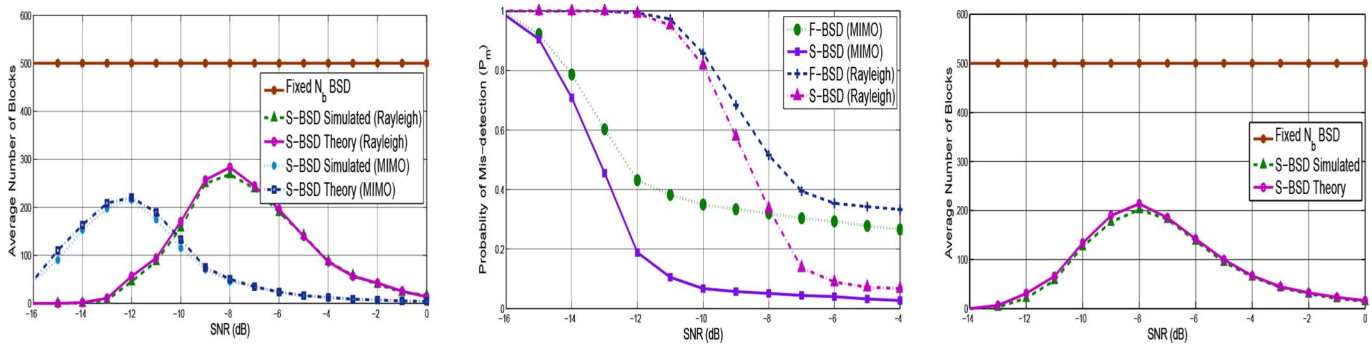
Theorem 3: The SPRT based S-BSD for the MIMO cognitive radio scenario in (11) above is similar to equation (4) with the aggregate test statistic $\tilde{\Lambda}_K(\mathbf{y})$ modified as,

$$\tilde{\Lambda}_K\left(\mathbf{y}\right) = \frac{TP\rho}{\sigma^2\left(\sigma^2+TP\rho\right)}\sum_{k=1}^{K} y_k + K\left(M-1\right)\ln\frac{\sigma^2}{\sigma^2+TP\rho}. \tag{12}$$

Proof: It can be seen that the test statistic $y_k = \sum_{r=1}^{M} |\boldsymbol{\beta}_k(f_1)\mathbf{x}_{k,r}|^2$ is distributed as a central χ_{2M}^2 random variable of $2M$ degrees of freedom with each component of variance $\frac{1}{2}\sigma^2$, $\frac{1}{2}(\sigma^2+TP\rho)$ corresponding to hypotheses $\mathcal{H}_0$, $\mathcal{H}_1$ respectively. The above simplification follows from the fact that $\|\boldsymbol{\beta}_k(f_1)\|^2 = 1$. Therefore, following a procedure similar to Theorem 2, the LLR $L(y_k)$ can be simplified as,

$$L\left(y_k\right) = y_k\left[\frac{1}{\sigma^2} - \frac{1}{\sigma^2+TP\rho}\right] + (M-1)\ln\frac{\sigma^2}{\sigma^2+TP\rho},$$

which simplifies to the combined statistic $\tilde{\Lambda}_K(\mathbf{y})$ given in equation (12). This is now employed in the SPRT framework in (5) to yield the corresponding sequential detector for the MIMO cognitive radio scenario. ■



(a) (b) (c)

Fig. 1. (a) Average number of blocks for MIMO and Rayleigh fading channel scenarios, (b) Probability of mis-detection for MIMO and Rayleigh fading channel scenarios, (c) Average number of blocks for an AWGN channel scenario.

On lines similar to that of Lemma 1, the simplified expression for the average number of blocks $\bar{K}$ for the MIMO scenario is given as,

$$\bar{K} = \mathrm{E}\{K\} = \Pr(\mathcal{H}_0)\,\frac{\alpha\ln\eta_1 + (1-\alpha)\ln\eta_0}{\frac{MTP\rho}{\sigma^2+TP\rho} + (M-1)\ln\frac{\sigma^2}{\sigma^2+TP\rho}} + \Pr(\mathcal{H}_1)\,\frac{(1-\delta)\ln\eta_1 + \delta\ln\eta_0}{\frac{MTP\rho}{\sigma^2} + (M-1)\ln\frac{\sigma^2}{\sigma^2+TP\rho}}. \tag{13}$$

Similar to the previous section, the corresponding results for the multi-user MIMO scenario can be derived by replacing the power P with $\tilde{P} = P|\mathbf{b}_k^H(f_j,\theta_j)\mathbf{v}_k(f_j,\theta_j)|^2$.

V. Optimal S-BSD for AWGN Cognitive Radio Scenarios

The system model for a conventional AWGN channel scenario can be derived from (1) by setting the channel coefficient $h_k = 1$.

Lemma 2: The SPRT based sequential Bartlett spectral detector (S-BSD) for the AWGN channel cognitive radio scenario is given by (4), with $\tilde{\Lambda}_K(\mathbf{y})$ modified as,

$$\tilde{\Lambda}_K(\mathbf{y}) = \sum_{k=1}^{K}\left(\frac{-\lambda_k}{M\sigma^2} + \ln I_0\left(\sqrt{\lambda_k y_k}\right)\right), \tag{14}$$

where $I_0(\cdot)$ denotes the 0th order modified Bessel function of the first kind [21].

Proof: From (1) with $h_k = 1$, it can be seen that the distribution of $\mathbf{x}_k$ corresponding to $\mathcal{H}_0$ and $\mathcal{H}_1$ is given as $\mathcal{CN}(\mathbf{0},\sigma^2\mathbf{I}_{LM})$, $\mathcal{CN}(a_k\mathbf{v}_k(f_1,\theta_1),\sigma^2\mathbf{I}_{LM})$ respectively. It follows that y_k is distributed as central, non-central χ_2^2 random variables with 2 degrees of freedom and each component of variance $\frac{1}{2}M\sigma^2$ under the hypotheses $\mathcal{H}_0$, $\mathcal{H}_1$ respectively where the non-centrality parameter is $\lambda_k = M^2|a_k|^2$. Hence, the expression for the aggregate test statistic $\tilde{\Lambda}_K(\mathbf{y})$ is given as,

$$\tilde{\Lambda}_K(\mathbf{y}) = \sum_{k=1}^{K} L(y_k) = \sum_{k=1}^{K}\ln\frac{\frac{1}{M\sigma^2}e^{\frac{-(y_k+\lambda_k)}{M\sigma^2}} I_0\left(\sqrt{\lambda_k y_k}\right)}{\frac{1}{M\sigma^2}e^{\frac{-y_k}{M\sigma^2}}},$$

which reduces to the expression in (14). This can now be employed in (5) to derive the corresponding sequential detector for the AWGN cognitive radio scenario. ■

The average number of blocks required for sequential detection for the S-BSD in an AWGN scenario can be derived as follows,

$$\bar{K} = \mathrm{E}\{K\} = \Pr(\mathcal{H}_0)\,\frac{\alpha\ln\eta_1 + (1-\alpha)\ln\eta_0}{\mathrm{E}\left\{\ln I_0\left(\sqrt{\lambda_k y_k}\right)|\mathcal{H}_0\right\} - \frac{\lambda_k}{M\sigma^2}} + \Pr(\mathcal{H}_1)\,\frac{(1-\delta)\ln\eta_1 + \delta\ln\eta_0}{\mathrm{E}\left\{\ln I_0\left(\sqrt{\lambda_k y_k}\right)|\mathcal{H}_1\right\} - \frac{\lambda_k}{M\sigma^2}}. \tag{15}$$

Further, the S-BSD and $\bar{K}$ for the multiple primary user AWGN scenario are obtained by replacing λ_k with $\tilde{\lambda}_k = \lambda_k|\mathbf{b}_k^H(f_j,\theta_j)\mathbf{v}_k(f_j,\theta_j)|^2$ in (14), (15) respectively.

VI. Simulation Results

This section presents simulation results to illustrate the performance of the proposed optimal BSD based sequential spectrum sensing scheme for the fading, MIMO and AWGN channel cognitive radio scenarios considered above. We consider a BPSK modulated primary user signal with $M = 3$ primary users at carrier frequencies $f = 180, 210, 240$ MHz along with $\alpha = 0.01$ and $\delta = 0.05$. The number of blocks for the non-sequential fixed block length F-BSD is set as $N_b = 500$, the number of antennas as $M = 4$ and $L = 128$. The target false alarm and mis-detection probabilities α, δ are now employed to fix the thresholds for the sequential detection scheme as described in Theorem 2. Fig. 1(a) shows the average number of blocks required for the SPRT based S-BSD and the non-sequential F-BSD detectors versus SNR for MIMO, Rayleigh fading wireless channel scenarios. It can be readily seen that the S-BSD requires a significantly lower number of blocks to achieve the given detection performance in comparison to the F-BSD. Further, the average number of blocks required for the S-BSD can be seen to coincide with the analytical values obtained from the expressions derived in (6), (13). Fig. 1(b). demonstrates the P_m versus SNR performance for the S-BSD and F-BSD detectors when the average number of blocks calculated from the proposed S-BSD at each SNR is used to compute the P_D and P_{FA} performance for the F-BSD. The figure clearly demonstrates that the sequential BSD has a lower probability of mis-detection when the average block sizes of the S-BSD and F-BSD are identical. An analogous result for the AWGN channel based cognitive radio scenario described in Section V is shown in Fig 1(c), and follows a similar trend.

VII. Conclusion

This letter proposes a novel Bartlett spectral detection scheme based on the SPRT for spectrum sensing in multi-antenna array Rayleigh, MIMO fading and AWGN channels in cognitive radio systems. The optimal sequential detectors and closed form expressions for the average number of blocks required by the various sequential detectors have also been derived for the above scenarios. Simulation results illustrate that the proposed S-BSD scheme achieves a significant improvement in the detection performance in comparison to the F-BSD. Further, the analytical results for the average number of blocks required by the various sequential detectors are seen to closely match the simulated values.

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