Leader-Follower Consensus in Mobile Sensor Networks

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***Abstract*—In this letter, we study the consensus-based leader-follower algorithm in mobile sensor networks, where the goal for the entire network is to converge to the state of a leader. We capture the mobility in the leader-follower algorithm by abstracting it as a Linear Time-Varying (LTV) system with random system matrices. In particular, a mobile node, moving randomly in a bounded region, updates its state when it finds neighbors; and does not update when it is not in the communication range of any other node. In this context, we develop certain regularity conditions on the system and input matrices such that each follower converges to the leader state. To analyze the corresponding LTV system, we partition the entire chain of system matrices into non-overlapping slices, and relate the convergence of the sensor network to the lengths of these slices. In contrast to the existing results, we show that a bounded length on the slices, capturing the dissemination of information from the leader to the followers, is not required; as long as the slice-lengths are finite and do not grow faster than a certain exponential rate.**

***Index Terms*—Data fusion, distributed algorithms, LTV systems, sensor networks.**

I. Introduction

A MOBILE sensor network is composed of a collection of mobile nodes with possibly limited sensing, communications, and computation capabilities. Since sensor mobility is becoming increasingly important in many practical applications, [1]–[4], e.g., monitoring a hazardous environment where a static network can not be deployed, mobile sensor networks have attracted a significant research attention in recent years.

In sensor networks, it is often desirable that the entire network reaches an agreement regarding a quantity of interest. This problem is referred to as *consensus*, with applications in, e.g., hypothesis testing, [5]–[7], estimation, [8], [9], and diffusion, [10]; consensus-related literature includes [11]–[16]. In some applications, sensors should be guided to a target state by introducing leaders, [17], [18]. The leader plays the role of an input whose influence is propagated throughout the network via a distributed algorithm, specifying the information exchange between a sensor and its neighbors, [19]. Related work on the conditions under which the sensor states converge to the leader state can be found in [20]–[24]; see [25] for running-consensus that incorporates time-varying measurements in the (non-mobile) sensor updates, and also [26], [27], for second-order consensus algorithms. In particular, Ref. [20] shows that the network converges to the state of the leader if there exists an infinite sequence of contiguous, non-overlapping and *bounded* time intervals with the property that the union of the graphs across each interval is strongly-connected.

Subsequently, Refs. [21], [22] extend the results in [20] to directed graphs, and prove that the coordination is possible if the union of the interaction graphs has a spanning tree frequently enough. Ref. [23] presents a graph-theoretic approach to study consensus in dynamically changing environments, where each graph can be represented by a stochastic matrix. Ref. [24] studies the convergence of the product of asymmetric stochastic matrices, and relates the convergence rate to the second largest eigenvalue of the product.

In this letter, we consider a leader-follower problem where the sensors move randomly in a bounded region of interest and have a limited communication range. We assume gossip-based communication, which is widely used due to its simplicity and robustness, [28]. In particular, we assume that only one sensor at each iteration exchanges information with nearby sensors and updates its state. Since the motion is random, a sensor may not find neighbors at all times and an arbitrary sensor may not communicate with a leader at any given time. This leads to different update scenarios, which we discuss thoroughly in Section II. We abstract this updating procedure by a Linear Time-Varying (LTV) system and develop the conditions under which this LTV system converges to the leader state regardless of the initial conditions. In contrast to a majority of the existing work, which deals with non-negative, stochastic matrices, the system matrices in a dynamic leader-follower algorithm are both *stochastic* or *sub-stochastic*, due to which the results focusing on stochastic matrices alone are not applicable.

In order to develop the results for sub-stochastic matrices, we introduce the notion of non-overlapping *slices*–the smallest product of consecutive system matrices that covers the system matrices *and* has an infinity norm of less than one. One such complete slice implies information propagation from the leader to every other node. Clearly, if this information dissemination is completed in a bounded time, i.e., the slices have a uniform bound on their lengths, then the information from the leader reaches the entire network infinitely often and the asymptotic behavior can be developed rather straightforwardly. An important contribution of this paper is to show that the (dynamic) leader-follower algorithm converges even when the slice lengths are unbounded; what is required is that the slices do not grow larger at a rate faster than a certain exponential growth. In other words, the information from the leader reaches the rest of the network in an unbounded number of steps, the rate of which is explicitly characterized.

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We now describe the rest of the letter. Section II formulates the problem, and describes the local updates and the (network) dynamics of the leader-follower algorithm. In Section III, we study the convergence of the sensor network to the leader state, and provide the sufficient conditions to achieve convergence. Simulation results are presented in Section IV, and finally Section V concludes the paper.

II. Problem Formulation

Consider a network with n followers (sensors) in the set Ω, and one leader. We assume that all of the nodes (leader and followers) move arbitrarily and the followers exchange information if they find a nearby node. For the ith follower, let $\mathcal{N}_i(k)$ be the set of neighbors (not including follower i) at time k. We also define $\mathcal{D}_i(k) = \mathcal{N}_i(k) \cup \{i\}$.

Leader-follower dynamics: The leader's state is fixed, whereas the state of a follower is influenced by its neighbors. Note that due to mobility, it is not guaranteed for a follower to find a leader among its neighbors at any time k. In fact, at a given time, it is possible that a follower does not have any neighbor. Let $x_i(k) \in \mathbb{R}$ be the state of the ith follower at time k and let $u \in \mathbb{R}$ be the leader state. The updating follower, $i \in \Omega$, implements the following:

(i) Follower i keeps its state when it has no neighbor:

$$x_i(k+1) = x_i(k). \tag{1}$$

(ii) With no leader among neighbors, the state-update is

$$x_i(k+1) = \sum_{j \in \mathcal{D}_i(k)} (P_k)_{i,j} x_j(k). \tag{2}$$

(iii) With the leader among neighbors, the state-update is

$$x_i(k+1) = \sum_{j \in \mathcal{D}_i(k) \cap \Omega} (P_k)_{i,j} x_j(k) + (B_k)_i u, \tag{3}$$

where $u \in \mathbb{R}$ is the leader state, $(P_k)_{i,j}$'s and $(B_k)_i$'s, are the coefficients assigned by the updating follower i to the followers and the leader, respectively. The above update can be abstracted by the following LTV system:

$$\mathbf{x}(k+1) = P_k \mathbf{x}(k) + B_k u, \qquad k \geq 0, \tag{4}$$

where $\mathbf{x}(k) \in \mathbb{R}^n$ is the state of the followers at time k; $P_k = \{(P_k)_{i,j}\}$ and $B_k = \{(B_k)_i\}$ are the time-varying system and input matrices, respectively.

Assumptions: We make the following assumptions:

A0: The state-update is a *linear-convex* combination, i.e.,

$$\sum_j (P_k)_{i,j} + (B_k)_i = 1, \quad \forall k, i \in \Omega. \tag{5}$$

A1: When there is no leader among the neighbors, then

$$0 < \beta_1 \leq (P_k)_{i,i} < 1, \quad \beta_1 \in \mathbb{R}, \forall i \in \Omega. \tag{6}$$

A2: When a leader is involved in an update, it always contributes a certain amount of information, i.e.,

$$0 < \beta_2 \leq (B_k)_i < 1, \quad \beta_2 \in \mathbb{R}, \forall i \in \Omega. \tag{7}$$

Remarks: The above assumptions are realistic and rather common in the related literature. While stochasticity is standard in sensor fusion and relevant applications, see e.g., [17], [18], [29], Eq. (6) implies that when a state update occurs without a leader, a non-zero self-weight is always assigned to the (updating) follower's current state. This assumption does not allow a follower to completely forget its past information. Note that Assumption **A1** is required to maintain sub-stochasticity of the process and is necessary for each follower that does not directly communicate with the leader. This is because this follower can lose information received (indirectly) from the leader by updating with neighbors that do not have any such information. Next, Eq. (7) restricts the amount of (unreliable) information received from other neighboring followers by guaranteeing that a certain information is always contributed by the leader. We note that it is possible for multiple followers to update at the same time and the assumption that only one follower updates is without loss of generality. Note that random motion in a bounded region is equivalent to saying that each follower communicates intermittently with other nodes with a non-zero probability. If a subset of followers is isolated, and never communicates with the rest of the network, this subset does not follow the convergence.

Under the above assumptions the LTV system matrices, $\{P_k\}$'s, are always non-negative, and may be *identity*-when no update occurs; *stochastic*-when there is no leader among the neighbors; and, *sub-stochastic*-when the neighbors include the leader. In the next section, we provide a framework to study the convergence of the leader-follower algorithm represented in Eq. (4), under Assumptions **A0-A2**.

III. Convergence

We now provide a related result on the convergence of an infinite product of stochastic and sub-stochastic matrices, [30], that will be used to study Eq. (4).

A. Infinite Product of (Sub-) Stochastic Matrices

In what follows, we discuss $\lim_{k\to\infty} P_k P_{k-1} \ldots P_0$, in the context of Eq. (4). A common approach to study an infinite product of such matrices is via the *Joint Spectral Radius (JSR)*, [31], which, in general, is NP-hard, even in special cases, [32]. We thus introduce an alternative approach, [30], which relates the norm properties of subsets of system matrices to the convergence of the infinite product of system matrices. In particular, we partition the entire sequence of system matrices into non-overlapping *slices*. We define a slice as the smallest product of system matrices such that: (i) each slice is initiated by a sub-stochastic matrix; (ii) each slice has a subunit infinity norm; and, (iii) the slices cover all of the system matrices. We denote the jth slice by M_j, with length $|M_j|$. Using slices, we introduce a new time index, $t \leq k$, and study

$$\lim_{t\to\infty} M_t M_{t-1} \ldots M_0, \tag{8}$$

instead of $\lim_{k\to\infty} P_k P_{k-1} \ldots P_0$. Slice construction is illustrated in Fig. 1, where the jth slice length may be defined as $|M_j| = m_j - m_{j-1}, m_{-1} = 0$.

For a given slice, M_j, we have the *largest* upper bound on the infinity norm of the slice as

$$\|M_j\|_\infty \leq 1 - {\beta_1}^{|M_j|-1} \beta_2, \tag{9}$$

which is strictly less than one, for any $|M_j| < \infty$, [30], given that β_1 and β_2 in the above follow **A0-A2**. Therefore, the convergence of the product of slices is characterized by the length of the slices, and by β_1 and β_2. For instance, it can be verified

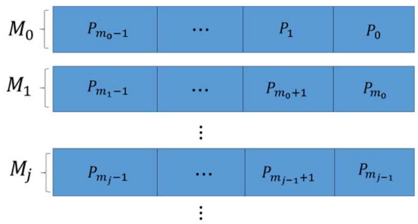

Fig. 1. Slice representation.

that $\|M_j\|_\infty \leq 1$, when either β_1 or β_2 is 0. On the other hand, when β_1, β_2, are non-zero, we get $\|M_j\|_\infty < 1$, for any finite slice length, $|M_j|$. We note that if each slice length has a uniform upper bound, i.e., $|M_j| < N < \infty, \forall j$, as is assumed in [20], [21], [22], then $\|M_j\|_\infty < N', \forall j$, for some $N' < \infty$, and the asymptotic stability of Eq. (4) is rather straightforward, e.g., by using the sub-multiplicative norm property. However, since we have an explicit expression for the infinity-norm bound in Eq. (9), a more general result can be developed that does not require such a uniform bound. We provide this result in the following theorem, and refer the reader to [30] for a detailed proof:

Theorem 1: With Assumptions **A0-A2**, Eq. (8) converges to zero if there exists a subset of slices, denoted by J_1, such that for every $i \in \mathbb{N}$, and $|M_j| < \infty, j \notin J_1$,

$$\exists M_j \in J_1 : |M_j| \leq \frac{1}{\ln(\beta_1)} \ln\left(\frac{1 - e^{(-\gamma_2 i^{-\gamma_1})}}{\beta_2}\right) + 1, \quad (10)$$

for $\gamma_1 \in [0, 1]$, $0 < \gamma_2$, chosen such that $\beta_2 > 1 - e^{-\gamma_2 i^{-\gamma_1}}$.

Unbounded connectivity: The complete proof of this theorem is beyond the scope of this short letter. In this letter, we derive an extension of Theorem 1 to the dynamic leader-follower algorithm. The theorem's statement can be interpreted with the help of the unbounded connectivity notion explained as follows. The infinite product of slices converges to zero, if for every $i \in \mathbb{N}$, there exist a slice in J_1, with length following Eq. (10), in no particular order. *This implies that the slice lengths can be unbounded as long as a well-behaved subset of slices exist whose lengths do not grow faster than the right hand side of Eq. (10).* Note that a longer slice length corresponds to the slower flow of information in the network.

B. Leader-Follower Convergence

We now study the *dynamic* leader-follower algorithm.

Theorem 2: Consider n followers and one leader moving in a finite and bounded region with follower state updates given in Eqs. (1)–(3). Then, for any random (or deterministic) motion that satisfies Eq. (10) on the corresponding slices, the followers (asymptotically) converge to the leader state.

Proof: The random motion of the agents in a finite, bounded region results in the following LTV system:

$$\mathbf{x}(k+1) = P_k \mathbf{x}(k) + B_k u, \quad k \geq 0, \quad (11)$$

where u is the state of the leader, and P_k is random and may be either identity, stochastic, or sub-stochastic. Hence,

$$\prod_{k=1}^{\infty} P_k = 0, \quad (12)$$

by Theorem 1. What is left to show is that when Eq. (12) holds and the leader state is fixed, all agents converge to the leader state. Using the slice notation, we may use a new time index, t, and rewrite Eq. (11) as

$$\mathbf{y}(t+1) = M_t \mathbf{y}(t) + N_t u, \quad t \geq 0, \quad (13)$$

such that $\mathbf{y}(0) = \mathbf{x}(0)$ and

$$\mathbf{y}(t) = \mathbf{x}(k), \quad k = \sum_{i=1}^{t} |M_i|, t \geq 1, \quad (14)$$

$$M_t = P_{\left(\sum_{i=0}^{t} |M_i|\right)-1} \dots P_{\left(\sum_{i=0}^{t-1} |M_i|\right)}, t > 0, \quad (15)$$

$$N_t = \sum_{m=0}^{|M_t|-1} \left(\prod_{j=1}^{m} P_{|M_t|-j}\right) B_{|M_t|-1-m}, \quad (16)$$

with $M_0 = P_{|M_0|-1} \dots P_0$. For simplicity, let us represent the tth slice in Eq. (15) as:

$$M_t \triangleq P_T P_{T-1} \dots P_0, \quad |M_t| = T + 1,$$

such that $(\sum_{i=0}^{t} |M_i| - 1) - (\sum_{i=0}^{t-1} |M_i|) = T + 1$. In addition, we can rewrite Eqs. (11) and (13) as

$$\begin{aligned}
\mathbf{x}(k+1) &= (P_k \dots P_0)\mathbf{x}(0) \\
&\quad + \sum_{m=0}^{k} \left(\prod_{j=1}^{m} P_{k-j+1}\right) B_{k-m} u, \\
\mathbf{y}(t+1) &= (M_t \dots M_0)\mathbf{y}(0) \\
&\quad + \sum_{m=0}^{t} \left(\prod_{j=1}^{m} M_{t-j+1}\right) N_{t-m} u,
\end{aligned}$$

respectively. In Eq. (13), $\mathbf{y}_{t+1}$ asymptotically converges to a limit, say $\mathbf{y}^*$, because u is a constant, and the spectral radius, $\rho(M_t)$, of the tth slice is strictly less than one, under the conditions of Theorem 1, i.e.,

$$\rho(M_t) \leq \|M_t\|_\infty < 1, \quad \forall t. \quad (17)$$

Thus, $\lim_{t\to\infty} \mathbf{y}_{t+1} = \mathbf{y}^*$. Note that for any given $t \geq 1$ we can find the corresponding k from Eq. (14), and we have

$$\mathbf{x}^* = \mathbf{y}^* = M_t \mathbf{y}^* + N_t u \rightarrow (I_n - M_t)\mathbf{y}^* = N_t u, \quad (18)$$

in which I_n denotes the $n \times n$ identity matrix. Therefore,

$$\mathbf{x}^* = (I_n - M_t)^{-1} N_t u. \quad (19)$$

Note that $(I - M_t)$ is invertible due to Eq. (17). In order to show that the limiting states of the follower are indeed the leader state, we need to show

$$(I_n - M_t)^{-1} N_t = \mathbf{1}_n \Rightarrow M_t \mathbf{1}_n + N_t = \mathbf{1}_n. \quad (20)$$

Note that since there is only one leader in the network, N_t is a $n \times 1$ column vector. By substituting M_t and N_t from Eqs. (15) and (16) in Eq. (20), we need to show that

$$(P_T \dots P_0)\mathbf{1}_n + \sum_{m=0}^{T} \left(\prod_{j=1}^{m} P_{T+1-j}\right) B_{T-m} = \mathbf{1}_n. \quad (21)$$

By expanding the left hand side of the above, we have

$$
\begin{aligned}
(P_T P_{T-1} \dots P_0)\mathbf{1}_n \quad &+ (P_T P_{T-1} \dots P_1)B_0 \\
&+ (P_T P_{T-1} \dots P_2)B_1 \\
&\;\vdots \\
&+ (P_T P_{T-1})B_{T-2} \\
&+ (P_T)B_{T-1} \\
&+ (B_T).
\end{aligned} \tag{22}
$$

The first line of the above expression can be simplified as

$$(P_T P_{T-1} \dots P_1)(P_0 \mathbf{1}_n + B_0), \tag{23}$$

in which $B_0 \neq 0$ is a $n \times 1$ vector corresponding to the first sub-stochastic update at the beginning of the slice, M_t. Also, B_0 has only one non-zero, say α_i, at the ith position if follower i updates with the leader at the beginning of the slice, M_t. From Eq. (5), which is natural in leader-follower settings, [33], it can be verified that

$$P_0 \mathbf{1}_n + B_0 = \mathbf{1}_n. \tag{24}$$

Therefore Eq. (22) reduces to the following

$$(P_T P_{T-1} \dots P_1)\mathbf{1}_n + (P_T \dots P_2)B_1 + \dots + (B_T). \tag{25}$$

After the first update, B_j's, $(1 < j \leq T-1)$, are non-zeros in case of sub-stochastic updates, and zeros otherwise. The procedure continues in a similar way for any a sub-stochastic update, i.e. update with the leader. Let us consider now the other case, where the update is stochastic. Suppose B_c is the next sub-stochastic update, and we have $B_j = 0, (1 \leq j < c)$. Eq. (25) then reduces to

$$(P_T \dots P_{c+1})(P_c P_{c-1} \dots P_1 \mathbf{1}_n + B_c) + \dots + (B_T). \tag{26}$$

Since between P_1 and P_c there is no sub-stochastic update, $P_{c-1} \dots P_1 \mathbf{1}_n = \mathbf{1}_n$, and we can rewrite Eq. (26) as

$$(P_T \dots P_{c+1})(P_c \mathbf{1}_n + B_c) + \dots + (B_T), \tag{27}$$

and the procedure continues as before (note the similarity between Eq. (23), and the first term on the left hand side of Eq. (27)). Finally

$$
\begin{aligned}
(P_T \dots P_0)\mathbf{1}_n + \sum_{m=0}^{M} \left(\prod_{j=1}^{m} P_{T+1-j} \right) B_{T-m} \\
= P_T \mathbf{1}_n + B_T \\
= \mathbf{1}_n,
\end{aligned} \tag{28}
$$

which leads to $\lim_{k \to \infty} \mathbf{x}(k) = \mathbf{x}^* = u$. ∎

IV. Simulations

In this section, we provide an illustrative example to demonstrate the concepts described in this letter. In Fig. 2, we show the dynamic leader-follower algorithm, with $n = 6$ mobile followers. We set the communication radius of all followers to $r = 1.5$, and each node can only explore a restricted region shown as a shaded area where the motion is restricted. Fig. 2 shows the random trajectories of each node for the first 25 iterations, as well as all possible update scenarios. In this setup only 2 followers are able to communicate directly to the leader, while other followers are never in the communication radius of the leader. The information from the leader can reach these followers (indirectly) only by propagation through the followers. This setup can be extended to arbitrary network configurations and sizes, where the communication and motion models ensure that the information reaches from the leader to each follower node. Finally, Fig. 3 illustrates the convergence of follower states to the state of the leader, chosen at $u = 3$.

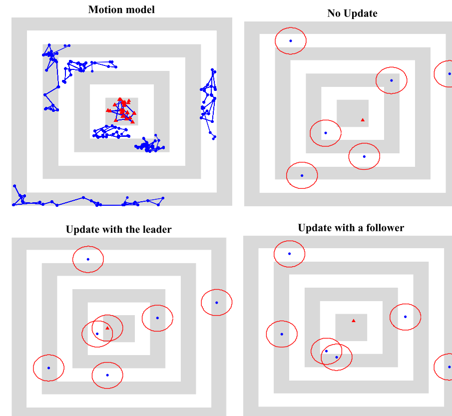

Fig. 2. Updates and motion in a leader-follower network of size 7; red triangle indicates the leader; blue circles represent the followers; red circles show the communication radius of each node.

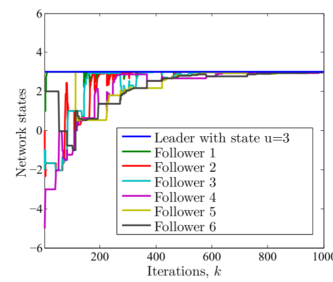

Fig. 3. Convergence of $n = 6$ followers in a dynamic leader-follower network with leader; blue line indicates the state of the leader.

V. Conclusions

In this letter, we study the leader-follower problem in mobile sensor networks, where the mobility of the sensors (followers) results in dynamic updating scenarios. Abstracting such updates as an LTV system with randomly appearing stochastic or sub-stochastic system matrices, we establish the conditions under which the corresponding algorithm converges to the state of a leader. In particular, we show that convergence is guaranteed if the motion of the sensors and the leader follow a certain information propagation rate; and certain weights chosen by each follower have a uniform lower bound. Furthermore, the rate at which the information goes from the leader to each follower does not have to be bounded as long as it is finite and does not grow faster than a certain exponential growth. Although we focus on the networks with only one leader, the results can be easily generalized to any network with multiple leaders, where the followers converge to a linear-convex combination of the leader states.

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