GLRT DETECTION WITH UNKNOWN NOISE POWER IN PASSIVE MULTISTATIC RADAR

Jun Liu, and Hongbin Li[*]

Stevens Institute of Technology
Hoboken, NJ 07030

Braham Himed

AFRL/RYMD
Dayton, OH 45433

ABSTRACT

This paper considers the problem of passive detection with a multistatic radar system involving a non-cooperative illuminator of opportunity (IO) and multiple receive platforms. An unknown source signal is transmitted by the IO, which illuminates a target of interest. These receive platforms are geographically dispersed, and collect independent target echoes due to the illumination by the same IO. We propose a generalized likelihood ratio test (GLRT) detector to deal with the passive detection problem in the case of unknown noise power. Moreover, a closed-form expression for the probability of false alarm of this GLRT detector is given. Numerical simulations demonstrate that the proposed GLRT detector generally outperforms its natural counterparts.

Index Terms— Passive radar, multistatic radar, target detection.

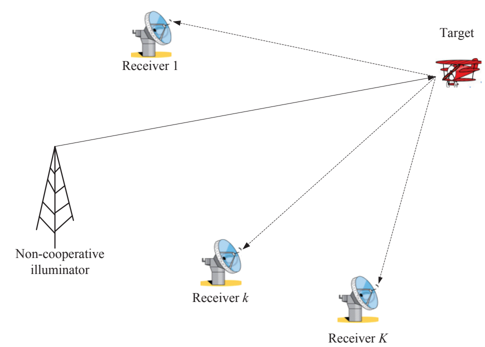

Fig. 1. Configuration of a multistatic passive radar system.

1. INTRODUCTION

Passive radar has been a topic of long-standing interest. It can detect and track a target of interest by exploiting readily available, non-cooperative illuminators of opportunity (IOs) [1–3]. The superiorities of the passive radar over an active radar lie in its stealth capability and low cost. In addition, many IOs are available for the passive radar, such as frequency modulation radio [4], digital video broadcasting-terrestrial (DVB-T) [5, 6], and second generation digital video broadcasting-terrestrial (DVB-T2) sources [7].

Due to the non-cooperative nature of the IO, the transmitted signal is out of control and generally unknown to a passive receiver. As a result, a conventional matched filter cannot be implemented for detection. In many passive radar systems, an additional separate channel, referred to as the reference channel (RC), is usually equipped to collect the transmitted signal as a reference for passive detection. For target detection, the reference signal can be heuristically employed to conduct delay-Doppler cross-correlation operation with the surveillance signal [1, 8, 9]. Nevertheless, the performance is significantly degraded when the reference signal is noisy. To deal with the lack of knowledge of the signal transmitted from the IO, a different approach is to employ multiple SCs in a passive radar system [10]. Since these SCs collect target echoes due to the illumination of the same IO, a correlation exists among the observations collected by the SCs, which can be employed for passive detection. In the following, we focus on target detection in a passive radar system with multiple spatially separated receivers. Each receiver collects target echoes, and serves as a SC for target detection. For a similar detection problem, a generalized coherence (GC) is proposed in [11, 12].

In this paper we develop a GLRT detector for the case of unknown noise power in passive multistatic radars, since in practice knowledge of the noise power is often unavailable a priori. It is shown that the proposed GLRT detector is associated with all eigenvalues of the Gram matrix of the received signals. A closed-form expression for the probability of false alarm of the proposed GLRT detector is obtained, which indicates that the proposed GLRT detector exhibits a CFAR property with respect to the noise power. Simulation results demonstrate that the proposed GLRT detector outperforms the GC detector in cases where the number of receive platforms is large (greater than 3).

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts $(\cdot)^T$ and $(\cdot)^\dagger$ denote transpose and complex conjugate transpose, respectively. $\mathbf{I}_p$ stands for a p-dimensional identity matrix. $|\cdot|$ represents the modulus of a complex number, $\|\cdot\|$ is the Frobenius norm, and $\jmath = \sqrt{-1}$. $\det\{\cdot\}$ and $\mathrm{tr}\{\cdot\}$ denote the determinant and trace of a matrix, respectively. C_n^m and $(n)_k$ are the binomial coefficient and the Pochhammer symbol, respectively. $u(\cdot)$ and $\Gamma(\cdot)$ denote the Heaviside step function and Gamma function, respectively. $\lambda_K(\mathbf{A}) \le \lambda_{K-1}(\mathbf{A}) \le \cdots \le \lambda_2(\mathbf{A}) \le \lambda_1(\mathbf{A})$ denote the ordered eigenvalues of K-dimensional matrix $\mathbf{A}$. The (i, j)th entry of matrix $\mathbf{A}$ is represented by $\mathbf{A}_{i,j}$.

2. SIGNAL MODEL

Consider a passive multistatic radar system as shown in Fig. 1, which involves one non-cooperative transmitter (i.e., IO) and K geographically dispersed receivers or sensors are deployed to collect the echoes of a target of interest due to the illumination of the IO.

Denote by $s(n)$ for $n = 1, 2, \ldots, N$ the unknown signal transmitted by the non-cooperative IO in the discrete time domain. Assume that in each receiver the direct signal from the IO has been

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removed by using a directional antenna or some signal processing techniques [8, 13]. The signal received in the kth receiver, denoted by $x_k(n)$, can be expressed as [10]

$$x_k(n) = \alpha_k s(n - n_k) \exp(\jmath \Omega_k n) + w_k(n), \quad (1)$$

where $n = 1, 2, \ldots, N$, $k = 1, 2, \ldots, K$, α_k is a scaling parameter that accounts for the target reflectivity as well as the propagation effects in the kth receive channel, n_k is the propagation delay of the target returns accounting for both the distance between the IO and the target and the distance between the target and the kth receive platform, Ω_k is the normalized Doppler frequency in the kth receive channel, and $w_k(n)$ is the Gaussian noise with zero mean and variance σ^2, i.e., $w_k(n) \sim \mathcal{CN}(0, \sigma^2)$. Suppose that $w_k(n)$ for $n = 1, 2, \ldots, N$ and $k = 1, 2, \ldots, K$ are identically and independently distributed (i.i.d.).

It is worth noting that the time delays (or frequency shifts) in the different channels may be distinct due to the geographical dispersion of the receivers. In practice, a set of time delay and frequency shift (e.g., (n_1, Ω_1)) in one of the received channels is selected as a reference set. Notice that the differences in the time delay (or frequency shift) between different received signals, instead of the original time delays (or frequency shifts) in all received signals, are of interest. These differences $(\breve{n}_k, \breve{\Omega}_k)$, where $\breve{n}_k = n_k - n_1$ and $\breve{\Omega}_k = \Omega_k - \Omega_1$, can be obtained by a cross-correlation operation between $x_k(n)$ and $x_1(n)$ [8]. Therefore, for a specific reference set (n_1, Ω_1), we can compensate for the time delays and Doppler shifts of the received signals in all other channels. A similar compensation operation can be found in [14] and [15]. In addition, although the time delay n_1 and the Doppler shift Ω_1 in the reference set are not known a priori, their estimates can be obtained using a grid search as in conventional active radars [16].

Let the null hypothesis (H_0) be such that the received data are free of the target echoes and the alternative hypothesis (H_1) be such that the received data contain the target echoes. After compensating for a particular hypothesised set, the passive detection problem can be formulated in terms of the following binary hypothesis test

$$\begin{cases} H_0 : \mathbf{x}_k = \mathbf{w}_k \\ H_1 : \mathbf{x}_k = \alpha_k \mathbf{s} + \mathbf{w}_k \end{cases} \qquad k = 1, 2, \ldots, K, \quad (2)$$

where

- $\mathbf{x}_k$ denotes the $N \times 1$ sample vector in the kth receiver (N is the number of samples);
- $\mathbf{s}$ is an $N \times 1$ sample vector, whose elements are unknown due to the non-cooperative nature of the IO;
- α_k is an unknown scaling parameter that accounts for the channel propagation effect and the target reflectivity;
- $\mathbf{w}_k$ is an $N \times 1$ noise vector in the kth receiver; they are modeled as independent circular complex Gaussian processes with zero mean and covariance matrix $\sigma^2 \mathbf{I}_N$, where σ^2 denotes the noise power, i.e., $\mathbf{w}_k \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$.

In practice, a long integration time is usually required in the passive detection due to the weakness of the target returns. Hence, we impose an assumption that $N > K$ in the passive detection problem (2).

3. GLRT DETECTION WITH UNKNOWN NOISE POWER

In this section, we consider the design of a GLRT detector for the case of unknown noise power. The GLRT detector in this case is to be obtained from

$$\frac{\max_{\{\alpha_k, \mathbf{s}, \sigma^2\}} f(\mathbf{X}|H_1)}{\max_{\{\sigma^2\}} f(\mathbf{X}|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \xi, \quad (3)$$

where ξ is the detection threshold,

$$\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K], \quad (4)$$

$f(\mathbf{X}|H_1)$ and $f(\mathbf{X}|H_0)$ are the probability density functions (PDFs) of the received signals under H_1 and H_0, respectively, i.e.,

$$f(\mathbf{X}|H_1) = \frac{1}{\pi^{KN} \sigma^{2KN}} \exp\left(-\frac{1}{\sigma^2} \sum_{k=1}^{K} \|\mathbf{x}_k - \alpha_k \mathbf{s}\|^2 \right), \quad (5)$$

and

$$f(\mathbf{X}|H_0) = \frac{1}{\pi^{KN} \sigma^{2KN}} \exp\left(-\frac{1}{\sigma^2} \sum_{k=1}^{K} \|\mathbf{x}_k\|^2 \right). \quad (6)$$

It can be shown that the MLE of α_k under H_1 is [14, eq. (5)]

$$\hat{\alpha}_k = \frac{\mathbf{s}^\dagger \mathbf{x}_k}{\mathbf{s}^\dagger \mathbf{s}}. \quad (7)$$

Inserting this MLE of α_k into (5) leads to

$$\max_{\{\alpha_k\}} f(\mathbf{X}|H_1) = \frac{1}{\pi^{KN} \sigma^{2KN}} \times \exp\left[-\frac{1}{\sigma^2} \left(\sum_{k=1}^{K} \|\mathbf{x}_k\|^2 - \frac{\mathbf{s}^\dagger \mathbf{X} \mathbf{X}^\dagger \mathbf{s}}{\mathbf{s}^\dagger \mathbf{s}} \right) \right]. \quad (8)$$

The maximization of (8) with respect to $\mathbf{s}$ is equivalent to maximizing the Rayleigh quotient $\frac{\mathbf{s}^\dagger \mathbf{X} \mathbf{X}^\dagger \mathbf{s}}{\mathbf{s}^\dagger \mathbf{s}}$. This maximum value is exactly the largest eigenvalue of $\mathbf{X}\mathbf{X}^\dagger$, i.e.,

$$\max_{\{\mathbf{s}\}} \frac{\mathbf{s}^\dagger \mathbf{X} \mathbf{X}^\dagger \mathbf{s}}{\mathbf{s}^\dagger \mathbf{s}} = \lambda_1(\mathbf{X}\mathbf{X}^\dagger) = \lambda_1(\mathbf{\Phi}), \quad (9)$$

where

$$\mathbf{\Phi} = \mathbf{X}^\dagger \mathbf{X}. \quad (10)$$

It is worth noting that the employment of the K-dimensional matrix $\mathbf{\Phi}$ instead of the N-dimensional matrix $\mathbf{X}\mathbf{X}^\dagger$ in (9) is more computationally effective in calculating the maximum eigenvalue. In addition, it should be pointed out that there exists an ambiguity in the estimation of the norm of the vector $\mathbf{s}$. It means that $\|\mathbf{s}\|$ cannot be uniquely determined. Nevertheless, this ambiguity does not affect the GLRT.

Substituting (9) into (8) produces

$$\max_{\{\alpha_k, \mathbf{s}\}} f(\mathbf{X}|H_1) = \frac{1}{\pi^{KN} \sigma^{2KN}} \times \exp\left[-\frac{1}{\sigma^2} \left(\sum_{k=1}^{K} \|\mathbf{x}_k\|^2 - \lambda_1(\mathbf{\Phi}) \right) \right]. \quad (11)$$

It is easy to show that the MLE of the noise power under H_1 is

$$\hat{\sigma}^2 = \frac{1}{KN} \left(\sum_{k=1}^{K} \|\mathbf{x}_k\|^2 - \lambda_1(\mathbf{\Phi}) \right). \quad (12)$$

Substituting (12) into (11) yields

$$\max_{\{\alpha_k,\mathbf{s},\sigma^2\}} f(\mathbf{X}|H_1) = \left[\frac{e\pi}{KN}\left(\sum_{k=1}^{K}\|\mathbf{x}_k\|^2 - \lambda_1(\mathbf{\Phi})\right)\right]^{-KN}. \tag{13}$$

According to (6), we obtain the MLE of the noise power under H_0 to be

$$\hat{\sigma}^2 = \frac{1}{KN}\sum_{k=1}^{K}\|\mathbf{x}_k\|^2. \tag{14}$$

Inserting (14) into (6), we have

$$\max_{\{\sigma^2\}} f(\mathbf{X}|H_0) = \left[\frac{e\pi}{KN}\sum_{k=1}^{K}\|\mathbf{x}_k\|^2\right]^{-KN}. \tag{15}$$

Applying (13) and (15) to (3) and making an equivalent transformation, we derive the GLRT detector as

$$\Xi = \frac{\lambda_1(\mathbf{\Phi})}{\sum_{k=1}^{K}\lambda_k(\mathbf{\Phi})} \underset{H_0}{\overset{H_1}{\gtrless}} \xi, \tag{16}$$

where ξ is a suitable modified version of the threshold in (3). Note that in the equivalent transformation in (16) we have used the result

$$\sum_{k=1}^{K}\lambda_k(\mathbf{\Phi}) = \mathrm{tr}(\mathbf{\Phi}) = \sum_{k=1}^{K}\|\mathbf{x}_k\|^2. \tag{17}$$

In the particular case where $K = 2$, the eigenvalues of $\mathbf{\Phi}$ can be explicitly expressed as elementary functions of the received signals, namely,

$$\lambda_1(\mathbf{\Phi}) = \frac{\|\mathbf{x}_1\|^2 + \|\mathbf{x}_2\|^2 + \sqrt{(\|\mathbf{x}_1\|^2 - \|\mathbf{x}_2\|^2)^2 + 4|\mathbf{x}_1^\dagger\mathbf{x}_2|^2}}{2}, \tag{18}$$

and

$$\lambda_2(\mathbf{\Phi}) = \frac{\|\mathbf{x}_1\|^2 + \|\mathbf{x}_2\|^2 - \sqrt{(\|\mathbf{x}_1\|^2 - \|\mathbf{x}_2\|^2)^2 + 4|\mathbf{x}_1^\dagger\mathbf{x}_2|^2}}{2}, \tag{19}$$

respectively. As a result, the test statistic Ξ in (16) for $K = 2$ can be explicitly written as the following equivalent form:

$$\tilde{\Xi} = \frac{\|\mathbf{x}_1\|^2 + \|\mathbf{x}_2\|^2 + \sqrt{(\|\mathbf{x}_1\|^2 - \|\mathbf{x}_2\|^2)^2 + 4|\mathbf{x}_1^\dagger\mathbf{x}_2|^2}}{\|\mathbf{x}_1\|^2 + \|\mathbf{x}_2\|^2} \underset{H_0}{\overset{H_1}{\gtrless}} \tilde{\xi}, \tag{20}$$

where $\tilde{\xi} = 2\xi$.

For the purpose of having a deeper insight into the structure of the proposed GLRT detector, we equivalently write (16) as

$$\frac{1}{\frac{1}{KN}\sum_{k=2}^{K}\lambda_k(\mathbf{\Phi})}\lambda_1(\mathbf{\Phi}) \underset{H_0}{\overset{H_1}{\gtrless}} \bar{\xi}, \tag{21}$$

where $\bar{\xi}$ is a suitable modified version of the threshold in (16). Interestingly, it can be seen from (12) and (17) that the MLE of the noise power under H_1 is

$$\hat{\sigma}^2 = \frac{1}{KN}\sum_{k=2}^{K}\lambda_k(\mathbf{\Phi}), \tag{22}$$

which is exactly the denominator of the test statistic in (21). This is to say, the test statistic in (21) can be interpreted as the maximum eigenvalue normalized by the estimated noise power.

3.1. Performance Analysis

In order to complete the construction of the test in (16), we should provide an approach to set the detection threshold. To this end, a closed-form expression for the probability of false alarm of the GLRT detector in (16) is derived, which can be employed to compute the detection threshold for any given probability of false alarm. According to [17], the probability of false alarm of the GLRT detector in (16) can be expressed as

$$\begin{aligned} P_{\mathrm{FA}} = 1 - \Gamma(KN)M_0^{-1}\sum_{k=1}^{K}\sum_{j=N-K}^{(N+K-2k)k} \\ \sum_{i=0}^{KN-j-2}\frac{\beta_{k,j}\,\mathrm{C}_{KN-j-2}^{i}\,(-k)^i}{\Gamma(KN-j-1)} \\ \times\left\{g_1(\xi,j,i)\left[u\left(\xi-\frac{1}{K}\right) - u\left(\xi-\frac{1}{k}\right)\right]\right. \\ \left. +\, g_2(k,j,i)u\left(\xi-\frac{1}{k}\right)\right\}, \end{aligned} \tag{23}$$

where $\frac{1}{K} \le \xi \le 1$,

$$M_0 = \prod_{k=1}^{K}[(K-k)!(N-k)!], \tag{24}$$

$$g_1(\xi,j,i) = \frac{1}{j+i+1}\left[\xi^{j+i+1} - K^{-(j+i+1)}\right], \tag{25}$$

and

$$g_2(k,j,i) = \frac{1}{j+i+1}\left[k^{-(j+i+1)} - K^{-(j+i+1)}\right]. \tag{26}$$

Note that the coefficients $\beta_{k,j}$ in (23) can be obtained by the following equality [18]:

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\det\{\mathbf{\Theta}(\xi)\} = \sum_{k=1}^{K}\sum_{j=N-K}^{(N+K-2k)k}\beta_{k,j}\xi^j\exp(-k\xi), \tag{27}$$

where the (n,m)th entry of $\mathbf{\Theta}$ is given by

$$\mathbf{\Theta}_{n,m}(\xi) = \gamma(N-K+n+m-1,\xi), \tag{28}$$

with the lower incomplete Gamma function $\gamma(n,x)$ defined as

$$\gamma(n,y) = \int_0^y t^{n-1}\exp(-t)\mathrm{d}t. \tag{29}$$

It is easy to determine $\beta_{k,j}$ in (27) by using most symbolic softwares such as Maple and Matlab (see [19, Algorithm 1]).

In the particular case where $K = 2$, the probability of false alarm of the GLRT detector in (20) can be explicitly written in terms of elementary functions, i.e.,

$$P_{\mathrm{FA}} = 1 - \frac{\Gamma(2N)}{\Gamma(N)\Gamma(N-1)}\left[h(\xi) - h(0.5)\right], \tag{30}$$

where $\frac{1}{2} \le \xi \le 1$, and

$$\begin{aligned} h(y) = \sum_{k=0}^{N-2}\mathrm{C}_{N-2}^{k}(-1)^{N-2-k} \\ \times\left(\frac{y^{2N-k-3}}{2N-k-3} - 4\frac{y^{2N-k-2}}{2N-k-2} + 4\frac{y^{2N-k-1}}{2N-k-1}\right). \end{aligned} \tag{31}$$

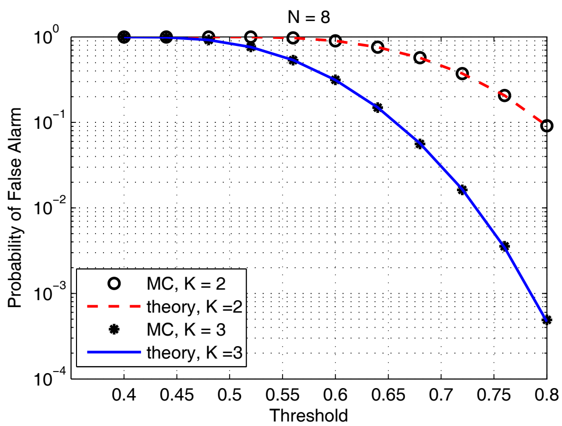

Fig. 2. Probability of false alarm of the GLRT detector in (16) versus the detection threshold for $N = 8$.

It can be seen from (23) that the probability of false alarm of the GLRT detection in (16) is independent of the noise power. It implies that the GLRT detection in (16) possesses the desirable CFAR property against the noise power. As to the detection performance, unfortunately, a closed-form expression for the detection probability of the GLRT detector in (16) is intractable.

4. SIMULATIONS RESULTS

In this section, numerical simulations are conducted to validate the above theoretical analysis and illustrate the performance of the proposed detector. The signal $\mathbf{s}$ transmitted from the IO is sampled from $\mathcal{CN}(\mathbf{0}, \mathbf{I})$. The signal-to-noise ratio (SNR) is defined by

$$\mathrm{SNR} = 10\log_{10}\frac{\frac{1}{KN}\sum_{k=1}^{K}\left(|\alpha_k|^2\sum_{n=1}^{N}|s(n)|^2\right)}{\sigma^2}. \quad (32)$$

For comparison purposes, the GC detector and the ED are introduced. The GC detector can be represented as [20]

$$\Lambda_{\mathrm{GC}} = 1 - \frac{\det\{\mathbf{\Phi}\}}{\prod_{k=1}^{K}\|\mathbf{x}_k\|^2} \underset{H_0}{\overset{H_1}{\gtrless}} \zeta_{\mathrm{GC}}, \quad (33)$$

where ζ_{GC} is the detection threshold. The ED can be expressed as

$$\Lambda_{\mathrm{ED}} = \frac{1}{\sigma^2}\sum_{k=1}^{K}\|\mathbf{x}_k\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \zeta_{\mathrm{ED}}, \quad (34)$$

where ζ_{ED} is the detection threshold. Note that a prior knowledge of the noise power has to be used to set the threshold of the ED.

The probability of false alarm of the proposed GLRT detector in (16) as a function of the detection threshold is presented in Fig. 2, where $N = 8$ and $\sigma^2 = 1$. It can be seen that there is exact agreement between the theoretical results and the simulation results.

Performance comparisons are made in Fig. 3, where $P_{\mathrm{FA}} = 0.01$. It is demonstrated in Fig. 3(a) that the proposed GLRT detector in (16) performs better than the GC detector, when multiple receivers (more than 3) are used. Additionally, the proposed GLRT detector in (16) also outperforms the ED, when the number of the receivers is large (e.g., $K \geq 6$ in this example). In Fig. 3(b) with $K = 8$, we can see that the proposed detector outperforms the GC, and provide performance better than the ED in the region of high SNR.

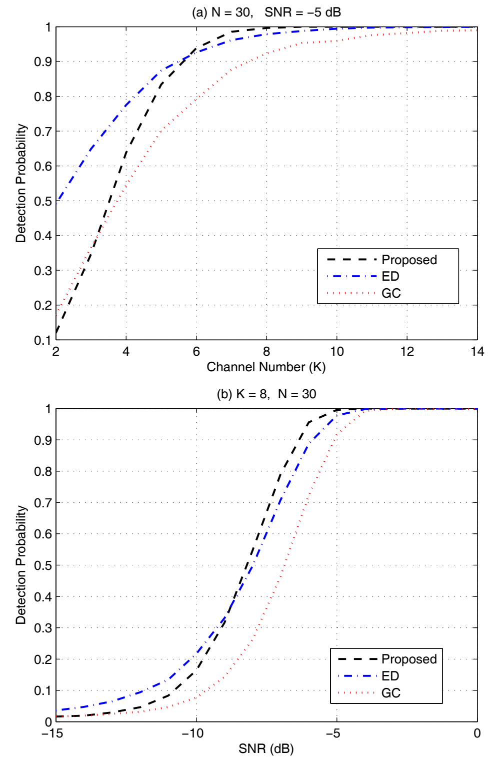

Fig. 3. Performance comparisons

5. CONCLUSION

We examined the problem of passive detection with a multistatic radar system consisting of a non-cooperative IO and multiple geographically distributed receive platforms. Due to the non-cooperative nature of the IO, the signal transmitted from the IO is unknown. A GLRT detector is proposed for the case in which both the transmitted signal and the noise power are unknown. It has the form of the ratio of the maximum eigenvalue to the sum of all eigenvalues of the Gram matrix or covariance matrix of the received signals. The proposed GLRT detector can also be transformed as an equivalent form of the maximum eigenvalue of the Gram matrix normalized by the estimated noise power. The probability of false alarm of the proposed GLRT detector is derived, implying that the proposed GLRT detector possesses the desirable CFAR property against the noise power. Simulation results demonstrate that the performance of the proposed GLRT detector increases as the number of receive platforms or/and data samples increases. In particular, when the number of receive platforms is large, the proposed GLRT detector outperforms the ED and the GC detector.

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