EFFICIENT UPDATE OF PERSISTENT PARTICLES IN THE SMC-PHD FILTER

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ABSTRACT

The paper is devoted to the implementation of the Sequential Monte Carlo Probability Hypothesis Density (SMC-PHD) filter. A measurement driven proposal for persistent target particles requires the predicted persistent target particles to be partitioned in a probabilistic manner using the received measurement set. Each partition is subsequently updated using a conveniently designed efficient proposal distribution (in this paper we apply the progressive correction). The performance of the described algorithm is demonstrated in the context of autonomous tracking of multiple moving targets using bearings-only measurements.

***Index Terms*—** Multi-target nonlinear filtering, particle filters, random set models

1. INTRODUCTION

PHD filter is a multi target nonlinear filter, derived as the first order statistical moment approximation of the exact multi-object Bayes filter formulated using random finite set models [1]. The mathematical elegance, the computational simplicity and ease of implementation have contributed to the widespread use of the PHD filter [2].

In the general nonlinear/non-Gaussian context, PHD filter cannot be solved analytically, and is typically implemented as a sequential Monte Carlo approximation, i.e. as a particle filter [3]. The conceptual framework for efficient particle PHD filter implementation has been cast in [4]: the proposal (importance) densities for drawing *persistent* (surviving) and *newborn* target particles need to dependent on the latest measurement set (which typically includes target-originated as well as false detections). How to construct these importance densities has been the topic of intensive research in the last decade [5], [6], [7], [8].

This paper is focused on the design of an importance density for drawing *persistent* target particles in the SMC-PHD filter. In the framework of standard particle filters, several methods for efficient important density design, using the latest measurement, have been proposed. Among them are the local-linearisation approach [9], progressive correction [10], particle flow [11], Laplace method [12], to name a few. Efficient importance distributions are particularly important in applications that involve high-dimensional state spaces [13] or highly informative models (i.e. models with small process or measurement noise) [14], [11]. Direct application of any of these methods to the update step of the PHD filter, using its pseudo-likelihood, does not perform well. Instead it is necessary for each element of the observation set to somehow identify and cluster its corresponding predicted (persistent) particles. In the previous work this has been done using *gating* and *data clustering* techniques [7], [8]. Both, however, are ad-hoc operations.

In this paper we argue that the selection (partitioning) of (persistent) particles can be done in a principled probabilistic manner, by simply exploiting the PHD update equation. Each partition of particles subsequently can be updated using its assigned measurement via the preferred method of efficient importance distribution design. For demonstration of the concept, in this paper we adopt the progressive correction (PC) [10]. The error performance of the described algorithm has been carried out numerically in the context of autonomous tracking of multiple moving targets using bearings-only measurements.

2. BACKGROUND

Suppose at time t_k, $k = 0, 1, 2\dots$, there are n_k objects with states $\mathbf{x}_{k,1}, \dots, \mathbf{x}_{k,n_k}$, taking values in the state space $\mathcal{X} \subset \mathbb{R}^{n_x}$. Both the number of targets n_k and their individual states in $\mathcal{X}$ are random and time-varying. The multi-target state at k, represented by a finite set $\mathbf{X}_k = \{\mathbf{x}_{k,1}, \dots, \mathbf{x}_{k,n_k}\}$ can conveniently be modelled as a random finite set on $\mathcal{X}$.

The sensor reports detected objects in the observation space $\mathcal{Z} \subset \mathbb{R}^{n_z}$. Detection process is uncertain, meaning that some of the objects in $\mathbf{X}_k$ are detected, while others are missed. In addition, detector may also report false detections or clutter. The measurement set at time t_k can be represented by a finite set $\mathbf{Z}_k = \{\mathbf{z}_{k,1}, \dots, \mathbf{z}_{k,m_k}\} \in \mathcal{F}(\mathcal{Z})$. Both cardinality $|\mathbf{Z}_k| = m_k$ and the position of individual elements of $\mathbf{Z}_k$ in the measurement space are random, thus $\mathbf{Z}_k$ can also be modelled by a random finite set.

Let us denote the posterior intensity function of the random finite set (RFS) $\mathbf{X}_k$ at discrete-time k by $D_{k|k}(\mathbf{x}|\mathbf{Z}_{1:k}) \stackrel{\text{abbr}}{=} D_{k|k}(\mathbf{x})$, where $\mathbf{Z}_{1:k} \equiv \mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_k$ is the accumulated sequence of measurement sets up to time k.

Assuming that individual objects evolve independently of each other via transitional density $\pi_{k+1|k}(\mathbf{x}|\mathbf{x}')$, the prediction equation of the PHD filter is given by [1], [6]:

$$D_{k+1|k}(\mathbf{x}) = \underbrace{\gamma_{k+1|k}(\mathbf{x})}_{D_{k+1|k,b}(\mathbf{x})} + \underbrace{p_S \int \pi_{k+1|k}(\mathbf{x}|\mathbf{x}')\, D_{k|k}(\mathbf{x}')\, d\mathbf{x}'}_{D_{k+1|k,p}(\mathbf{x})} \quad (1)$$

The first term on the right-hand side (RHS) of (1), $D_{k+1|k,b}(\mathbf{x})$, is the intensity function of the RFS of object births between time k and $k+1$. The second term, $D_{k+1|k,p}(\mathbf{x})$, is the intensity function of objects that survive (or persist) from time k to $k+1$; p_S is the probability of object survival.

Assuming that the predicted RFS, whose intensity function is $D_{k+1|k}(\mathbf{x})$, is Poisson, and that measurements of individual objects are mutually independent, the intensity function, after the update

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with measurement set $\mathbf{Z}_{k+1}$, can be expressed as [1], [6]:

$$D_{k+1|k+1}(\mathbf{x}) = D_{k+1|k+1,b}(\mathbf{x}) + D_{k+1|k+1,p}(\mathbf{x}) \tag{2}$$

where

$$D_{k+1|k+1,p}(\mathbf{x}) = [1 - p_{\text{D}}]D_{k+1|k,p}(\mathbf{x}) + \sum_{\mathbf{z}\in\mathbf{Z}_{k+1}} \frac{p_{\text{D}} g_{k+1}(\mathbf{z}|\mathbf{x}) D_{k+1|k,p}(\mathbf{x})}{B(\mathbf{z})} \tag{3}$$

$$D_{k+1|k+1,b}(\mathbf{x}) = \sum_{\mathbf{z}\in\mathbf{Z}_{k+1}} \frac{g_{k+1}(\mathbf{z}|\mathbf{x})\gamma_{k+1|k}(\mathbf{x})}{B(\mathbf{z})} \tag{4}$$

with

$$B(\mathbf{z}) = \kappa_{k+1}(\mathbf{z}) + \langle g_{k+1}(\mathbf{z}|\cdot), \gamma_{k+1|k}\rangle + p_{\text{D}}\langle g_{k+1}(\mathbf{z}|\cdot), D_{k+1|k,p}\rangle \tag{5}$$

$D_{k+1|k+1,p}(\mathbf{x})$ and $D_{k+1|k+1,b}(\mathbf{x})$ are intensity functions of persistent and newborn objects, respectively. Notation $\langle a, b\rangle = \int a(\mathbf{x})\, b(\mathbf{x})\, d\mathbf{x}$ stands for the inner product between two real-valued functions, while $\kappa_k(\mathbf{z})$ is the intensity function of the clutter RFS at time k. The clutter is modelled by a Poisson RFS whose intensity function is $\kappa_k(\mathbf{z}) = \lambda_k\, c_k(\mathbf{z})$, where λ_k is the mean clutter rate, while $c_k(\mathbf{z})$ is the spatial distribution of clutter over $\mathcal{Z}$. Function $g_k(\mathbf{z}|\mathbf{x})$ represents the likelihood of measurement $\mathbf{z} \in \mathbf{Z}_k$ resulting from an individual object in state $\mathbf{x}$. Inner product $\langle g_{k+1}(\mathbf{z}|\cdot), \gamma_{k+1|k}\rangle = p_b$ in (5) is typically an input parameter of the PHD filter, which indicates the intensity of the target birth process. In order to simplify notation, we drop subscript k in $\kappa_k(\mathbf{z})$ and $g_k(\mathbf{z}|\mathbf{x})$.

The PHD filter recursion (1)-(4) is initialised with $D_{0|0}(\mathbf{x})$. In the absence of any prior, we can set $D_{0|0}(\mathbf{x}) = 0$, meaning that initially there are no objects in the surveillance volume.

In the general nonlinear/non-Gaussian context, n_x dimensional integrals in (1) and (5) are intractable. We solve PHD recursions approximately using the sequential Monte Carlo method. Suppose $D_{k|k}(\mathbf{x})$ is approximated by a particle system $\mathcal{P}_k \equiv \{(w_k^{(i)}, \mathbf{x}_k^{(i)})\}_{i=1}^{N_k}$ as follows:

$$D_{k|k}(\mathbf{x}) \approx \sum_{i=1}^{N_k} w_k^{(i)}\, \delta_{\mathbf{x}_k^{(i)}}(\mathbf{x}) \tag{6}$$

where $\mathbf{x}_k^{(i)}$ is ith particle and $w_k^{(i)} \geq 0$ its weight; $\delta_{\mathbf{w}}(\mathbf{x})$ is the standard Dirac delta concentrated at $\mathbf{w}$. The sum of the weights approximates the expected number of objects, i.e. $\nu_{k|k} \approx \sum_{i=1}^{N_k} w_k^{(i)} \geq 0$.

Following [6] we distinguish between the persistent (surviving) objects and newborn objects. The particle system $\mathcal{P}_k$ is predicted to time $k+1$ using the transitional density $\pi_{k+1|k}(\mathbf{x}|\mathbf{x}')$, resulting in the particle system $\mathcal{P}_{k+1|k,p} = \{(w_{k+1|k,p}^{(i)}, \mathbf{x}_{k+1|k,p}^{(i)})\}_{i=1}^{N_k}$ which approximates the predicted intensity function of persistent objects $D_{k+1|k,p}(\mathbf{x})$ defined in (1). This step is carried out for $i = 1, \ldots, N_k$, as follows:

$$\mathbf{x}_{k+1|k,p}^{(i)} \sim \pi_{k+1|k}(\mathbf{x}_{k+1}|\mathbf{x}_k^{(i)}) \tag{7}$$

$$w_{k+1|k,p}^{(i)} = p_{\text{S}} w_k^{(i)}. \tag{8}$$

The focus of the paper is efficient importance sampling scheme, which uses the measurement set $\mathbf{Z}_{k+1}$, and implements the update of persistent target particles (3). An efficient importance sampling scheme for newborn target particles has been discussed in [6].

3. THE PROPOSED ALGORITHM

3.1. Partitioning of particles

For every $\mathbf{z} \in \mathbf{Z}_{k+1}$ we need to find its corresponding particles and then to design for them an efficient proposal density. The first step, hence, is to partition the set of predicted persistent target particles $\mathcal{P}_{k+1|k,p}$ based on the measurement set $\mathbf{Z}_{k+1}$. The result is $m_{k+1}+1$ sets of particle indices, denoted $\mathcal{I}_{k+1|k}^j$, $j = 0, 1, \ldots, m_{k+1}$, which define the clusters of weighted particles:

$$\mathcal{C}_{k+1|k}^j = \left\{(w_{k+1|k,p}^{(i)} \mathbf{x}_{k+1|k,p}^{(i)}); i \in \mathcal{I}_{k+1|k}^j\right\} \tag{9}$$

such that

$$\bigcup_{j=0}^{m_{k+1}} \mathcal{I}_{k+1|k}^j = \{1, 2, \ldots, N_k\} \text{ and } \bigcup_{j=0}^{m_{k+1}} \mathcal{C}_{k+1|k}^j = \mathcal{P}_{k+1|k,p}. \tag{10}$$

Partitioning can be implemented based on the update equation (3), which consists of a sum of $m_{k+1}+1$ terms. Each term in the sum on the RHS of (3) represents the probability that measurement $\mathbf{z} \in \mathbf{Z}_{k+1}$ is due to an object in state $\mathbf{x}$. Then we can define for each particle-measurement pair the probability that measurement $\mathbf{z}_j \in \mathbf{Z}_{k+1}$ is due to the object in state $\mathbf{x}_{k+1|k,p}^{(i)}$ [3, p.78]:

$$p_{ij} = \frac{p_{\text{D}}\, g(\mathbf{z}_j|\mathbf{x}_{k+1|k,p}^{(i)})\, w_{k+1|k,p}^{(i)}}{\kappa(\mathbf{z}_j) + p_b + p_{\text{D}} \sum_{\ell=1}^{N_k} g(\mathbf{z}_j|\mathbf{x}_{k+1|k,p}^{(\ell)})\, w_{k+1|k,p}^{(\ell)}} \tag{11}$$

where $i = 1, \ldots, N_k$, $j = 1, \ldots, m_{k+1}$. From (11) it is clear that $0 \leq p_{ij} \leq 1$.

Based on the update equation (3) we can also define the probability that an object in state $\mathbf{x}_{k+1|k,p}^{(i)}$ has not been detected. If we refer to this case as $j = 0$, we denote this probability as [3, p.78]:

$$p_{i0} = (1 - p_{\text{D}}) w_{k+1|k,p}^{(i)}. \tag{12}$$

Partitioning of $\mathcal{P}_{k+1|k,p}$ now proceeds based on probabilities (11) and (12). For each particle with index $i = 1, 2, \ldots, N_k$ of $\mathcal{P}_{k+1|k,p}$, one computes p_{ij} for all $j = 0, 1, \ldots, m_{k+1}$. A probability distribution over elements of $\mathbf{Z}_{k+1}$, plus the empty set, can then be formed for particle i as

$$p_i(j) = \frac{p_{ij}}{\sum_{\ell=0}^{m_{k+1}} p_{i\ell}}, \quad j = 0, 1, \ldots, m_{k+1}. \tag{13}$$

One can now draw index $j^i \in \{0, 1, \ldots, m_{k+1}\}$ as $j^i \sim p_i(j)$. The particle with index i is then assigned to index partition $\mathcal{I}_{k+1|k}^{j^i}$ and thus to the cluster of weighted particles $\mathcal{C}_{k+1|k}^{j^i}$. Note that the described partitioning procedure may result in some index partitions $\mathcal{I}_{k+1|k}^j$ being empty (for example, if $\mathbf{z}_j \in \mathbf{Z}_{k+1}$ is a false detection, $\mathcal{I}_{k+1|k}^j$ is likely to be empty).

3.2. Updating persistent target particles

The pseudo-code of this algorithm is given in Alg.1. Let us denote a component of $D_{k+1|k,p}(\mathbf{x})$ which is approximated by the cluster of particles $\mathcal{C}_{k+1|k}^j$ by $q_{k+1|k}^j(\mathbf{x})$. According to (3), the Bayes update of $q_{k+1|k}^j(\mathbf{x})$ using the assigned measurement $\mathbf{z}_j \in \mathbf{Z}_{k+1}$ is as follows:

$$q_{k+1}^j(\mathbf{x}) = \frac{p_{\text{D}}\, g(\mathbf{z}_j|\mathbf{x})\, q_{k+1|k}^j(\mathbf{x})}{\kappa(\mathbf{z}_j) + p_b + p_{\text{D}} \int g(\mathbf{z}_j|\mathbf{x})\, q_{k+1|k}^j(\mathbf{x}) d\mathbf{x}} \tag{14}$$

A slight modification of any standard particle filter can implement update (14). This step is carried out in line 5 of Alg.1, where PFU stands for *particle-filter update*. The output of PFU is a cluster $\mathcal{C}^j_{k+1} = \{(w^{(\ell)}_{k+1,p}\mathbf{x}^{(\ell)}_{k+1,p})\}^L_{\ell=1}$, which is added to the set $\mathcal{P}_{k+1,p}$ in line 6. The sum of the updated weights in cluster j is less or equal to 1 and represents the probability of existence, that is, the probability that a target, whose probability density function (PDF) is approximated by cluster $\mathcal{C}^j_{k+1}$ (with normalised weights) exists. The probability of existence is computed in line 7 of Alg.1: if it is above a reporting threshold η, a weighted mean of the particles in cluster $\mathcal{C}^j_{k+1}$ is computed in line 9 and included in the multi-object state estimate $\hat{\mathbf{X}}_{k+1}$ in line 10, of Alg.1.

The particles in cluster $\mathcal{C}^0_{k+1|k}$ are treated differently, since they are not assigned any measurement from $\mathbf{Z}_{k+1}$, see the loop between lines 14 and 19 in Alg.1. Recall that every measurement creates new particles (the so-called newborn target particles) which, potentially, could result in the ever-growing number of particles over time. The if-then clause in line 16 of Alg. 1 is introduced to prevent that from happening. The particles in cluster $\mathcal{C}^0_{k+1|k}$ whose weights are smaller than a certain threshold ξ are eliminated from propagating further in time (e.g. those initially created on false detections). Particle elimination threshold ξ (line 15) must be chosen so that the particles on undetected true existing objects are not eliminated. This is very important in applications such as bearings-only tracking: by elimination of predicted particles on an undetected true existing object, the observability of its state would be lost. Standard SMC-PHD filters [4], [6] would be inappropriate for this application (as we demonstrate in Sec.4): they control the growth in the number of particles by resampling *all* particles, which is likely to eliminate the predicted particles on undetected true existing object.

Algorithm 1 Update Persistent Particles

1: **Input**: $\mathcal{P}_{k+1|k,p}$, $\{\mathcal{I}^j_{k+1|k}\}^{m_{k+1}}_{j=0}$, $\mathbf{Z}_{k+1}$
2: $\mathcal{P}_{k+1,p} = \emptyset$; $\hat{\mathbf{X}}_{k+1} = \emptyset$
3: **for** $j = 1, \dots, m_{k+1}$ **do**
4:   **if** $\mathcal{I}^j_{k|k-1} \neq \emptyset$ **then**
5:     $[\mathcal{C}^j_{k+1}] = PFU[\mathcal{C}^j_{k+1|k}, \mathbf{z}_j]$ ▷ Update (14)
6:     $\mathcal{P}_{k+1,p} = \mathcal{P}_{k+1,p} \cup \mathcal{C}^j_{k+1}$
7:     $p^j_e = \sum^B_{b=1} w^{(b)}_{k+1,p}$ ▷ Prob. of exist.
8:     **if** $p^j_e > \eta$ **then**, ▷ η is reporting threshold
9:       $\hat{\mathbf{x}}^j_{k+1} = \sum^B_{b=1} \frac{w^{(b)}_{k+1,p}}{p^j_e}\mathbf{x}^{(b)}_{k+1,p}$
10:       $\hat{\mathbf{X}}_{k+1} = \hat{\mathbf{X}}_{k+1} \cup \{\hat{\mathbf{x}}^j_{k+1}\}$
11:     **end if**
12:   **end if**
13: **end for**
14: **for** every $i \in \mathcal{I}^0_{k+1|k}$ **do**
15:   **if** $w^{(i)}_{k+1|k,p} > \xi$ **then** ▷ ξ is threshold
16:     $w^{(i)}_{k+1} = (1 - p_{\text{D}})w^{(i)}_{k+1|k,p}$
17:     $\mathcal{P}_{k+1,p} = \mathcal{P}_{k+1,p} \cup \{(w^{(i)}_{k+1}, \mathbf{x}^{(i)}_{k+1|k,p})\}$
18:   **end if**
19: **end for**
20: **Output**: $\mathcal{P}_{k+1,p}$, $\hat{\mathbf{X}}_{k+1}$

Two methods of the PFU (line 5 of Alg. 1) have been implemented for the sake of demonstration: the bootstrap-type filter update [15], and the progressive correction (PC) method [14], [16], which belong to the class of efficient important density methods. In order to explain the PC method, let us rewrite (14) using simpler notation:

$$\pi(\mathbf{x}) = \mathcal{L}(\mathbf{z}|\mathbf{x}) \cdot \pi_0(\mathbf{x}) \tag{15}$$

where $\pi_0(\mathbf{x}) \equiv q^j_{k+1|k}(\mathbf{x})$ is the prior distribution, $\pi(\mathbf{x}) \equiv q^j_{k+1}(\mathbf{x})$ is the posterior, and

$$\mathcal{L}(\mathbf{z}|\mathbf{x}) = \frac{p_{\text{D}}\, g(\mathbf{z}|\mathbf{x})}{\kappa(\mathbf{z}) + p_b + p_{\text{D}} \int g(\mathbf{z}|\mathbf{x})\, q^j_{k+1|k}(\mathbf{x}) d\mathbf{x}} \tag{16}$$

is the likelihood function. Importance sampling using the prior PDF as a proposal (i.e. bootstrap-type update) works well only if there is a significant overlap between the likelihood and the prior PDF. This happens when the information contained in the measurement is relatively small. For highly informative measurements or high-dimensional state spaces, this would not be the case. In these situations it is more efficient to construct a series of intermediate distributions of which the first is the prior, while the final should be similar to the posterior.

Let S denote the number of stages of the PC and $\pi_s(\mathbf{x})$, $s = 1, \dots, S$ the intermediate distribution that we sample from at stage s. Note that $\pi_S(\mathbf{x}) \equiv \pi(\mathbf{x})$. A suitable distribution to draw samples from at stage s is $\pi_s(\mathbf{x}) \propto \mathcal{L}(\mathbf{z}|\mathbf{x})^{\Phi_s}\pi_0(\mathbf{x})$, where $\Phi_s = \sum^s_{i=1}\phi_i$, with $\phi_i \in (0, 1]$ and $\Phi_S = 1$. Since Φ_s is an increasing function bounded by one, the intermediate likelihood used at stage $s < S$, i.e. $\mathcal{L}(\mathbf{z}|\mathbf{x})^{\Phi_s}$, is broader than the true likelihood $\mathcal{L}(\mathbf{z}|\mathbf{x})$, particularly in the early stages. In this manner particles are moved in stages from the prior to the region of the state space suggested by the true likelihood, i.e. the correction imposed by the measurement is gradually introduced on the prior. The correction factors $\phi_1, \dots, \phi_S$ can be selected in advance (as fixed values) or computed adaptively for each stage s [14], [16].

4. NUMERICAL RESULTS

This section compares three SMC-PHD filters in the context of multi-target bearings-only tracking using highly informative models. The tracking scenario is shown in Fig.1: the observer is manoeuvreing while three targets are moving with constant velocity. The end of each trajectory is indicated in Fig.1 by a square. The state vector of ith each target is adopted as

$$\mathbf{x}^t_{k,i} = \begin{bmatrix} x^t_{k,i} & \dot{x}^t_{k,i} & y^t_{k,i} & \dot{y}^t_{k,i} \end{bmatrix}^\intercal \tag{17}$$

where $(x^t_{k,i}, y^t_{k,i})$ is its position and $(\dot{x}^t_{k,i}, \dot{y}^t_{k,i})$ its velocity, in Cartesian coordinates. The observer state vector $\mathbf{x}^o_k$, which is known, is similarly defined. The motion model will be written for the relative state vector, which for ith target is defined as:

$$\mathbf{x}_{k,i} := \mathbf{x}^t_{k,i} - \mathbf{x}^o_k = \begin{bmatrix} x_{k,i} & \dot{x}_{k,i} & y_{k,i} & \dot{y}_{k,i} \end{bmatrix}^\intercal. \tag{18}$$

where $i = 1, \dots, n_k = 3$. Target motion is modelled by a constant velocity (CV) model, that is the transitional density $\pi_{k+1|k}(\mathbf{x}|\mathbf{x}')$ is a Dirac delta function, i.e. $\pi_{k+1|k}(\mathbf{x}_{k+1}|\mathbf{x}_k) = \delta_{\mathbf{F}\mathbf{x}_k - \mathbf{U}_{k+1,k}}(\mathbf{x})$, where $\mathbf{F}$ is the transition matrix and $\mathbf{U}_{k+1,k}$ is a known deterministic matrix that takes into account the effect of observer accelerations. Note that this is a highly informative motion model (no process noise). We adopt:

$$\mathbf{F} = \mathbf{I}_2 \otimes \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad \mathbf{U}_{k+1,k} = \begin{bmatrix} x^o_{k+1} - x^0_k - T\dot{x}^o_k \\ \dot{x}^o_{k+1} - \dot{x}^o_k \\ y^o_{k+1} - y^0_k - T\dot{y}^o_k \\ \dot{y}^o_{k+1} - \dot{y}^o_k \end{bmatrix} \tag{19}$$

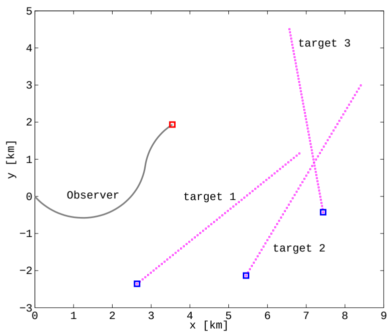

Fig. 1. Bearings-only tracking scenario

where $\otimes$ is the Kronecker product and $T = t_{k+1} - t_k$ is the sampling interval (in simulations T is set to 30 seconds).

A set of bearings measurements of existing targets typically includes some false detections. For a bearings measurement $z \in Z_{k+1}$, which originates from a target in the (relative) state $\mathbf{x}$, the likelihood function $g(z|\mathbf{x})$ is specified as:

$$g(z|\mathbf{x}) = \mathcal{N}(z; h(\mathbf{x}), \sigma_v^2), \text{ where } h(\mathbf{x}) = \arctan \frac{x_k}{y_k} \qquad (20)$$

and $\mathcal{N}(\cdot; \mathbf{m}, \mathbf{P})$ is the Gaussian distribution with mean $\mathbf{m}$ and covariance $\mathbf{P}$. The standard deviation of target-originated measurements is set to $\sigma_v = 0.05^o$; hence the measurements are very precise, i.e. the measurement model is highly informative. The probability of detection is $p_{\text{D}} = 0.95$. The spatial distribution of false detections $c_k(z)$ is uniform over the measurement space $(-\pi, \pi]$, while the mean number of false detections per scan is set to $\lambda_k = 1$.

The error performance of bearings-only PHD filters is measured using the optimal sub-pattern assignment (OSPA) error [17]. Fig. 2 shows the mean OSPA error (in km), obtained by averaging over 100 Monte Carlo runs. The three contesting SMC-PHD filters are: (1) the SMC-PHD filter developed in [18], [6] (this algorithm does not partition the particles in the update step); its OSPA error is shown by a dotted line; (2) the proposed SMC-PHD filter, where the update of persistent target particles (line 5 in Alg.1) is carried out using the bootstrap-type method; its OSPA errors curve is indicated by a dashed line; (3) the proposed SMC-PHD filter (OSPA error in solid line), where the update in line 5 in Alg.1 is carried out depending on the effective sample size N_{eff} of the particle partition $\mathcal{C}^j_{k+1|k} = \{(w^{(m)}_{k+1|k,p}, \mathbf{x}^{(m)}_{k+1|k,p})\}_{m=1}^{M_j}$:

$$N_{\text{eff}} = \frac{\left[\sum_{m=1}^{M_j} w^{(m)}_{k+1|k,p}\right]^2}{\sum_{m=1}^{M_j} \left(w^{(m)}_{k+1|k,p}\right)^2}$$

If $N_{\text{eff}} < 0.35\, M_j$, the update (line 5 in Alg.1) is carried out using the PC, otherwise the bootstrap-type method is applied. The parameters of the SMC-PHD filter are as follows: reporting threshold $\eta = 1 - p_{\text{D}}$; particle preserving threshold $\xi = (1 - p_{\text{D}})/M_j$; for each measurement a total of N newborn particles are placed; resampling is also carried out N times. The value of N in Fig.2.(a) is $N = 200$, while in Fig.2.(b) is $N = 400$. Progressive correction is always carried out $S = 4$ times, using the correction factors $\phi_1 = 0.05$, $\phi_2 = 0.1$, $\phi_3 = 0.2$ and $\phi_4 = 0.65$.

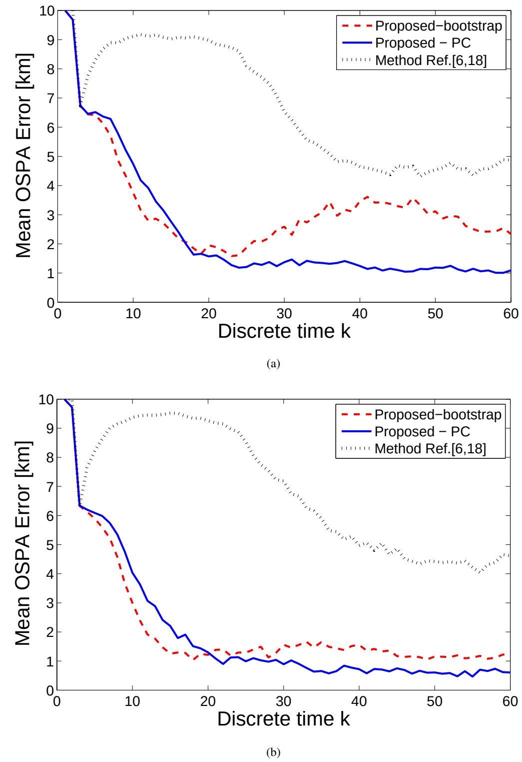

Fig. 2. Mean OSPA errors for three particle PHD filters: (a) $N = 200$; (b) $N = 400$

Observe from Fig.2 that the SMC-PHD filter of [18], [6] is clearly inappropriate in the considered scenario (as anticipated earlier). For a small number of particles (Fig.2.(a), $N = 200$), the bootstrap-type update occasionally fails, which is reflected in the higher OSPA error after discrete-time $k = 20$. During this interval, the bearings-rates of the three targets are very high, and consequently their state estimation becomes a serious challenge. The PC deals with the difficulties posed by the high bearings-rate easily and its mean OSPA error in Fig.2.(a) is steadily decreasing. By doubling the number of particles (Fig.2.(b), $N = 400$), the performance of the two particle PHD filters becomes similar, although the PC method is still characterised by somewhat smaller steady-state error.

5. CONCLUSIONS

The paper proposed a framework for efficient implementation of particle PHD filters for nonlinear filtering of multiple (possibly appearing/disappearing) targets. The main idea is to partition the particles based on the received measurement set, and then to update each partition separately using the associated measurement. This allows for the efficient particle filtering schemes to be used in the particle PHD filter. The method is demonstrated in the context of multi-target bearings only tracking, using highly informative models. When the effective sample size of a partition of particles is too small, an efficient proposal is generated using the method of importance sampling with progressive correction. The result is a robust multi-target state estimator that performs well even using a small number of particles.

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