BAYESIAN PATH ESTIMATION USING THE SPATIAL ATTRIBUTES OF A ROAD NETWORK

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ABSTRACT

We consider the problem of estimating the path taken by an object in a road network from sparse, noisy position measurements. Path estimation is posed in a Bayesian framework which allows the incorporation of prior information about vehicle movements. A carefully designed importance sampler is used to approximate the posterior path probabilities. The algorithm is demonstrated on simulated data.

***Index Terms*—** path estimation, Bayesian estimation, Monte Carlo approximation

1. INTRODUCTION

Map-aided positioning has many applications in transportation and navigation and has therefore been the subject of extensive research interest. The particular problem considered here is that of determining the path taken by an object using a sequence of, usually sparse, position measurements. In this context, sparse means that position measurements are not available along all edges of the road network. A comprehensive account of the methods proposed for path estimation, also called map matching, can be found in [1, 2]. In [1] path estimation algorithms are categorised into four groups: geometric [3], topological [4], probabilistic and advanced. Here we pose path estimation in an estimation theoretic framework, specifically that of Bayesian estimation. As such, our approach belongs in the class of probabilistic methods. The Bayesian approach provides a principled way of incorporating prior information and uncertainty about the measurements. The result is a posterior distribution over the paths which is useful not only for path estimation but also for quantifying how well the path can be estimated.

A disadvantage of the Bayesian approach is that it is not practical to compute the posterior path probabilities exactly and developing a computationally efficient approximation is challenging. Several approaches have been developed to address this problem. The multiple hypothesis tracker (MHT) [5] has been adapted for path estimation in several papers [6–9]. The MHT involves enumerating and evaluating hypotheses regarding the edges occupied by an object at measurement sampling times [7, 8] or the path taken between measurements [6]. Only [6] considers sparse data but does not develop a rigorous Bayesian model comprising a prior for the unknown parameters and a likelihood. Markov chain Monte Carlo (MCMC) methods for path estimation have been proposed in [10–12]. The particular MCMC approach used in these papers is Metropolis-Hastings sampling, a broad class of techniques in which the basic idea is to propose a sample which is then accepted probabilistically.

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We develop a full Bayesian model in which a prior distribution is used to incorporate information about vehicle movement. A measurement model captures the uncertainty in the position measurements and their attribution to particular objects. A sequential importance sampling procedure is used to approximate the posterior path probabilities. Importance sampling involves drawing samples from an importance density and then weighting them appropriately. The key to efficient importance sampling, in the sense of providing accurate approximation with a moderate sample size, lies in the construction of the importance density. A vital property is that samples be drawn conditional on the measurements [13]. We develop a sequential sampling procedure for the object path and the object locations at each measurement sampling instant. We show that, under the assumption that the road network is composed of straight lines, we can design a measurement-directed importance density from which it is easy to sample. As a result, almost optimal Bayesian path estimation can be achieved with moderate sample sizes.

The approach to path estimation proposed here is most closely related to the MCMC methods developed in [10–12]. Our contributions relative to this existing work are as follows. First, instead of using MCMC sampling, which produces dependent samples and requires convergence diagnostics [14], we use an importance sampler. Importance sampling produces independent samples and does not require a burn-in period before convergence is achieved. Second, care has been taken to ensure that samples are drawn in the important part of the parameter space and computationally costly numerical approximations are avoided. The result of these two contributions is that accurate approximation is obtained with small sample sizes.

The paper is organised as follows. In Section 2 the path estimation problem is formulated in the Bayesian framework. An importance sampling algorithm for approximating the posterior path probabilities is described in Section 3. The performance of this algorithm is evaluated in a simulation scenario in Section 4.

2. THE PATH ESTIMATION PROBLEM

A transportation network is structured as a directed, weighted graph $G = (V,\ E)$, with vertices V connected by edges $E \subseteq V \times V$ and edge weights $\ell : E \to \mathbb{R}^+$ representing distance along edges. A graph embedding maps the vertices in the graph to point locations in the plane and the edges in the graph to polyline geometries in the plane. A path ρ is a sequence of vertices $v_1, \dots, v_q$ such that $(v_i,\ v_{i+1}) \in E$ for $i = 1, \dots, q-1$.

It is desired to estimate the path taken by an object using m imprecise position measurements $\mathbf{y}_1, \dots, \mathbf{y}_m$, $\mathbf{y}_j \in \mathbb{R}^2$, obtained at times $t_1, \dots, t_m$. We pose the problem in the Bayesian framework. In particular, the problem is formulated in a sequential fashion in which we recursively estimate the path taken up to time t_j for $j = 1, \dots, m$. A sequential approach is favoured because it allows Bayesian inference to be performed in a computationally efficient manner. This is explained in more detail in Section 3.

Let ρ_j denote the path taken up to time t_j. The notation $\rho_{a:b}$ is used to denote the sequence $\rho_a, \dots, \rho_b$. Our approach to path estimation involves computation of the joint posterior density:

$$\pi_m(\rho_{1:m}) \propto g(\mathbf{y}_{1:m}|\rho_{1:m})\pi_0(\rho_{1:m}), \tag{1}$$

where $\pi_m(\cdot)$ denotes the path distribution conditional on the measurements $\mathbf{y}_{1:m}$, $g(\cdot|\rho_{1:m})$ is the density of the measurements taken at times $t_{1:m}$ conditional on the paths $\rho_{1:m}$ and $\pi_0(\cdot)$ is the path prior density. The latter two densities are developed below.

2.1. Measurement model

Let $\ell(e)$ denote the length of the edge $e \in E$. The position of an object on the road network is represented by $s = (r, e) \in \cup_{d\in E}[0, \ell(d)] \times \{d\} = S$ where e is the edge occupied by the object and r is the distance along the edge. Points on the road network are mapped to positions in the observation space via the function $\mathbf{x} : S \to \mathbb{R}^2$. It is assumed that the position measurements $\mathbf{y}_{1:m}$ are independent and identically distributed random variables. Each variable $\mathbf{y}_j$ follows a two-dimensional joint Gaussian distribution with mean $\mathbf{x}(s_j)$ and covariance $\mathbf{R}$. The position measurements then satisfy

$$g(\mathbf{y}_{1:m}|\rho_{1:m}) = \int \prod_{j=1}^{m} \mathsf{N}(\mathbf{y}_j;\, \mathbf{x}(s_j),\, \mathbf{R})\pi_0(s_{1:m}|\rho_{1:m})\, \mathrm{d}s_{1:m}, \tag{2}$$

where $\mathsf{N}(\cdot;\, \boldsymbol{\mu},\, \boldsymbol{\Sigma})$ denotes the normal density with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$ and $\pi_0(s_{1:m}|\rho_{1:m})$ is the prior density of the object location given the path. Note that the integration in (2) is over the space S^m and therefore involves summation over the edges and integration over the length of each edge. It remains to specify the prior density $\pi_0(s_{1:m}|\rho_{1:m})$. This is addressed in the next subsection.

2.2. Prior density

Computing the posterior density (1) requires the path prior and the prior for the object location conditional on the path. The notation $\pi_0(\cdot)$ is used for all priors. The particular prior being referred to is determined by the arguments. The path prior factorises as

$$\pi_0(\rho_{1:m}) = \pi_0(\rho_1) \prod_{j=2}^{m} \pi_0(\rho_j|\rho_{1:j-1}). \tag{3}$$

At time t_1 only a single measurement is available so the path ρ_1 is composed of a single edge. Any distribution over the edge space E could be selected. The prior for the path ρ_j at time t_j, $j \geq 2$ must begin with the same edges as the path ρ_{j-1}. It seems reasonable to choose the prior probabilities of the paths satisfying this basic requirement on the assumption that vehicles will take the shortest and most direct route to their destination. Let $L(\rho_1,\, \rho_2)$ be the length of the extension between ρ_1 and ρ_2 and $T(\rho_1,\, \rho_2)$ denote the turn penalties imposed in the extension of ρ_1 to ρ_2. Roughly speaking, the turn penalty should increase as the number of turns and their severity increases. The prior for the path extensions is then

$$\pi_0(\rho_j|\rho_{1:j-1}) \propto 1/[L(\rho_{j-1},\, \rho_j) + T(\rho_{j-1},\, \rho_j)]. \tag{4}$$

The location prior factorises as

$$\pi_0(s_{1:m}|\rho_{1:m}) = \pi_0(s_1|\rho_1) \prod_{j=2}^{m} \pi_0(s_j|s_{j-1},\, \rho_j). \tag{5}$$

Recall that ρ_1 is simply an edge. We choose the location prior at time t_1 to be a uniform distribution over this edge. The recursion of the location from time t_{j-1} to time t_j should take into account the assumed characteristics of vehicles moving through the road network. For instance, if the object is moving in a busy city it is unlikely to be moving quickly. We consider a starting location $s_1 = (r_1,\, e_1)$, path ρ and finishing location $s_2 = (r_2,\, e_2)$. The path ρ must be such that it includes the edge e_1. If the edge e_2 is not the same as the ending edge of ρ then the location s_2 has zero probability. Assuming that e_2 is the ending edge of ρ, let k denote the number of edges in ρ between e_1 and the ending edge. The ith such edge is denoted as ε_i and has length $\ell_i = \ell(\varepsilon_i)$. The time taken to travel along the ith edge of ρ is assumed to be drawn from the density $\mathsf{N}(\cdot; \overline{T}_i, \kappa_i)$. The dependence of these quantities on ρ is not reflected in the notation for the sake of brevity. Then, for $e_2 = \varepsilon_k$, the location prior can be found as

$$\pi_0(s_2|\rho, s_1) = \frac{\mathsf{N}\left(t_j - t_{j-1}; \widetilde{T} + \dfrac{r_2\overline{T}_k}{\ell_k}, \widetilde{\kappa} + \dfrac{r_2^2\kappa_k}{\ell_k^2}\right)\chi_{[0,\ell_k]}(r_2)}{C}, \tag{6}$$

where $\chi_A(\cdot)$ is the indicator function on the set A and

$$\widetilde{T} = (\ell_1 - r_1)\overline{T}_1/\ell_1 + \sum_{i=2}^{k-1} \overline{T}_i \tag{7}$$

$$\widetilde{\kappa} = (\ell_1 - r_1)^2\kappa_1/\ell_1^2 + \sum_{i=2}^{k-1} \kappa_i \tag{8}$$

$$C = \int_0^{\ell_k} \mathsf{N}\left(t_j - t_{j-1};\, \widetilde{T} + r\overline{T}_k/\ell_k,\, \widetilde{\kappa} + r^2\kappa_k/\ell_k^2\right)\, \mathrm{d}r. \tag{9}$$

3. BAYESIAN PATH ESTIMATION ALGORITHM

Given the prior densities and measurement model described above inference over the paths can, in principle, be performed by computing (1). In practice, this cannot be done exactly and approximations are sought. An efficient importance sampling approach is proposed in which the posterior density is approximated by drawing samples from an importance density and assigning appropriate weights. The importance sampling approximation is developed under the following assumptions:

A1 The road network is composed of straight line segments.

A2 The measurement noise covariance matrix is diagonal.

Assumption A1 can be satisfied with arbitrary accuracy by suitably selecting the vertices of the network. For instance, on curved segments the vertices should be close to each other. Assumption A2 is made to simplify the derivations and can be relaxed.

3.1. Importance sampling

It is convenient to re-formulate the problem as one of having to estimate both the paths $\rho_{1:m}$ and the object locations $s_{1:m}$. The joint posterior of the paths and locations is

$$\pi_m(\rho_{1:m},\, s_{1:m}) \propto \prod_{j=1}^{m} \mathsf{N}(\mathbf{y}_j;\, \mathbf{x}(s_j),\, \mathbf{R})\, \pi_0(\rho_{1:m},\, s_{1:m}). \tag{10}$$

The object locations $s_{1:m}$ may be viewed as auxiliary variables, the sampling of which simplifies the sampling of the paths. The importance sampling approximation to the posterior is obtained by drawing $(\rho^i_{1:m},\, s^i_{1:m}) \sim q(\cdot)$ for $i = 1, \ldots, n$, where n is the sample size, and computing weights

$$w^i \propto \frac{\prod_{j=1}^{m} \mathsf{N}(\mathbf{y}_j;\, \mathbf{x}(s^i_j),\, \mathbf{R})\, \pi_0(\rho^i_{1:m},\, s^i_{1:m})}{q(\rho^i_{1:m},\, s^i_{1:m})}. \tag{11}$$

Quantities of interest related to the paths can be computed from the samples and associated weights. For instance, the posterior probability of a path ρ can be approximated as

$$\hat{\pi}_m(\rho) = \sum_{\{i:\rho^i_m=\rho\}} w^i. \tag{12}$$

The most probable path can be used as an estimate of the path taken by the object, i.e., $\hat{\rho} = \arg\max_\rho \hat{\pi}_m(\rho)$.

Selecting a suitable importance density is the key to obtaining an accurate Monte Carlo approximation with a reasonable sample size. In the following sections we construct an importance density by considering the measurements sequentially. The structure of the problem is exploited to obtain an importance density which makes effective use of the measurements and remains easy to draw samples from.

3.2. The optimal sequential sampling density

Sequential sampling of the paths and locations is achieved by selecting the importance density to factorise as

$$q(\rho_{1:m},\, s_{1:m}) = q(\rho_1,\, s_1) \prod_{j=2}^{m} q(\rho_j,\, s_j | \rho_{1:j-1},\, s_{1:j-1}). \tag{13}$$

Following [13], the importance density in which each factor of (13) is proportional to the corresponding factor in the posterior (1) is referred to as the optimal importance density. At time t_1, sample paths are drawn from

$$q(\rho_1) \propto \pi_0(\rho_1) \int \mathsf{N}(\mathbf{y}_1;\, \mathbf{x}(s_1),\, \mathbf{R}) \pi_0(s_1|\rho_1)\, \mathrm{d}s_1 \tag{14}$$

$$q(s_1|\mathbf{y}_1) \propto \mathsf{N}(\mathbf{y}_1;\, \mathbf{x}(s_1),\, \mathbf{R}) \pi_0(s_1|\rho_1). \tag{15}$$

The importance density at time t_j, $j \geq 2$ is defined by the two conditional densities:

$$q(\rho_j|\rho_{1:j-1},\, s_{1:j-1}, \mathbf{y}_{1:j}) \propto \pi_0(\rho_j|\rho_{j-1})\gamma_j(\rho_j) \tag{16}$$

$$q(s_j|\rho_{1:j},\, s_{1:j-1},\, \mathbf{y}_{1:j}) \propto \mathsf{N}(\mathbf{y}_j;\, \mathbf{x}(s_j),\, \mathbf{R}) \pi_0(s_j|s_{j-1},\, \rho_j), \tag{17}$$

where

$$\gamma_j(\rho) = \int \mathsf{N}(\mathbf{y}_j;\, \mathbf{x}(s_j),\, \mathbf{R}) \pi_0(s_j|s_{j-1},\, \rho)\, \mathrm{d}s_j. \tag{18}$$

The importance weight for the ith sample can be found from (11) and (14)-(17) as

$$w^i \propto \prod_{j=2}^{m} \sum_{\rho_j} \pi_0(\rho_j|\rho^i_{j-1}) \gamma_j(\rho_j). \tag{19}$$

The optimal sampling density described here cannot be used in general because the integral (18) and the density (17) are not necessarily tractable. In the following subsection a computationally efficient but accurate approximation is developed.

3.3. An approximation to the optimal sequential sampling density

We develop an approximation to the optimal sequential importance density under assumptions A1 and A2. Since each edge e is a straight line segment it can be characterised by a starting point $\boldsymbol{\xi}_e = [x_e, y_e]'$, orientation θ_e and length $\ell(e)$. The following two results are used.

Proposition 1. *For $\mathbf{y} = [x, y]'$ and $s = (r, e)$, the likelihood can be written as*

$$\mathsf{N}(\mathbf{y};\, \mathbf{x}(s),\, \mathbf{R}) = u_e \mathsf{TN}(r;\, \mu_e,\, v,\, [0, \ell(e)]), \tag{20}$$

where $\mathsf{TN}(\cdot;\, \mu,\, \sigma^2,\, A)$ is the density of a normal random variable with mean μ and variance σ^2 truncated over the set A, and

$$u_e = K_e \times \begin{cases} \sec\theta_e \mathsf{N}(y - y_e;\, (x - x_e)\tan\theta_e,\, v\sec^2\theta_e), \\ \qquad\qquad\qquad\qquad \cos\theta_e \neq 0, \\ \operatorname{cosec}\theta_e \mathsf{N}(x - x_e;\, (y - y_e)\cot\theta_e, v \operatorname{cosec}^2\theta_e), \\ \qquad\qquad\qquad\qquad \cos\theta_e = 0. \end{cases} \tag{21}$$

with

$$K_e = \int_0^{\ell(e)} \mathsf{N}(r; \mu_e, v)\, dr \tag{22}$$

Proof. See [15]. □

The location prior given in (6) cannot be combined with the likelihood to give a closed-form sampling density because the variance depends on the location r_2. To construct the importance density we use the following approximation to the location prior density, for $s_2 = (r_2,\, e_2)$, $e_2 = \varepsilon_k$,

$$\begin{aligned} \hat{\pi}_0(s_2|\rho,\, s_1) &= \frac{\mathsf{N}(t_j - t_{j-1};\, r_2\bar{T}_k/\ell_k + \widetilde{T},\, \widetilde{\kappa} + \kappa_k)\chi_{[0,\, \ell_k]}(r_2)}{C_1} \\ &= \mathsf{TN}(r_2;\, \alpha_k,\, \lambda_k,\, [0,\, \ell_k]), \end{aligned} \tag{23}$$

where

$$C_1 = \int_0^{\ell(\varepsilon_k)} \mathsf{N}(r;\, \alpha_k,\, \lambda_k)\, \mathrm{d}r \tag{24}$$

$$\alpha_k = \ell_k(t_j - t_{j-1} - \widetilde{T})/\bar{T}_k \tag{25}$$

$$\lambda_k = \ell_k^2(\widetilde{\kappa} + \kappa_k)/\bar{T}_k^2. \tag{26}$$

Recall that ℓ_k is the length of the final edge in the path ρ. The approximation (23) removes the dependence of the variance on the location. This doesn't prevent asymptotic convergence as $n \to \infty$ provided samples are weighted according to (11).

Using the Gaussian product lemma [16] and Proposition 1, an approximation to the optimal sampling density which makes use of (23) can be found as, for $s_2 = (r_2,\, e_2)$, $e_2 = \varepsilon_k$,

$$\begin{aligned} q(s_2|\rho,\, s_1, \mathbf{y}) &\propto \mathsf{N}(\mathbf{y};\, \mathbf{x}(s_2),\, \mathbf{R})\hat{\pi}_0(s_2|\rho,\, s_1) \\ &\propto \mathsf{TN}(r_2;\, \nu_{e_2},\, \zeta_k,\, [0,\, \ell_k]), \end{aligned} \tag{27}$$

where

$$\nu_e = (v\alpha_k + \lambda_k\mu_e)/(v + \lambda_k) \tag{28}$$

$$\zeta_k = \lambda_k v/(v + \lambda_k). \tag{29}$$

The path is chosen by sampling from

$$q(\rho|\varrho,\, \mathbf{y}) \propto \pi_0(\rho|\varrho)\hat{\gamma}(\rho), \tag{30}$$

where $\hat{\gamma}(\rho)$ is an approximation of (18),

$$\hat{\gamma}(\rho) = u_{\varepsilon_k}\mathsf{N}(\mu_{\varepsilon_k};\, \alpha_k,\, v + \lambda_k)C_2/C_1, \tag{31}$$

with

$$C_2 = \int_0^{\ell(\varepsilon_k)} \mathsf{N}(r;\, \nu_k,\, \zeta)\,\mathrm{d}r. \tag{32}$$

For the first measurement, recall that ρ_1 is composed of only a single edge so that the location prior is, for $s_1 = (r_1,\, e_1)$, $e_1 = \rho_1$,

$$\pi_0(s_1|\rho_1) \propto \chi_{[0,\, \ell(\rho_1)]}(r_1). \tag{33}$$

Then, under the assumption that the road network is made of straight line segments, the location sampling density is, for $s_1 = (r_1,\, e_1)$, $e_1 = \rho_1$,

$$\begin{aligned} q(s_1|\rho_1,\, \mathbf{y}_1) &\propto \mathsf{N}(\mathbf{y}_1;\, \mathbf{x}(s_1),\, \mathbf{R})\chi_{[0,\, \ell_{\rho_1}]}(r_1) \\ &\propto \mathsf{TN}(r_1;\, \mu_{\rho_1},\, v,\, [0,\, \ell_{\rho_1}]). \end{aligned} \tag{34}$$

where we have used Proposition 1. The path sampling probabilities are

$$\begin{aligned} q(\rho_1) &\propto \int \mathsf{N}(\mathbf{y}_1;\, \mathbf{x}(s_1),\, \mathbf{R})\pi_0(s_1|\rho_1)\,\mathrm{d}s_1\, \pi_0(\rho_1) \\ &= K_{\rho_1} u_{\rho_1} \pi_0(\rho_1)/\ell(\rho_1), \end{aligned} \tag{35}$$

Finally, the sample weights are

$$w^i \propto \prod_{j=2}^{m} \frac{\pi_0(s_j^i|\rho_j^i,\, s_{j-1}^i)}{\hat{\pi}_0(s_j^i|\rho_j^i,\, s_{j-1}^i)} \sum_{\rho_j} \pi_0(\rho_j|\rho_{j-1}^i)\hat{\gamma}_j(\rho_j). \tag{36}$$

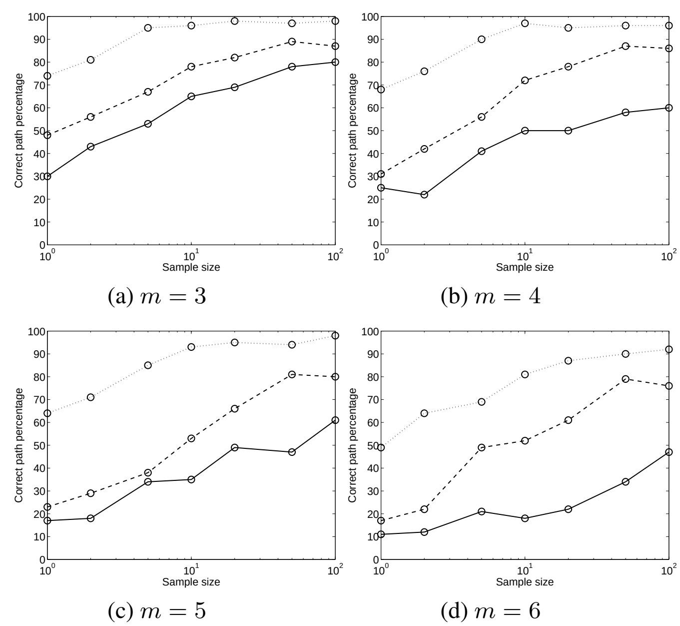
Note that the approximations used for the location priors at times $t_2, \ldots, t_m$ are accounted for in the weight update. These approximations over-estimate the uncertainty in the location prior to a degree which decreases as the length of the path increases. The k-shortest paths algorithm [17] is used to construct a set of candidate paths between each pair of measurements. Sampling is performed only over these paths rather than the whole set of paths.

4. PERFORMANCE ANALYSIS

The road network used to test the performance of the path estimation algorithm is a regular grid consisting of 8×8 nodes with a uniform edge length of 100 m. In the prior, the duration along each edge is Gaussian distributed with mean $\overline{T}_i = 10$ s and $\kappa_i = 4$. Measurements of the object positions are Gaussian distributed with zero bias and standard deviation 5 m.

We consider the effect of path length and measurement sampling rate on algorithm performance, as measured by the number of times the correct path is estimated over 100 realisations. Paths and position measurements are randomly generated for each realisation. The correct path percentage is plotted against sample size in Fig. 1 for $m = 3, 4, 5$ and 6 measurements with measurement sampling rates of 10, 15 and 20 s. The following points are of interest:

- Since the mean travel duration for an edge is 9 s, a sampling rate of 10 s should result in a measurement being obtained along most edges traversed by the object. Not surprisingly, the correct path is then estimated with high probability, at least for sample sizes greater than 10. For the larger sampling period, where position measurements are not available along all edges, performance is significantly worse.
- It may be expected that the ability to correctly estimate the path would depend primarily on the sparsity of the measurements. While this may be true of the achievable performance, algorithm performance is significantly affected by the length of the path. This can be seen by comparing the results obtained for $m = 3, 4, 5$ and 6. Performance clearly deteriorates as the number of measurements, and therefore the length of the path, increases. This can be attributed to an increase in the dimension over which paths, and positions, are sampled.
- The deterioration in performance as m increases becomes less marked as the sample size increases. This can be seen, for instance, by comparing the results obtained for samples sizes of 10 and 100 with a measurement sampling period of 15 s. For $n = 10$, the correct path probability achieved for $m = 6$ is 66.7% of that achieved for $m = 3$ while for $n = 100$ the corresponding percentage is 87.4%. This suggests that the performance of the optimal Bayes' estimator, obtained as the sample size $n \to \infty$, may not be affected by path length.



(a) $m = 3$ (b) $m = 4$ (c) $m = 5$ (d) $m = 6$

Fig. 1: Probability of correct path estimation plotted against sample size for (a) $m = 3$, (b) $m = 4$, (c) $m = 5$ and (d) $m = 6$ with sampling periods of 10 s (dotted), 15 s (dashed) and 20 s (solid).

5. CONCLUSION

Path estimation of an object moving in a road network has been formulated in the Bayesian framework. An efficient importance sampler, which exploits the structure in the prior density and likelihood, has been proposed to approximate the posterior path probabilities. The algorithm has been shown to perform well in simulation scenarios. Future work will involve testing with real data.

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