HISTOGRAM-PMHT WITH AN EVOLVING POISSON PRIOR

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ABSTRACT

The Histogram-Probabilistic Multi-Hypothesis Tracker (H-PMHT) is an efficient multi-target tracking approach to the Track-Before-Detect (TkBD) problem. However, it cannot adequately deal with fluctuating targets and this can degrade track management performance. By assuming an alternative measurement model based on a Poisson distribution, the H-PMHT algorithm can be re-derived to incorporate a time-correlated estimate of the component mixing terms, allowing for an improved measure for track quality.

***Index Terms*—** Track-Before-Detect

1. INTRODUCTION

The H-PMHT is an efficient TkBD algorithm [1, 2] that is based on the extension of the Probabilistic Multi-Hypothesis Tracker (PMHT) [3] to intensity modulated data. The H-PMHT inherently assumes a multi-target scenario but retains linear complexity with the number of targets.

The H-PMHT algorithm is based on the generation of a synthetic histogram by quantising the energy in the sensor data followed by the application of Expectation-Maximisation (EM) [4] mixture modeling to describe the underlying data sources. The quantisation step converts the continuous-valued measurement data in each pixel into an integer-value, which is interpreted as a *count of the number of shots* that fell in each pixel. The counts in each pixel are assumed to follow a multinomial distribution.

The H-PMHT can be applied to a wide range of problems as long as an appropriate state estimator exists to perform the maximisation step of the EM algorithm. In the case when both the target dynamics and measurement models are assumed to be linear with Gaussian noise, a Kalman Filter (KF) can be incorporated into the H-PMHT to perform the state estimation component of the algorithm [5–7]. More recently, H-PMHT has been applied to non-linear non-Gaussian problems using non-linear state estimation techniques [8, 9].

The H-PMHT employs EM methods to estimate the target states as well as each target's contribution to the overall mixture model. The target mixing proportion terms can be interpreted as the received power from each target, and can be used as a natural test statistic for track quality. Under multinomial assumptions, the H-PMHT models the mixing proportions as unknown parameters that can be time-varying or constant with time. The H-PMHT also assumes that the number of targets is known and remains invariant with time. These assumptions are too restrictive for most practical applications as targets commonly appear and disappear from surveillance regions and target signal-to-noise ratio (SNR) can fluctuate with time.

In conventional target tracking, this problem was addressed by introducing a time-correlated SNR estimate into the PMHT by imposing a dynamics model on the component mixing proportions [10]. Although an improvement in tracking accuracy was observed, the coupling of the mixing terms resulted in an exponential complexity with the number of targets. Recently, Davey [11] proposed an alternative derivation of the PMHT based on a Poisson distribution on the number of point measurements. Under a Poisson assignment model, Davey derived a time-correlated SNR estimate based on [12], to more accurately estimate track existence. An important feature of this new algorithm is that it also retains linear complexity with the number of targets.

This paper extends Davey's work to TkBD applications and is the first to include a time-correlated SNR measure in the H-PMHT to more accurately estimate track existence. As the multinomial assumption in the H-PMHT is consistent with a Poisson Point Process [13], it is possible to re-derive the H-PMHT with a Poisson assumption on the number of quantised measurements. The other benefit to this derivation is that the resulting algorithm retains linear complexity with the number of targets. This new algorithm is referred to as the Poisson H-PMHT and is shown to be consistent with the standard H-PMHT derived under multinomial assumptions.

2. H-PMHT

Assume a scenario in which a sensor observing M targets collects images $\mathbf{Z}_t = \{z_t^i\}_{i=1}^{I}$, at discrete times $t = 1 \dots T$ where z_t^i denotes the energy in the ith pixel of the sensor image at time t and I denotes the total number of observed pixels. Let x_t^m denote the state of component m at time t

for $m = 0 \dots M$. As a component can be attributed to either a clutter or target object, let $m = 0$ denote the clutter component with known distribution $G_0(\tau)$. Assume that the remaining components $m = 1, \dots, M$ are target objects that evolve according to a known process that may be non-linear and stochastic.

In the first step of the H-PMHT algorithm, the energy in each measurement pixel i is quantised to an arbitrary quantisation level. Define the quantised vector corresponding to $\mathbf{Z}_t$ as $\mathbf{N}_t = \{\mathbf{n}_t^i\}_{i=1}^I$, where $\mathbf{n}_t^i$ denotes the quantised energy or count of the number of shots in pixel i at time t. The variables to be estimated are the component states $\mathbf{X} = \boldsymbol{x}_{1:T}^{0:M}$ and their associated mixing proportions $\Pi = \pi_{1:T}^{0:M}$, for all components m and time scans t. The target mixing proportions can be interpreted as the received power of component m at time t.

Measurement Model: The counts in the quantised image are assumed to be multinomial distributed where each shot is assumed to be an independent identically distributed (iid) random variable following a distribution defined by the density function, $f(\tau|\boldsymbol{x}_t^{0:M};\pi_t^{0:M})$. The probability of any shot falling into pixel i is given by,

$$f^i\left(\boldsymbol{x}_t^{0:M};\pi_t^{0:M}\right) = \pi_t^0 h^i(\emptyset) + \sum_{m=1}^{M} \pi_t^m h^i(\boldsymbol{x}_t^m), \qquad (1)$$

where $h^i(\boldsymbol{x}_t^m)$ denotes the probability that a shot due to target m falls in pixel i and is defined as the integral of the point spread function $h\left(\tau|\boldsymbol{x}_t^m\right)$ over B_i, the spatial extent of pixel i. Similarly, $h^i(\emptyset)$ denotes the probability of a clutter shot falling in pixel i: the integral of $G_0(\tau)$ over pixel i. The mixing proportions π_t^m form a probability vector, i.e. $\pi_t^m \geq 0$ and $\sum_{m=0}^{M} \pi_t^m = 1$.

The calculation of the maximum likelihood estimates (MLEs) are infeasible due to the exponential complexity of enumerating over all data associations of shots to targets. The solution is to employ the EM method, which is a general iterative procedure for calculating the MLE given missing data. The sensor image does not identify which component of the mixture gave rise to each shot or the precise location of the shot within the pixel. In addition, H-PMHT allows for unobserved pixels that are notionally sensor pixels for which no data was collected. These variables are treated as missing data and EM is used to marginalise them out of the problem and optimise for $\mathbf{X}$ and Π. Note that the use of the quantised measurement $\mathbf{N}_t$ rather than $\mathbf{Z}_t$ is only an intermediate step in the derivation of the algorithm. Having defined an EM algorithm at a prescribed quantisation, the derivation then takes the limit of the quantisation and the original sensor data $\mathbf{Z}_t$ is recovered.

At a given time step t, let $\hat{\boldsymbol{x}}_t^m$ and $\hat{\pi}_t^m$ denote the states and mixing proportions at the previous EM iteration, respetively. Then, the H-PMHT algorithm employs an iterative procedure to determine the probability of the missing data (E-step) and then refines the component and mixing proportions estimates (M-step). The key steps in the H-PMHT algorithm can be summarised as follows:

E-Step: The E-step evaluates the conditional expectation of the logarithm of the complete data likelihood with respect to the missing data. This is given by the auxiliary function, which can be decomposed into two separate expressions for individually estimating $\mathbf{X}$ and Π:

$$Q_X^m = \log\left\{p(\boldsymbol{x}_0^m)\right\} + \sum_{t=1}^{T} \frac{||\mathbf{Z}_t||}{F_t} \log\left\{p(\boldsymbol{x}_t^m|\boldsymbol{x}_{t-1}^m)\right\}$$
$$+ \sum_{t=1}^{T}\sum_{i=1}^{I} \frac{\hat{\pi}_t^m \bar{z}_t^i}{f^i\left(\hat{\boldsymbol{x}}_t^{1:M};\hat{\pi}_t^{0:M}\right)} \int_{B_i} h(\tau|\hat{\boldsymbol{x}}_t^m) \log\left\{h(\tau|\boldsymbol{x}_t^m)\right\} d\tau, \qquad (2)$$

$$Q_{t\pi} = \sum_{m=0}^{M}\sum_{i=1}^{I} \frac{\hat{\pi}_t^m \bar{z}_t^i}{f^i\left(\hat{\boldsymbol{x}}_t^{1:M};\hat{\pi}_t^{0:M}\right)} h^i(\hat{\boldsymbol{x}}_t^m) \log\{\pi_t^m\}, \qquad (3)$$

where $F_t = \sum_{i=1}^{I} h^i\left(\hat{\boldsymbol{x}}_t^{1:M}\right)$, which is generally very close to unity and $||\mathbf{Z}_t|| = \sum_{i=1}^{I} \boldsymbol{z}_t^i$ denotes the total energy received from the image. In addition, define $\bar{z}_t^i$ to be

$$\bar{z}_t^i = \begin{cases} \boldsymbol{z}_t^i & i \in \mathcal{O}, \\ ||\mathbf{Z}_t|| & i \in \bar{\mathcal{O}}, \end{cases} \qquad (4)$$

where $\mathcal{O}$ is the set of all observed pixels and $\bar{\mathcal{O}}$ is the set of all unobserved pixels, which may be empty [9].

M-Step: The auxiliary function Q_X^m is maximised using an appropriate state estimator. An analytic expression for the mixing proportion estimates can be found by employing the Lagrangian multiplier method to maximise (3) subject to the normalisation constraint $\sum_{m=0}^{M} \pi_t^m = 1$ such that,

$$\pi_t^m = p_t^m \Big/ \sum_{m=0}^{M} p_t^m, \qquad (5)$$

where $p_t^m = \hat{\pi}_t^m \sum_{i=1}^{I} \frac{\bar{z}_t^i h^i(\hat{\boldsymbol{x}}_t^m)}{f^i\left(\hat{\boldsymbol{x}}_t^{1:M};\hat{\pi}_t^{0:M}\right)}$.

3. POISSON H-PMHT

The standard H-PMHT makes the implicit assumption that the number of shots $||\mathbf{N}_t|| = \sum_{i=1}^{I} \mathbf{n}_t^i$ i.e. number of multinomial trials is known. This is important as it implies that the unconditional distribution of the counts $\mathbf{N}_t$ in the H-PMHT measurement image can be factored into two distributions: a multinomial distribution for the counts $\mathbf{N}_t$ across pixel categories i given that the sample size $||\mathbf{N}_t||$ is known; and a Poisson distribution for the overall total number of counts, where $||\mathbf{N}_t^m||$, the total number of measurement shots received from each component m, is Poisson distributed by the thinning property [14].

Based on this, the H-PMHT can be re-derived using an alternative measurement model by assuming that $||\mathbf{N}_t^m||$ is Poisson distributed with rate intensity parameter λ_t^m. The component mixing terms can now be estimated using λ_t^m. Let $\mathbf{\Lambda} = \lambda_{1:T}^{0:M}$ denote the collection of Poisson mixing rates for all components m and times t. These mixing rates are now unknown random variables that need to be estimated. Thus under a Poisson measurement model, the variables to be estimated are the component states $\mathbf{X}$ and their associated Poisson mixing rates $\mathbf{\Lambda}$. The unobserved measurement shots, assignments of shots to components, and the precise locations of each shot in each pixel are still considered to be missing data.

Under the Poisson measurement model, the procedure for evaluating the probability of missing data under the EM is the same as in the standard H-PMHT, except that the density $f(\tau|\boldsymbol{x}_t^{0:M}; \pi_t^{0:M})$ and per pixel probability $f^i\left(\boldsymbol{x}_t^{0:M}; \pi_t^{0:M}\right)$ in the standard H-PMHT are now replaced with an intensity function and per-pixel shot intensity. For further details regarding the derivation of the Poisson H-PMHT, refer to [15].

Measurement Model: In a similar way to the standard H-PMHT, an expression for the underlying intensity in terms of a mixture model can be formed. The shots now have a distribution defined by the *intensity* function, $\mathbb{f}_t(\tau|\boldsymbol{x}_t^{1:M}; \lambda_t^{0:M})$. The intensity of shots falling into pixel i is now given by,

$$\mathbb{f}_t^i\left(\boldsymbol{x}_t^{0:M}; \lambda_t^{0:M}\right) = \lambda_t^0 h^i(\emptyset) + \sum_{m=1}^{M} \lambda_t^m h^i(\boldsymbol{x}_t^m). \qquad (6)$$

We can see that the multinomial mixing proportions π_t^m in the original H-PMHT mixture density have been replaced with the Poisson mixing rates λ_t^m. The key steps for the Poisson H-PMHT algorithm can be summarised as follows:

E-Step: Again, we can decompose the auxiliary function into two separate expressions for estimating the component states $\mathbf{X}$ and the Poisson mixing rates $\mathbf{\Lambda}$. The state auxiliary function Q_X^m remains unchanged and is given by (2). We can show that the auxiliary function for $\mathbf{\Lambda}$ simplifies to,

$$Q_\lambda^m = \log\left\{p(\lambda_0^m)\right\} + \sum_{t=1}^{T}\Big[\log\{p(\lambda_t^m|\lambda_{t-1}^m)\} + \log\left\{\lambda_t^m\right\}\sum_{i=1}^{I}\mu_t^{im}\boldsymbol{z}_t^i - \lambda_t\Big]. \qquad (7)$$

M-Step: As in the standard H-PMHT, Q_X^m can be implemented using an appropriate state estimator. To estimate λ_t^m, consider a model for the prior $p(\lambda_0^m)$ and its evolution with time, $p(\lambda_t^m|\lambda_{t-1}^m)$. The well known conjugate prior for the Poisson distribution is the gamma distribution. We can show that the posterior distribution for λ_t simplifies to,

$$p(\lambda_t|\mathbf{N}_t) \propto \mathit{Gamma}(\lambda_t; \alpha_{t|t-1} + ||\mathbf{N}_t||, \beta_{t|t-1} + 1), \qquad (8)$$

where $\alpha_{t|t-1}$ and $\beta_{t|t-1}$ are the predicted parameters of the gamma distribution. It is important to realise that under this formulation, λ_t is *not* gamma distributed with updated parameters α_t and β_t, but the values of λ_t that maximise the auxiliary function (7) are the expected values of a gamma distribution, with the following mean $E[\lambda_t] = \alpha_t/\beta_t$ and variance $\text{Var}[\lambda_t] = \alpha_t/\beta_t^2$. As the EM method performs a maximisation, it is more appropriate to consider the mode of the gamma distribution, which is defined as $\operatorname{argmax} p(\lambda_t) = (\alpha_t - 1)/\beta_t$.

Granström [12] provides a framework for the prediction and update for the parameters of the measurement rate λ_t. In the prediction stage, Granström defines a forgetting factor η that determines how much weight is applied to the past estimates of α and β. As $\eta \to \infty$, the predictions place more weight on past estimates and the updated estimates will be highly correlated with time. This can be beneficial in scenarios in which the target SNR is buried under a high level of noise, as it is allows for a better average estimate of a target's true SNR. In the case when the limit of $\eta \to 0$, the predicted gamma parameters also go to zero, and the updated estimates will be effectively uncorrelated with time. In this case, the Poisson mixing rates can be approximated by,

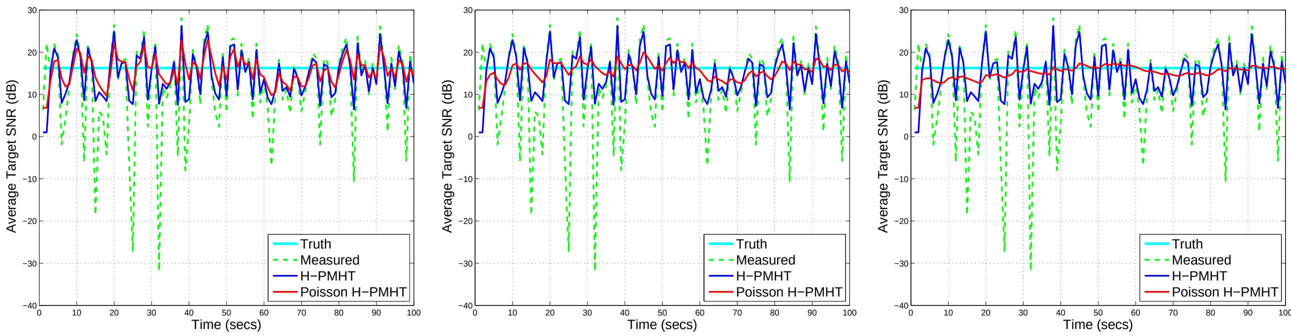
$$\lambda_t^m = \hat{\lambda}_t^m \sum_{i=1}^{I} \frac{\boldsymbol{z}_t^i h^i(\hat{\boldsymbol{x}}_t^m)}{\mathbb{f}_t^i\left(\hat{\boldsymbol{x}}_t^{1:M}; \hat{\lambda}_t^{0:M}\right)}, \qquad (9)$$

where $\hat{\lambda}_t^m$ is the Poisson mixing rate at the previous EM iteration. Observe that if we replace the intensity function $\mathbb{f}_t^i\left(\boldsymbol{x}_t^{1:M}; \lambda_t^{0:M}\right)$ with the probability $f^i\left(\boldsymbol{x}_t^{1:M}; \pi_t^{0:M}\right)$, (9) is equivalent to the un-normalised multinomial mixing proportions p_t^m in the standard H-PMHT. We can see that the H-PMHT algorithm formed under a Poisson measurement model generalises the H-PMHT under multinomial assumptions through the parameter η.

As we have imposed a dynamics model on the Poisson mixing rates, it is expected that the smoothed estimates of λ_t^m will provide a more robust measure of track quality than its multinomial counterpart π_t^m. Note that the multinomial mixing proportions π_t^m are dependent through a normalisation process, while the Poisson mixing rates λ_t^m are calculated independently due to the Poisson thinning property. This is important for implementation, as it allows the H-PMHT algorithm under the Poisson measurement model to retain linear complexity with the number of targets.

4. SIMULATIONS

This section demonstrates and verifies the performance of the Poisson H-PMHT for a simulated single-target linear Gaussian scenario. It is assumed that the target amplitude follows a Swerling I model where the average target SNR was set to 17 dB. As we are only considering a linear Gaussian scenario, it is sufficient to verify the Poisson H-PMHT using a KF for the target state estimation step. A KF implementation can be easily integrated into the algorithm in the same way as in the



(a) $\eta = 1$ (b) $\eta = 3$ (c) $\eta = 10$

Fig. 1. **Swerling I Scenario:** Comparison of the average target SNR for a single run for the standard H-PMHT and Poisson H-PMHT for varying forgetting factor η.

standard H-PMHT. For both algorithms, it was found that ten iterations was sufficient to ensure EM convergence.

The average target SNR estimates for the Poisson H-PMHT were compared with the standard H-PMHT. Figure 1 shows the estimated SNR in dB for each algorithm for a single run. The true average target SNR is shown as a solid cyan line and the instantaneous measured target is in dashed green. The Poisson H-PMHT was implemented using a forgetting factor of $\eta = 1, 3$ and 10 to investigate the effect of varying the contribution of past estimates on updated estimates. Note that the performance of the standard H-PMHT is independent of the forgetting factor η. Both algorithms give similar performance when $\eta = 1$, however as η increases, the SNR estimates from the Poisson H-PMHT become smoother, giving a better estimate of the true average target SNR.

When the target amplitude model features instantaneous fluctuations, the Poisson H-PMHT clearly provides a more stable prediction of the average target SNR for large η values. Due to the dynamics model imposed on the Poisson mixing rates λ_t^m, the SNR estimates are slower to respond to random fluctuations in the observed target SNR. This can be beneficial for track confirmation as it allows for a more stable test statistic for track quality. In contrast, the H-PMHT average SNR estimates are susceptible to variations in the measurement noise as it assumes there is no correlation with time.

By allowing for a forgetting factor term, the Poisson H-PMHT is also able to reduce the track SNR variance estimates by performing smoothing over a larger time window. This is evident in Figure 2, which shows the estimated track SNR variance versus forgetting factor η, averaged over 100 Monte Carlo runs and over all time scans. As stated earlier, the performance of the standard H-PMHT is independent of η and thus its variance remains constant in Figure 2. In contrast, the Poisson H-PMHT decreases as η increases and consistently gives smaller variance estimates than the standard H-PMHT. Figure 2 also shows that as $\eta \to 0$, the performance of the Poisson H-PMHT converges to the standard H-PMHT. This further verifies that for small η values, the

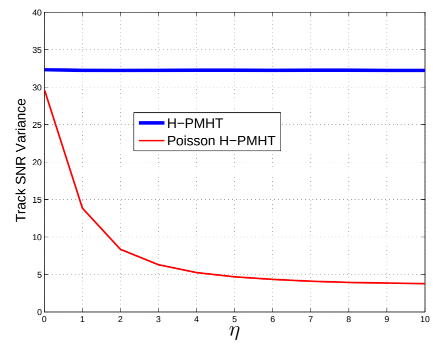


Fig. 2. Track SNR variance versus forgetting factor η for the standard H-PMHT and Poisson H-PMHT averaged over 100 Monte Carlo runs and assuming a Swerling I amplitude model.

Poisson H-PMHT performance is equivalent to the standard H-PMHT.

5. SUMMARY

In this paper, we presented an alternative derivation of the H-PMHT based on a Poisson measurement model and showed that the resulting algorithm is consistent with the standard H-PMHT derived under multinomial assumptions. This new algorithm is referred to as the Poisson H-PMHT and imposes a dynamics model on the Poisson measurement rate parameter to allow for a randomly evolving mean target amplitude state with instantaneous fluctuations. This enhancement can be used to improve track management performance. The Poisson H-PMHT is shown to be a generalisation of the standard H-PMHT through a forgetting factor term.

For a simulated scenario featuring a fluctuating target amplitude model, the Poisson H-PMHT is shown to provide a more consistent measure for track quality with smaller variance estimates through the forgetting factor term.

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