GENERAL SOLUTION AND APPROXIMATE IMPLEMENTATION OF THE MULTISENSOR MULTITARGET CPHD FILTER

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ABSTRACT

Random finite set (RFS) based filters such as the *cardinalized probability hypothesis density* (CPHD) filter have been successfully applied to the problem of single sensor multitarget tracking. Various multisensor extensions of these filters have been proposed in the literature, but exact update equations for the multisensor CPHD filter have not been identified. In this paper, we provide the update equations and propose an approximate implementation. The exact implementation of the multisensor CPHD filter is infeasible even for very simple scenarios. We develop an algorithm that greedily searches for the most likely groups of measurement subsets. This enables a computationally tractable implementation. Numerical simulations are performed to compare the proposed filter implementation with other random finite set based filters.

***Index Terms*—** random finite sets, CPHD filter, multisensor processing, multitarget tracking

1. INTRODUCTION

In this paper we address the problem of multitarget state estimation using the measurements generated by multiple sensors. In the *random finite set* (RFS) framework the multitarget state and sensor measurements are modelled as realizations of random finite sets. Various filters have been developed within this framework for the case when measurements are generated by a single sensor. Prominent examples include the *probability hypothesis density* (PHD) filter [1] and the *cardinalized probability hypothesis density* (CPHD) filter [2]. Implementations of these filters have been successfully applied to single sensor tracking [3–5], but there has been less progress in developing accurate and computationally tractable multisensor filters.

The general multisensor PHD filter was first developed by Mahler [6], but equations were only presented for the two-sensor case. Delande et al. [7] extended the result to apply to any number of sensors. Exact implementation is infeasible due to the combinatorial nature of the filter. Further simplifications or approximations are required for a computationally tractable algorithm. Delande et al. [8, 9] have provided filter equations with reduced computational complexity when there is limited overlap in the fields-of-view of the different sensors. They describe a particle filter based implementation. Jian et al. [10] proposed to implement the general multisensor PHD filter by repetitive application of a two-sensor PHD filter, but the proposed extension to multiple sensors is unclear. In the *iterated-corrector PHD filter* [11] the multisensor information is processed in a sequential manner. Measurements from the first sensor are processed by the single sensor PHD filter. The output PHD produced by this step is used as the predicted PHD when processing measurements from the second sensor and so on. A drawback of this approach is that the final result depends on the order in which sensors are processed [12]. An iterated-corrector CPHD filter can be constructed in a similar fashion. To address the issue of sensor order dependence, Mahler proposed the *product multisensor PHD and CPHD filters* [13]. Although the final results are independent of sensor order, Ouyang and Ji [14] have reported that Monte Carlo implementations of these filters are unstable and the problem worsens as the number of sensors increases. We have observed a similar instability in Gaussian mixture model based implementations.

We make two main contributions in this paper. First, we provide exact update equations for the general multisensor CPHD filter. Due to space restrictions we do not include the derivations, but we provide them in [15]. Exact implementation of the filter is computationally infeasible. In our second contribution, we develop an approximate, computationally tractable implementation using a Gaussian mixture model. The filter estimates at each time step both the number of targets and the target state values. We use greedy algorithms to identify the most likely groupings of the subsets of measurements associated with each target. By processing only these groupings, we can drastically reduce the computational overhead without experiencing significant degradation in tracking performance.

1.1. Problem Statement

We describe the multisensor multitarget tracking problem in this section. Let the individual target state at time k be denoted by $\mathbf{x}_{k,i} \in R^{g_\mathbf{x}}$ for the i^{th} target. The multitarget state is given by the set $X_k = \{\mathbf{x}_{k,1}, \dots \mathbf{x}_{k,n_k}\}$ where $n_k \geq 0$ is the unknown number of targets present at time k. The single target state is assumed to evolve according to the Markovian transition kernel $f_{k+1|k}(\mathbf{x}_{k+1,i}|\mathbf{x}_{k,i})$. Let $b_k(\mathbf{x})$ be the target birth intensity function at time k and $p_{sv,k}(\mathbf{x})$ be the single target survival probability.

Information about the multitarget state is available from s independent sensors. Let $Z_k^j = \{\mathbf{z}_{1,k}^j, \mathbf{z}_{2,k}^j, \dots, \mathbf{z}_{m_{j,k}}^j\}$ be the measurement set generated by the j-th sensor at time step k. The measurement set can be empty. Denote by $Z_k^{1:s}$ the collection of measurement sets generated by all sensors at time k, i.e., $Z_k^{1:s} = \{Z_k^1, Z_k^2, \dots, Z_k^s\}$. If a target is present at location $\mathbf{x}$, sensor j detects it with probability of detection $p_{d,k}^j(\mathbf{x})$ and generates a measurement $\mathbf{z}$ with probability density (likelihood function) given by $h_{j,k}(\mathbf{z}|\mathbf{x})$. Denote the probabilty of a missed detection as $q_{d,k}^j(\mathbf{x}) = 1 - p_{d,k}^j(\mathbf{x})$.

We are interested in forming an estimate $\widehat{X}_k$ of the multitarget state at time k given all the measurements up until time k by all the s sensors denoted by $Z_{1:k}^{1:s} = \{Z_1^{1:s}, Z_2^{1:s}, \dots, Z_k^{1:s}\}$. More generally, we would like to estimate the posterior multitarget state distribution $f_{k|k}(X_k|Z_{1:k}^{1:s})$. In a CPHD filter setting, we assume that the posterior multitarget state distribution can be approximated using an *independent and identically distributed cluster* (IIDC) process.

2. GENERAL MULTISENSOR CPHD FILTER

In this section we present the general multisensor CPHD filter. The filter derivation is based on the following modelling assumptions: *(i)* the predicted multitarget distribution at time $k+1$ is IIDC; *(ii)* the sensor observation processes are independent conditional on the multitarget state X_{k+1}, and the sensor clutter processes are IIDC; *(iii)* each target generates at most one measurement per sensor at each time instant; and *(iv)* each measurement is either associated with one target or is generated by clutter.

We introduce some notation and quantities before presenting the filter equations. Let $c_{k+1,j}(\mathbf{z})$ be the clutter spatial distribution and $C_{k+1,j}(y)$ be the *probability generating function* (PGF) of the clutter cardinality distribution of the j-th sensor at time $k+1$. Let $D_{k+1|k}(\mathbf{x})$ denote the predicted PHD function and $r_{k+1|k}(\mathbf{x})$ denote the normalized predicted PHD function at time $k+1$ (normalized so that it integrates to one). Let the PGF of the predicted cardinality distribution $p_{k+1|k}(n)$ be denoted by $G_{k+1|k}(y)$. For brevity we drop the time index and denote

$$c_{k+1,j}(\mathbf{z}) \equiv c_j(\mathbf{z}), \quad C_{k+1,j}(y) \equiv C_j(y), \quad p^j_{d,k+1}(\mathbf{x}) \equiv p^j_d(\mathbf{x})$$
$$G_{k+1|k}(y) \equiv G(y), \quad q^j_{d,k+1}(\mathbf{x}) \equiv q^j_d(\mathbf{x})$$
$$h_{j,k+1}(\mathbf{z}|\mathbf{x}) \equiv h_j(\mathbf{z}|\mathbf{x}), \quad m_{j,k+1} \equiv m_j$$

Note that abbreviated notation is used only for convenience and the above quantities are in general functions of time. Throughout the entire paper, for functions $a(\mathbf{x})$ and $b(\mathbf{x})$, the notation $a[b]$ is defined as $a[b] = \int a(\mathbf{x})\, b(\mathbf{x})\, d\mathbf{x}$. We use the notation $[\![1,s]\!]$ to denote the set of integers from 1 to s.

2.1. CPHD prediction step

The prediction step of the CPHD filter for the multisensor case is the same as that for the single sensor case. The posterior probability hypothesis density at time k is $D_{k|k}(\mathbf{x})$ and the posterior cardinality distribution is $p_{k|k}(n)$. The predicted probability hypothesis density function at time $k+1$ is given by [2,5]

$$D_{k+1|k}(\mathbf{x}) = b_{k+1}(\mathbf{x}) + \int_{R^{g_\mathbf{x}}} p_{sv}(\mathbf{w}) f_{k+1|k}(\mathbf{x}|\mathbf{w}) D_{k|k}(\mathbf{w}) d\mathbf{w}\,. \tag{1}$$

The predicted cardinality distribution at time $k+1$ is given by [2,5]

$$p_{k+1|k}(n) = \sum_{j=0}^{n} p_b(n-j) \sum_{l=j}^{\infty} \binom{l}{j} \frac{(D_{k+1|k}[p_{sv}])^j (D_{k+1|k}[1-p_{sv}])^{l-j}}{(D_{k+1|k}[1])^l} p_{k|k}(l)\,,$$

where n, j and l are non-negative integers and p_b is the cardinality distribution of the birth process.

2.2. CPHD update step

We now provide the CPHD filter update equations. For derivation of these equations please refer to the technical report [15]. Let W be any subset of the measurement set $Z^{1:s}_{k+1}$ such that it contains at most one measurement per sensor. Think of W as a possible subset of measurements at different sensors generated by the same target. Let P be a grouping of disjoint subsets W. We denote the collection of all such possible groupings by $\mathcal{P}$. We denote by $|P|$ the number of subsets W in P, and define

$$|P|_j = \left|\{\mathbf{z} \in Z^j_k : W \in P \text{ with } \mathbf{z} \in W\}\right|\,. \tag{2}$$

The quantities $C_j^{(v)}(y) = \frac{d^v C_j(y)}{dy^v}$ and $G_{k+1|k}^{(v)}(y) = \frac{d^v G_{k+1|k}(y)}{dy^v}$ are the v^{th}-order derivatives of the PGFs of the clutter cardinality distribution and the predicted cardinality distribution, respectively.

We use γ to denote the probability, under the predictive PHD, that a target is detected by no sensor, and we thus have:

$$\gamma = r_{k+1|k}\left[\prod_{j=1}^{s} q^j_d\right]. \tag{3}$$

For concise specification of the update equations, it is useful to have notation to describe the cardinality component of the weight associated with a grouping P. Let us define the following quantities:

$$\psi_P = \left(\prod_{j=1}^{s} C_j^{(m_j - |P|_j)}(0)\right) G^{(|P|)}(\gamma)\,, \tag{4}$$

$$\psi^*_P = \left(\prod_{j=1}^{s} C_j^{(m_j - |P|_j)}(0)\right) G^{(|P|+1)}(\gamma)\,, \tag{5}$$

$$\psi^n_P = \frac{n!}{(n-|P|)!}\left(\prod_{j=1}^{s} C_j^{(m_j - |P|_j)}(0)\right) \gamma^{n-|P|}\,. \tag{6}$$

Here ψ_P is the cardinality component of the weight assuming that at least $|P|$ targets are present; ψ^*_P assumes additionally that at least one of these is not detected; and ψ^n_P is the appropriate weight when n is the true cardinality of the multitarget set.

Since the measurement subset W includes at most one measurement from each sensor, we can associate with it an index set $T_W \subseteq [\![1,s]\!]$ which is the collection of sensor indexes which contribute measurements to W. Define:

$$d_W = \frac{r_{k+1|k}\left[\left(\prod_{i\in T_W} p^i_d\, h_i(\mathbf{z}^i)\right)\left(\prod_{j\notin T_W} q^j_d\right)\right]}{\prod_{i\in T_W} c_i(\mathbf{z}^i)}\,, \tag{7}$$

$$\rho_W(\mathbf{x}) = \frac{\left(\prod_{i\in T_W} p^i_d(\mathbf{x})\, h_i(\mathbf{z}^i|\mathbf{x})\right)\left(\prod_{j\notin T_W} q^j_d(\mathbf{x})\right)}{r_{k+1|k}\left[\left(\prod_{i\in T_W} p^i_d\, h_i(\mathbf{z}^i)\right)\left(\prod_{j\notin T_W} q^j_d\right)\right]}\,, \tag{8}$$

$$\alpha_0 = \frac{\sum_{P\in\mathcal{P}}\left(\psi^*_P \prod_{W\in P} d_W\right)}{\sum_{P\in\mathcal{P}}\left(\psi_P \prod_{W\in P} d_W\right)}\,, \quad \alpha_P = \frac{\psi_P \prod_{W\in P} d_W}{\sum_{P\in\mathcal{P}}\left(\psi_P \prod_{W\in P} d_W\right)}\,. \tag{9}$$

The probability hypothesis density update expression for the general multisensor CPHD filter is then given by:

$$\boxed{\frac{D_{k+1|k+1}(\mathbf{x})}{r_{k+1|k}(\mathbf{x})} = \alpha_0 \prod_{j=1}^{s} q^j_d(\mathbf{x}) + \sum_{P\in\mathcal{P}} \alpha_P \left(\sum_{W\in P} \rho_W(\mathbf{x})\right).} \tag{10}$$

The cardinality distribution update expression is given by

$$\boxed{\frac{p_{k+1|k+1}(n)}{p_{k+1|k}(n)} = \frac{\sum_{\substack{P\in\mathcal{P}\\ |P|\le n}}\left(\psi^n_P \prod_{W\in P} d_W\right)}{\sum_{P\in\mathcal{P}}\left(\psi_P \prod_{W\in P} d_W\right)}.} \tag{11}$$

3. APPROXIMATE IMPLEMENTATION OF THE GENERAL CPHD FILTER

In this section, we propose a computationally tractable implementation of the general multisensor CPHD filter. An exact implementation of the general multisensor CPHD filter is infeasible because of the prohibitively large size of the collection $\mathcal{P}$. Hence we propose the following two-step approximation to identify elements of $\mathcal{P}$ which make significant contribution to the update expressions. The first approximation is to select a few measurement subsets W for each Gaussian component that are best explained by that component. The second approximation step is to greedily construct groupings of these subsets which are significant for the update step.

3.1. Selecting the best measurement subsets

For this step we assume the following Gaussian mixture model representation of the normalized predicted PHD function

$$r_{k+1|k}(\mathbf{x}) = \sum_{i=1}^{J_{k+1|k}} w_{k+1|k}^{(i)} \mathcal{N}^{(i)}(\mathbf{x}), \tag{12}$$

where $\mathcal{N}^{(i)}(\mathbf{x})$ is a Gaussian density function. Consider the measurement subset W and the associated index set T_W as defined earlier. For the i^{th} Gaussian component and the measurement subset W we can associate a weight function $\beta^{(i)}(W)$ defined as

$$\beta^{(i)}(W) = \frac{w_{k+1|k}^{(i)} \mathcal{N}^{(i)} \left[\left(\prod_{j \in T_W} p_d^j h_j(\mathbf{z}^j) \right) \left(\prod_{j \notin T_W} q_d^j \right) \right]}{\prod_{j \in T_W} c_j(\mathbf{z}^j)}.$$

This weight can be intuitively thought of as the ratio of the likelihood that W was generated by the single target represented by the i^{th} Gaussian component and the likelihood that W was generated by the clutter process. When the measurement subset W is truly generated by the i^{th} Gaussian component, the weight $\beta^{(i)}(W)$ is high. We use $\beta^{(i)}(W)$ to rank measurement subsets for each Gaussian component and retain only a fraction of them with highest weights.

For each Gaussian component, we select the measurement subsets by randomly ordering the sensors and incrementally incorporating information from each sensor in turn. We retain a maximum of $W_{\max}$ subsets at each step. The algorithm can be visualized in the form of a trellis diagram. Figure 1 provides a pictorial representation. The nodes of the trellis correspond to the enumerated sensor observations $(1, 2, \dots)$ or the no detection case (0). Each column of the trellis corresponds to observations from one of the sensors. The sensor number is indicated at top of each column. Each path through the trellis corresponds to a different measurement subset. The sequential sensor processing can be demonstrated as follows. A measurement subset (path) retained after processing observations from sensor 3 is shown as a solid line passing through nodes 0, 1 and 1 corresponding to sensors 1, 2 and 3 respectively. When processing sensor 4 information, this path is extended for each node of sensor 4 as represented by the dashed line. The weights of these new measurement subsets (paths) are calculated using the expression for $\beta^{(i)}(W)$ but limited to only the first 4 sensors. This is done for each existing path in the sensor-measurement space and $W_{\max}$ measurement subsets with highest weights are retained.

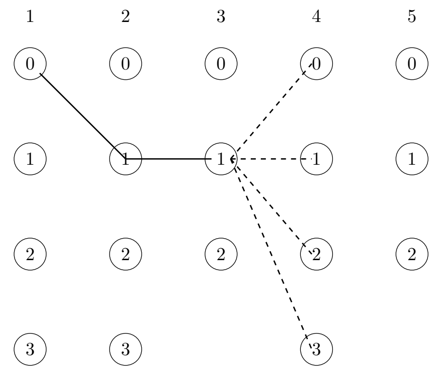


Fig. 1. Trellis diagram

3.2. Grouping of subsets

The algorithm to select groupings of subsets is similar to the above algorithm used to identify the best measurement subsets. To understand the algorithm, we can interpret the trellis diagram in Figure 1 as follows: Each column of the trellis corresponds to the set of measurement subsets identified by a Gaussian component with the component number indicated at the top for each column. The node 0 corresponds to the empty measurement subset $W = \varnothing$ which is always included for each component. We note that not all paths through this trellis correspond to a valid grouping because of the constraint that the measurement subsets within a group should be disjoint. With each valid group (path) P we associate the weight measure $\delta_P = \prod_{W \in P} d_W$ with $d_\varnothing = 1$.

We greedily identify groupings of subsets by incrementally incorporating measurement subsets from the different components. We process the components in decreasing order of their associated weights. While performing extension of paths, only those leading to a valid group are considered. After processing each component, we retain a maximum of $P_{\max}$ paths corresponding to the ones with highest δ_P. These selected groups of measurement subsets are used in the update equations (10) and (11) to compute the posterior PHD and cardinality distribution.

4. NUMERICAL SIMULATIONS

We next compare Gaussian mixture model based implementations of the different multisensor PHD and CPHD filters. We compare the iterated-corrector PHD (IC-PHD [11]), the product PHD (P-PHD [13]), the general multisensor PHD (G-PHD [6, 7]), the iterated-corrector CPHD (IC-CPHD [11]), the product CPHD (P-CPHD [13]) and the general multisensor CPHD (G-CPHD) proposed in this paper. The simulation setup is similar to the one in [12].

Target dynamics and parameters: The target tracks are simulated using the constant velocity model [12]. The single target state is a quadruple consisting of its x and y coordinates and its velocities along those axes. The targets are moving inside a $2000m \times 2000m$ square region for 100 time steps. Two targets are present initially and a third target arrives at time step $k = 66$ and stays until the end. The single target survival probability is assumed to be constant over the whole region with $p_s = 0.99$. The cardinality distribution of target births p_b is assumed Poisson with mean 0.2. The target birth intensity function $b_{k+1}(\mathbf{x})$ is a two component Gaussian mixture with weights 0.1 each and centered at $[250, 250, 0, 0]$ and

$[-250, -250, 0, 0]$ with covariance matrix diag([100, 100, 25, 25]). All targets originate from either of these two locations.

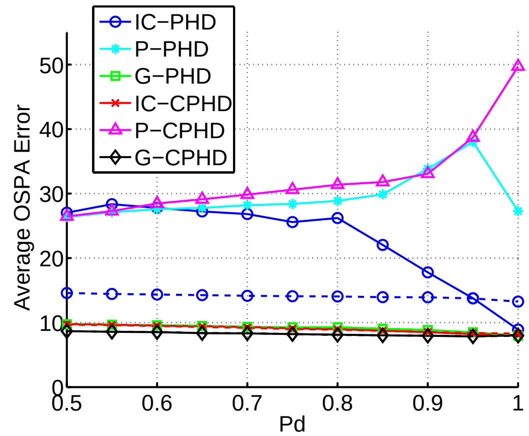
Observation model and parameters: Three sensors (s = 3) make observations about the targets' positions (x and y coordinates). For each sensor, the measurement noise when a target is detected is additive Gaussian with zero mean and covariance matrix $\sigma_r^2\mathbf{I}$ where $\mathbf{I}$ is the identity matrix and $\sigma_r^2 = 100m^2$. The clutter process for each sensor is Poisson with rate $\lambda = 10$ and a uniform clutter density over the monitoring region. Two of the sensors have a fixed probability of detection $p_d = 0.95$. The probability of detection of the third sensor is changed gradually from 0.5 to 1 in the simulations. We consider two different sensor orderings while processing each of the above filters. In Case 1 the sensor with variable probability of detection is processed last while in Case 2 it is processed first.

Filter implementation details: The algorithms use a Gaussian mixture approximation of the PHD, as first employed in [4, 5]. The simulations were conducted using MATLAB. Pruning and merging of Gaussian components is performed after processing each sensor for the iterated-corrector filters. Many of the components have very small weights and pruning them after processing each sensor has no significant effect on the tracking accuracy but greatly reduces computation time. For other filters, pruning and merging is conducted at the end of each time iteration as they process measurement subsets and we do not have access to the intermediate Gaussian components.

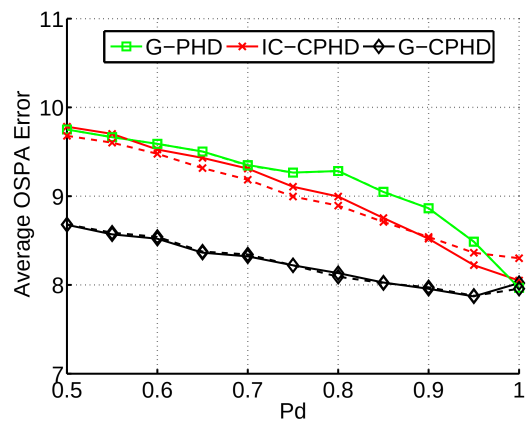
We develop novel implementations of the general multisensor PHD and CPHD filters to make them computationally tractable. For both filters, the first step is to find measurement subsets using the algorithm in Section 3.1 and we use $W_{\text{max}} = 8$. Implementation of the general multisensor PHD filter proceeds by finding all possible partitions from the given collection of measurement subsets. This problem can be mapped to the exact cover problem in computer science [16]. An efficient algorithm called Dancing Links has been suggested by Knuth [17] for solving this problem. We use an open source implementation of this algorithm in the C programming language [18]. For the general multisensor CPHD filter, we perform grouping of measurement subsets using the algorithm described in Section 3.2. We select a maximum of $P_{\text{max}} = 25$ groups of measurement subsets in our implementation. For the CPHD filters, the cardinality distribution is assumed to be zero for $n > 20$.

Results and discussion: We use the OSPA error metric [19] to compare the tracking estimates of different filters. For the OSPA metric, we set the cardinality penalty factor $c = 100$ and power $p = 1$. For the PHD filters, we estimate the number of targets by rounding the sum of weights of the Gaussian components to the nearest integer. For the CPHD filters, we estimate the number of targets as the peak of the posterior cardinality distribution. For all the filters, the target state estimates are the centres of the Gaussian components with highest weight in the posterior PHD. After each time step, we restrict the number of Gaussian components to a minimum of four and a maximum of four times the estimated number of targets.

The average OSPA error (calculated from 50 Monte Carlo simulations) is shown in Figure 2(a) for the different filters as a function of the probability of detection p_d of the variable sensor. Both the product PHD and the product CPHD filters have significantly higher error because of the unstable nature of their update equations. The IC-PHD filter is sensitive to sensor ordering; processing the sensor with low probability of detection last (Case 1) leads to a significant deterioration in its performance. For the remaining filters the impact of sensor ordering is minimal. A portion of Figure 2(a) is expanded and shown in Figure 2(b) for clarity. The G-PHD filter and the IC-CPHD filter have comparable performance. The average OSPA error is lowest for the G-CPHD filter proposed in this paper with about



(a)



(b)

Fig. 2. (a): Average OSPA error versus the probability of detection p_d of the variable sensor. The solid and dashed lines correspond to Case 1 and Case 2, respectively. (b): A zoomed-in version of the figure in (a) focusing on the IC-CPHD filter and the general multisensor PHD and CPHD filters.

10% improvement when compared to the G-PHD or IC-CPHD filters for lower values of p_d. The G-CPHD filter improves over the G-PHD filter because of the additional cardinality information. It also outperforms the IC-CPHD filter because it jointly processes groups of multisensor measurement subsets.

5. CONCLUSIONS

In this paper, we propose update equations for the probability hypothesis density and the cardinality distribution of the general multisensor CPHD filter. Since the exact equations are computationally intractable we propose an approximate Gaussian mixture model implementation of the filter. We achieve computational tractability by restricting the number of measurement subsets for each Gaussian component and further using a greedy algorithm to identify likely candidate groupings of the subsets. We demonstrated superiority of the proposed filter using numerical multitarget tracking simulations.

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