DISTRIBUTED KALMAN FILTERING WITH QUANTIZED SENSING STATE

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ABSTRACT

This paper studies a Quantized Gossip-based Interactive Kalman Filtering (QGIKF) algorithm implemented in a wireless sensor network, where the sensors exchange their quantized states with neighbors via inter-sensor communications. We show that with the information loss due to quantization, the network can still achieve weak consensus, i.e., the estimation error variance sequence at a randomly selected sensor can converge weakly (in distribution) to a unique invariant measure. To prove the weak convergence, we first interpret the error variance sequence evolution as the interacting particle, then formulate the sequence as a Random Dynamical System (RDS), and finally prove that it is stochastically bounded.

***Index Terms*—** Distributed signal processing, Kalman filter, quantization, gossip

1. INTRODUCTION

We consider a dynamic scalar estimation system with multiple sensors constructing a wireless sensor network. We seek a totally distributed sensing scheme where a reliable estimate of the scalar state process is computed at each sensor. Some sensors make observations highly corrupted by noises or even no observations due to the locations or other factors, which lead to unreliable estimations at individual sensors if without node collaborations. To mitigate this issue, the sensors may collaborate with each other, where the collaboration is achieved by inter-sensor communications within the neighborhood [1, 2]. In wireless sensor networks, quantization is usually required before the data is exchanged through inter-sensor communications, since the limited sources, such as bandwidth and power, prevent the exchange of high precision data (say, real-valued analog data) among the sensors.

Here we propose a Quantized Gossip-based Interactive Kalman Filtering (QGIKF) scheme, which is a fully distributed Kalman filtering solution with each sensor executing a local Kalman filter and at each epoch the state of a sensor (including its local estimation and error variance) being swapped with one of its neighbors via an inter-sensor communication channel. Before swapping over on the channel, the data is quantized. For quantized Kalman filtering in the literature, in [3], the observation innovation is quantized by either an iterative binary quantizer or a single-shot batch quantizer, and a recursive state estimator is introduced. In [4], the quantized Kalman filters based on quantized observations and quantized innovations are proposed, and the tradeoff between energy consumption and estimation accuracy is studied. For other quantization based estimation problems, in [5], an optimal quantization level and transmit power scheduling strategy for the decentralized estimation at local sensors in an inhomogeneous sensor network is proposed so as to minimize the total transmit power. In [6], a distributed adaptive quantization scheme is proposed to estimate the parameters, where each individual sensor node dynamically adjusts the threshold of its quantizer. In this paper, we quantize the local estimation and the corresponding error variance. The quantization procedure adds some noise on the swapped signal, such that the received state from the neighbor loses certain information. This makes the problem more challenging and different from our previous work [7, 8], where we assumed that the state of a sensor is perfectly transmitted to its neighbor, and we prove that the estimation error covariance sequence at a randomly selected sensor converges weakly (in distribution) to a unique invariant measure by following a GIKF scheme. Then a natural question to ask is whether or not the estimation error variance sequence could still achieve weak convergence with the information loss due to quantization. To seek a positive answer, in this paper we first interpret the error variance sequences as interacting particles and model each sequence evolution as a Random Dynamic System (RDS); we then prove the stochastic boundedness of the error variance sequence. Following the properties of RDS, finally we prove the weak convergence of the error variance sequence, i.e., the network achieves weak consensus.

Notation: We use $\mathbb{T}, \mathbb{T}_+, \mathbb{R}, \mathbb{R}_+$ to denote the integers, non-negative integers, reals, and non-negative reals, respectively.

The rest of the paper is organized as follows. The problem is set up in Section II. The RDS formulation is presented in Section III and our main results are presented in Section IV. Finally, Section V concludes the paper.

2. PROBLEM SETUP

2.1. System and Observation Model

We consider a discrete-time linear Gaussian dynamic scalar system observed by a network of N sensors. The system model is

$$x_{k+1} = Fx_k + w_{k+1}, \tag{1}$$

where $\{x_k\}$ is the system state sequence with an initial state x_0 distributed as a zero mean Gaussian variable with variance $\widehat{P}_{0|-1}$, and $\{w_k\}$ is the system noise sequence, which is an uncorrelated zero mean Gaussian sequence with variance Q independent of x_0.

The observation signal model at sensor n, $1 \le n \le N$, is given as

$$y_k^n = C_n x_k + v_k^n, \tag{2}$$

where C_n is the observation scaling factor and the observation noise $\{v_k^n\}$ is another uncorrelated zero mean Gaussian sequence with variance R_n. These noise sequences at different sensors are independent of each other, and independent of both the system noise sequence $\{w_k\}$ and the initial system state x_0. Suppose we require a fully distributed solution where a reliable estimate of the system state process is computed at each sensor. The observations at different sensors are different in quality due to various locations of the sensors or other factors, and some sensors may even have no observability at all as $C_n = 0$. In order to reach certain global agreement locally, the sensors need to collaborate through inter-sensor communications. We now first establish the inter-sensor communication model, then discuss the quantization scheme and the QGIKF algorithm.

2.2. Communication Model

First we model the network as an undirected graph $(V, \mathcal{E})$, where V denotes the set of N sensors and $\mathcal{E}$ denotes the set of edges (valid communication links), which means that if the sensors n and l can communicate, $\mathcal{E}$ contains the edge (n, l). The graph could be represented by its $N \times N$ adjacency matrix $\mathcal{A}$ as

$$\mathcal{A}_{nl} = \begin{cases} 1 & \text{if } (n,l) \in \mathcal{E} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

We assume that the diagonal elements of $\mathcal{A}$ are identically 1, indicating that a sensor n can always communicate to itself. Note that $\mathcal{E}$ is the maximal allowable set of allowable communication links in the network across times; however, at a particular instant, each sensor may choose to communicate only to one or a subset of its neighbors. For definiteness, we assume the following generic communication model, which subsumes the widely used gossip protocol for real-time embedded architectures [9] and the graph matching based communication protocols for internet architectures [10]. Define the set $\mathcal{M}$ of symmetric 0-1 $N \times N$ matrices:

$$\mathcal{M} = \left\{ A \;\middle|\; \mathbf{1}^T A = \mathbf{1}^T,\;\; A\mathbf{1} = \mathbf{1},\;\; A \le \mathcal{A} \right\}^1. \tag{4}$$

Let $\mathcal{D}$ be a probability distribution on the space $\mathcal{M}$. The sequence of time-varying adjacency matrices $\{A(k)\}$, governing the inter-sensor communication, is then set as an i.i.d. sequence in $\mathcal{M}$ with distribution $\mathcal{D}$. We define the symmetric stochastic matrix $\overline{A} = \mathbb{E}[A(k)] = \int_{\mathcal{M}} A d\mathcal{D}(A)$, and assume it is irreducible and aperiodic, which implies the connectivity of the network, i.e., the network is assumed fully connected.

2.3. Quantization Scheme

The quantization scheme adopted in this paper is the dithered quantization [11],where a controlled noise or dither is first added to randomize the value before a uniform quantizer is applied. The detailed quantization process is stated as follows. The dither v as a random variable is added to the value x to be quantized; then we adopt a uniform quantizer with a quantization step Δ and apply the countably infinite quantization alphabet [12] given by $\mathcal{Q} = \{k\Delta | k \in \mathbb{Z}\}$. We leave the case with finite quantization alphabets to our full journal version, where the infinite quantization alphabet case could serve as a performance benchmark to quantify the effect of quantization noise.

The quantizing function $q(\cdot) : \mathbb{R} \to \mathcal{Q}$ is given as

$$q(x) = \arg\min_{k\Delta} |k\Delta - (x + v)|. \tag{5}$$

Then, the quantization noise ε is defined as $\varepsilon = q(x) - x$, while the quantization error e is $e = q(x) - (x + v)$. We see that $\varepsilon = e + v$. Here we adopt the non-subtractive dithered quantization [13], which is more practical compared to the subtractive dithered quantization where $\varepsilon = e$ due to the assumption that the receiver knows the dither signal and subtracts it from the reconstructed value.

If the dither v satisfies the Schuchman conditions [11], the quantization error e is i.i.d. uniformly distributed on $[-\frac{\Delta}{2}, \frac{\Delta}{2})$ and independent of the input value x. A sufficient condition for v to satisfy the Schuchman conditions is that v is i.i.d. uniformly distributed on $[-\frac{\Delta}{2}, \frac{\Delta}{2})$ and independent of the input value x. In the sequel, we assume v i.i.d. uniformly distributed on $[-\frac{\Delta}{2}, \frac{\Delta}{2})$ and independent of the input value x; thus e is i.i.d. uniformly distributed on $[-\frac{\Delta}{2}, \frac{\Delta}{2})$ and independent of the input value x.

[1] The inequality $A \le \mathcal{A}$ is interpreted as component-wise.

2.4. QGIKF Algorithm

With the above quantization scheme, we now introduce the quantized gossip-based interacting Kalman filtering (QGIKF) scheme for distributed estimation of the state process x_k over time. Let the filter at sensor n be initialized by the pair $\left(\widehat{x}_{0|-1}, \widehat{P}_{0|-1}\right)$, where $\widehat{x}_{0|-1}$ denotes the prior estimate of x_0 (with no observation information) and $\widehat{P}_{0|-1}$ is the corresponding error variance. Also, $\left(\widehat{x}^n_{k|k-1}, \widehat{P}^n_{k|k-1}\right)$ denote the prediction of x_k at sensor n based on information till time $k-1$ and the corresponding conditional error variance, respectively. The pair $\left(\widehat{x}^n_{k|k-1}, \widehat{P}^n_{k|k-1}\right)$ is also referred to as the state of sensor n at time $k-1$. To define the estimate update rule for the QGIKF, let $n_k^{\rightarrow}$ be the neighbor of sensor n at time k w.r.t. the adjacency matrix $A(k)$. We assume that all inter-sensor communications for time k occur at the beginning of the slot, after the state $\left(\widehat{x}^n_{k|k-1}, \widehat{P}^n_{k|k-1}\right)$ is quantized according to the dithered quantization scheme with output $q\left(\widehat{x}^n_{k|k-1}, \widehat{P}^n_{k|k-1}\right)$. The paired communicating sensors receive the quantized state from the other and swap their previous states, i.e., if at time k, $n_k^{\rightarrow} = l$, sensor n replaces its previous state $\left(\widehat{x}^n_{k|k-1}, \widehat{P}^n_{k|k-1}\right)$ by $q\left(\widehat{x}^l_{k|k-1}, \widehat{P}^l_{k|k-1}\right)$ and sensor l replaces its previous state $\left(\widehat{x}^l_{k|k-1}, \widehat{P}^l_{k|k-1}\right)$ by $q\left(\widehat{x}^n_{k|k-1}, \widehat{P}^n_{k|k-1}\right)$. After the above communication is over and a new observation is made, by the recursion algorithm of Kalman filtering, the estimate update at sensor n at the end of the slot k executes as $\widehat{x}^n_{k+1|k} = F\widehat{x}^n_{k|k}$, where

$$\widehat{x}^n_{k|k} = q\left(\widehat{x}^{n_k^{\rightarrow}}_{k|k-1}\right) + K^n_k \left[y^n_k - C_n q\left(\widehat{x}^{n_k^{\rightarrow}}_{k|k-1}\right)\right], \tag{6}$$

with K^n_k as the Kalman gain [14]. Then for the estimation error variance, we have

$$\widehat{P}^n_{k+1|k} = \mathbb{E}\left[\left(x_{k+1} - \widehat{x}^n_{k+1|k}\right)^2 \middle| q\left(\widehat{x}^{n_k^{\rightarrow}}_{k|k-1}\right), q\left(\widehat{P}^{n_k^{\rightarrow}}_{k|k-1}\right), n_k^{\rightarrow}, y^n_k\right].$$

Due to the limited space, we skip the detail process to calculate $\widehat{P}^n_{k+1|k}$, which follows the logic of deriving the error variance recursion in the classical Kalman filtering theory [14]. In brief, $\widehat{P}^n_{k+1|k}$ can be recursively computed as

$$\widehat{P}^n_{k+1|k} = F^2 \widehat{P}^n_{k|k} + Q \tag{7}$$

where

$$\begin{aligned} \widehat{P}^n_{k|k} = {} & q\left(\widehat{P}^{n_k^{\rightarrow}}_{k|k-1}\right) - 2q\left(\widehat{P}^{n_k^{\rightarrow}}_{k|k-1}\right) C_n K^{n*}_k \\ & + (K^{n*}_k)^2 \left[C_n^2 q\left(\widehat{P}^{n_k^{\rightarrow}}_{k|k-1}\right) + R_n\right] + Z^n_k \end{aligned} \tag{8}$$

in which $Z^n_k = \frac{\Delta^2}{6}(1 - K^{n*}_k C_n)^2$, and the optimal Kalman gain K^{n*}_k is

$$K^{n*}_k = \frac{\left[q\left(\widehat{P}^{n_k^{\rightarrow}}_{k|k-1}\right) + \frac{\Delta^2}{6}\right] C_n}{C_n^2 q\left(\widehat{P}^{n_k^{\rightarrow}}_{k|k-1}\right) + C_n^2 \frac{\Delta^2}{6} + R_n}. \tag{9}$$

To prove the property that the estimation error variance at a randomly selected sensor converges in distribution to a unique invariant distribution, we first study the following algorithm with a suboptimal Kalman filter gain K^n_k, then show that the convergence property is automatically verified in the optimal case after establishing it in the suboptimal case.

For the suboptimal case, we choose the gain K_k^n as

$$K_k^n = q\left(\widehat{P}_{k|k-1}^{n_k^{\rightarrow}}\right) C_n \left[C_n^2 q\left(\widehat{P}_{k|k-1}^{n_k^{\rightarrow}}\right) + R_n\right]^{-1}. \quad (10)$$

Then, according to (7) and (8), we have

$$\widehat{P}_{k+1|k}^n = F^2 q\left(\widehat{P}_{k|k-1}^{n_k^{\rightarrow}}\right) + Q + F^2 Z_k^n - F^2 q\left(\widehat{P}_{k|k-1}^{n_k^{\rightarrow}}\right)^2 C_n^2 \left[C_n^2 q\left(\widehat{P}_{k|k-1}^{n_k^{\rightarrow}}\right) + R_n\right]^{-1}. \quad (11)$$

In the sequel, we will study the asymptotic property of the error variance sequence $\left\{\widehat{P}_{k+1|k}^n\right\}$ in (11) to show that the network achieves weak consensus.

3. RDS FORMULATION

First, to simplify the notation in (11), we define the function f_n as

$$f_n(X) = F^2 q(X) + Q - F^2 q(X)^2 C_n^2 \left[C_n^2 q(X) + R_n\right]^{-1} + \frac{\Delta^2}{6} F^2 \left(1 - C_n^2 q(X)\left[C_n^2 q(X) + R_n\right]^{-1}\right)^2. \quad (12)$$

Then the sequence of error variance $\widehat{P}_{k+1|k}^n$ at sensor n iterates according to $\widehat{P}_{k+1|k}^n = f_n\left(\widehat{P}_{k|k-1}^{n_k^{\rightarrow}}\right)$.

3.1. Interacting Particle Representation

To track the sequence $\left\{\widehat{P}_{k|k-1}^n\right\}$, we adopt the following interacting particle process to represent it. We will show that by the interacting particle representation, we can completely characterize and track the evolution of the sequence $\left\{\widehat{P}_{k|k-1}^n\right\}$ for $n = 1, ..., N$.

Note that the inter-sensor communication link formation process given by the sequence $\{A(k)\}$ can be represented by N particles moving on the graph as a Markov chain. The state of the n-th particle at time k is denoted by $p_n(k)$, where $p_n(k)$ takes value in the state space $[1, ..., N]$, and the transition of the n-th particle is given by

$$p_n(k) = (p_n(k-1))_k^{\rightarrow},\ p_n(0) = n. \quad (13)$$

Recall that $n_k^{\rightarrow}$ is the neighbor selection of sensor n at time k. Therefore, the n-th particle can be considered as originating from node n and then travelling on the graph according to the link formation process $\{A(k)\}$.

For each n, the process $\{p_n(k)\}$ is a Markov chain on $V = [1, \cdots, N]$ with transition probability matrix $\overline{A}$. For each of the Markov chains $\{p_n(k)\}$, we define a sequence of iteration $P_n(k)$ with initial state $P_n(0) = \widehat{P}(0)$ as

$$P_n(k+1) = f_{p_n(k)}\left(P_n(k)\right). \quad (14)$$

Note that the sequence $\{P_n(k)\}$ is governed by the Markov chain $\{p_n(k)\}$, and from the perspective of the particle, $\{P_n(k)\}$ can be considered as a particle originating at sensor n and hopping around the network as a Markov chain with transition probability $\overline{A}$, whose state $P_n(k)$ evolves according to function (14). With the Markov chain $\{p_n(k)\}$, the relation between $\left\{\widehat{P}_{k|k-1}^n\right\}$ and $\{P_n(k)\}$ could be shown as

$$(P_1(k), \cdots, P_N(k)) = \left(\widehat{P}_{k|k-1}^{p_1(k)}, \cdots, \widehat{P}_{k|k-1}^{p_N(k)}\right), \quad (15)$$

from which we see that the properties of the sequence of interest $\left\{\widehat{P}_{k|k-1}^n\right\}$ could be obtained by studying the corresponding sequence $\{P_n(k)\}$. Hence, in the sequel, we will study the sequence $\{P_n(k)\}$ to show the weak convergence.

3.2. An Auxiliary Sequence: RDS Formulation

Since the Markov chains $\{p_n(k)\}$ are non-stationary, the standard analysis based on a random dynamical system (RDS) given in [15] cannot be applied to analyzing $\{P_n(k)\}$. We need an auxiliary sequence of $\widetilde{P}(k)$ with a stationary Markov chain $\{\widetilde{p}(k)\}$, which are defined as: $\{\widetilde{p}(k)\}$ is a Markov chain with the transition matrix $\overline{A}$ and uniform initial distribution $\mathbb{P}[\widetilde{p}(0) = n] = 1/N$, $n = 1, \cdots, N$. And $\widetilde{P}(k)$, with random initial condition $\widetilde{P}(0)$, is defined as

$$\widetilde{P}(k+1) = f_{\widetilde{p}(k)}(\widetilde{P}(k)). \quad (16)$$

Now in order to proceed the asymptotic analysis of the auxiliary sequence of $\widetilde{P}(k)$, we can construct a RDS (θ^R, φ^R) equivalent to the auxiliary sequence of $\widetilde{P}(k)$ in the sense of distribution. The construction process is similar to that in our previous paper [7]; so the details of which are skipped here.

Based on RDS theory, the sequence $\left\{\varphi^R\right\}$ is distributionally equivalent to the sequence of $\widetilde{P}(k)$. At this stage, we can analyze the asymptotic distributional properties of the sequence of $\widetilde{P}(k)$ by utilizing the properties in RDSs, which is presented in the next section.

4. MAIN RESULTS

In this section, we first establish some intermediate results and then two lemmas. Afterwards, by the previously established relation between the RDS and the error variance sequences, we can show the weak convergence of these sequences.

The following proposition states two bounded properties to be used later for proving the lemmas.

Proposition 1

(i) Define an auxiliary sequence $\{P'_w(k)\}_{1\le k\le l}$ with initial condition $P'_w(1) = X$ and the iterations as

$$P'_w(k+1) = F^2 P'_w(k) + Q - F^2 C_{n_k}^2 P'_w(k)^2 (C_{n_k}^2 P'_w(k) + R_{n_k})^{-1} + \frac{\Delta^2}{6} F^2. \quad (17)$$

Define another auxiliary sequence $\{P''_w(k)\}_{1\le k\le l}$ with the same initial condition $P''_w(1) = X$ and the iterations as

$$P''_w(k+1) = F^2\left(P''_w(k) + \Delta\right) + Q - F^2 C_{n_k}^2 \left(P''_w(k) + \Delta\right)^2 \times \left[C_{n_k}^2\left(P''_w(k) + \Delta\right) + R_{n_k}\right]^{-1} + \frac{\Delta^2}{6} F^2. \quad (18)$$

Then, we recursively have

$$P''_w(l+1) < P'_w(l+1) + \frac{F^2(F^{2l}-1)\Delta}{(F^2-1)}. \quad (19)$$

(ii) For $f_n(X)$ defined in (12), we have that $f_n(X)$ is upper-bounded as

$$f_n(X) < F^2[X + Y(\Delta)] + Q, \quad (20)$$

where the function $Y(\Delta) = \frac{\Delta^2}{6} + \Delta$.

The proof is based on the monotonically non-decreasing property of function $h(X)$: $h(X) = F^2X + Q - F^2C_{n_k}^2X^2(C_{n_k}^2X + R_{n_k})^{-1} + \frac{\Delta^2}{6}F^2$, which is left to the full journal version.

Lemma 2

(i) Denote $w = (n_1, \cdots, n_l)$ as a walk on the graph $(V, \mathcal{E})$. If we define the function g_w by

$$g_w(X) = f_{n_l} \circ f_{n_{l-1}} \circ \cdots \circ f_{n_1}(X), \tag{21}$$

where $f_n(X)$ is defined in (12), there exists a constant $\alpha > 0$ such that $g_w(X) \le \alpha,\ \forall X \ge 0$.

(ii) The sequence $\{\widetilde{P}(k)\}$ is stochastically bounded, i.e.,

$$\lim_{J\to\infty} \sup_{k\in\mathbb{T}_+} \mathbb{P}\left(\widetilde{P}(k) > J\right) = 0. \tag{22}$$

The proof of Lemma 2 is according to the results in Proposition 1, which is skipped.

With the property of stochastic boundedness of the sequence $\{\widetilde{P}(k)\}$, Lemma 6.1 in [16], and Theorem 27 in [7], we can conclude that only claim b) in Theorem 27 of [7] holds, i.e., there exists a unique almost equilibrium $u^{\overline{A}}(w)$ defined on a θ^R-invariant [2] set $\Omega^* \in \mathcal{F}^R$ with $\mathbb{P}(\Omega^*) = 1$, such that for any random variable $v(w)$ possessing the property $0 \le v(w) \le \eta u^{\overline{A}}(w)$ for all $w \in \Omega^*$ and deterministic η, the following holds:

$$\lim_{k\to\infty} \varphi(k, \theta_{-k}^R w, v(\theta_{-k}^R w)) = u^{\overline{A}}(w),\ \ w \in \Omega^*. \tag{23}$$

Further incorporating Lemma 17 in [7], we have the following theorem regarding the weak convergence of the sequence $\left\{\widetilde{P}(k)\right\}$:

Theorem 3 Under the assumption of full network connectivity, there exists a unique probability measure $\mu^{\overline{A}}$ (functional of the stochastic matrix $\overline{A}$), such that for each $n \in \{1, \cdots, N\}$, the sequence $\left\{\widetilde{P}(k)\right\}$ converges weakly to $\mu^{\overline{A}}$ from every initial condition $P_n(0)$:

$$\widetilde{P}(k) \Rightarrow \mu^{\overline{A}},\ \ \forall n \in \{1, \cdots, N\}. \tag{24}$$

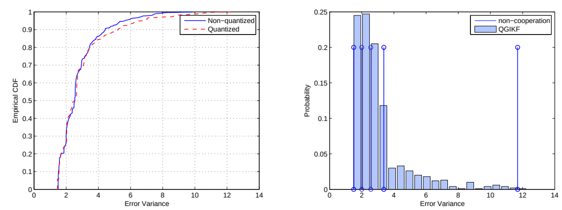
After establishing Theorem 3, following the logic in the proof for Theorem 10 in [7], we now present the key result characterizing the convergence property of the sequence $\left\{\widehat{P}_{k|k-1}^n\right\}$.

Theorem 4 Under the assumption of full network connectivity, denote n as the index of the sensors (uniformly) randomly selected from the whole set of sensors $\{1, \cdots, N\}$. Then the sequence $\left\{\widehat{P}_{k|k-1}^n\right\}$ converges weakly to the unique probability measure $\mu^{\overline{A}}$ as in Theorem 3, i.e.,

$$\widehat{P}_{k|k-1}^n \Rightarrow \mu^{\overline{A}}. \tag{25}$$

For the optimal algorithm with the error variance sequence $\left\{\widehat{P}_{k|k-1}^n\right\}$ in (7) taking the optimal gain K_k^{n*} in (9), the convergence or consensus property over the network can be easily established based on the above analysis over the suboptimal case. We can define the corresponding functions in the optimal algorithm, in the same way as f_n defined in (12). Then with the interacting particle representation, we can construct the RDS formulation for the corresponding auxiliary sequence. Since we have established the stochastically bounded nature in the suboptimal case, the stochastically bounded nature of $\left\{\widehat{P}_{k|k-1}^n\right\}$ in the optimal algorithm is automatically established. Finally, with the properties in the RDS, we can prove the convergence property for the optimal algorithm.

[2]A set $A \in F$ is called θ^R-invariant if $\theta^R A = A$ for all $k \in \mathbb{T}$.



(a) (b)

Fig. 1. (a) Empirical CDFs for the measures $\mu^{\overline{A}}$ of QGIKF and non-quantized GIKF. (b) Histogram of the measure $\mu^{\overline{A}}$ in QGIKF and the error variance distribution of the non-cooperation algorithm.

5. SIMULATION RESULTS

The simulation is based on a network of 5 sensors with an adjacency matrix satisfying the connectivity requirement of the network. The parameters C_n and R_n in the observation model (2) are selected differently for various sensors, such that we could have different estimation error variances when each sensor running its own local Kalman filter without cooperation. The quantization step Δ is set as 1. The QGIKF algorithm runs with $1,000$ recursions to ensure the convergence. We simulate the optimal estimation error variance of the QGIKF algorithm with gain K_k^{n*} in (9) for $5,000$ times and calculate the corresponding empirical cumulative distribution function (CDF).

In Fig. 1(a), we show the comparison between the empirical CDFs for the measure $\mu^{\overline{A}}$ in the QGIKF and that in the non-quantized GIKF of [7]. Since the QGIKF involves more error or information loss due to the quantization, the performance of the estimation error variance with QGIKF is worse than that of the non-quantized GIKF. In Fig. 1(b), we show the performance of QGIKF versus the non-cooperation scheme, i.e., each sensor runs its own local Kalman filter and there is no information exchange among the sensors. The histogram in probability of Fig. 1(b) displays the statistic of the measure $\mu^{\overline{A}}$ obtained in the QGIKF (shown in Theorem 4) and the statistic of the error variance in the non-cooperation scheme, by uniformly selecting the index of sensors. Compared with the non-cooperation scheme, the QGIKF demonstrates much more chances to provide a lower estimation error variance, which validates the advantage of cooperation even with quantization incorporated in the inter-sensor communications.

6. CONCLUSION

We introduced the QGIKF scheme with consideration of the limited bandwidth and power for inter-sensor communications, to estimate the state sequence of a scalar dynamic system. By formulating the problem as an RDS, we show that the network can still achieve weak consensus, even with the information loss due to quantization. Future work will focus on how to extend the problem to incorporate more practical finite-alphabet quantizers and to study the quantized vector dynamic systems.

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