TRANSMIT CODE DESIGN FOR EXTENDED TARGET DETECTION IN THE PRESENCE OF CLUTTER

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ABSTRACT

In this paper, we consider the problem of transmit code design to optimize the detection probability of an extended target embedded in Gaussian signal-dependent interference. We model the target with *a priori* known target impulse response (TIR) structure along with an unknown reflection factor. As is known, the performance of the generalized likelihood ratio test (GLRT) detector, for the case at hand, monotonically increases with the signal-to-interference-plus-noise ratio (SINR). Hence, we deal with the code design problem via maximizing the SINR. We also enforce a peak-to-average power ratio (PAR) constraint on the sought code. We devise a cyclic method to tackle the highly non-convex design problem. Numerical results highlight the effectiveness of the proposed method to improve the detection performance of the system.

***Index Terms*—** Code design, detection performance, extended target model, signal-dependent interference.

1. INTRODUCTION

Transmit code design can provide significant performance improvements for active sensing systems by suitably exploiting some prior knowledge about the target and the surrounding environment [1]. As such, a lot of research activities have been focused on the waveform design problem [2–5]. In fact, waveform optimization leads to performance improvements, in terms of target detection, target identification and classification, as well as tracking.

The problem of extended target detection has been the topic of several studies for the last decades [6–9]. Through this rich literature, many papers have been conducted to optimize the transmit waveform in order to improve the detection probability (P_d) (see e.g., [2, 10–14]). In [2], a solid paper in this field, a signal-to-noise ratio (SNR)-based waveform design approach has been introduced to improve the detection probability of an extended target in a noise-only environment. There, the target impulse response (TIR) has been modeled to be deterministic and the SNR criterion is optimized. More challenges appear when considering signal-dependent interference. In [10], using the SNR-based procedure, an iterative approach has been proposed to optimize the detection probability associated with a deterministic target embedded in signal-dependent interference environment; but there is no guarantee for the convergence of the algorithm [13]. Moreover, in [11], illumination waveforms matched to stochastic targets have been devised in the presence of signal dependent interference. The signals have been synthesized according to SNR and mutual information (MI) optimization. Finally, in [12], a parametric waveform design approach under an energy constraint has been devised, where the clutter and noise are assumed complex Gaussian processes with known statistics.

In this paper, we devise a novel code design method to optimize the detection probability associated with the extended target in the presence of clutter (i.e., a signal-dependent interference). We assume *a priori* known TIR structure with an unknown reflection factor. The performance of the generalized likelihood ratio test (GLRT) detector is determined via signal-to-interference-plus-noise ratio (SINR) of the detector. Therefore, we deal with the code design problem via maximization of the SINR. In order to obtain practically interesting waveforms, we also consider constrained code designs and impose the peak-to-average power ratio (PAR) constraint. The cast design problems are non-convex. We devise a cyclic method to tackle the constrained design problem.

The rest of the paper is organized as follows. The problem formulation is presented in Section 2. Section 3 is dedicated to deal with the transmit code design. Numerical examples are provided in Section 4. Finally, conclusions are drawn in Section 5.

2. PROBLEM FORMULATION

2.1. Target Model

In high range resolution (HRR) radar systems, the target physical extent is much larger than the range cell size; and hence it no longer holds true to model the target as a point-like scatterer. Precisely, the target can be described as a set of multiple dominant scattering centers; i.e., portions of the target that

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yield significant returns are located into a number of isolated range cells along the radar line-of-sight (LoS) [15]. In this context, the target can be represented as a linear system with a finite impulse response (i.e., its TIR). In what follows, the TIR of the extended target of interest is denoted by $\alpha \cdot t(n)$, where $t(n)$ is the deterministic TIR structure, which can be available through track files or other sources of sensing, and $\alpha \in \mathbb{C}$ is the unknown reflection factor which accounts for the attenuation and delay coefficient of the propagation environment. We assume that $t(n)$ has a support interval of length Q, viz., $t(n) = 0$ unless $n \in \{0, \ldots, Q-1\}$. The backscattered signal associated with such a (stationary) target is then characterized by the convolution of the transmit signal with the TIR (assuming the linear environment).

2.2. Signal Model

We consider a monostatic radar system that transmits an N-dimensional fast-time code $\boldsymbol{s} = [s(0), \ldots, s(N-1)]^T \in \mathbb{C}^N$. At the receive side, the received signal is down-converted to the baseband, underwent the pulse matched filter and then sampled. At the time instance n, the received signal associated with the extended target of interest (with the TIR $t(n)$), embedded in a clutter environment, can be expressed as

$$r(n) = [\alpha \cdot t(n) + c(n)] \otimes s(n) + v(n), \tag{1}$$

where $\otimes$ denotes the convolution operator, $c(n)$ denotes the clutter impulse response (CIR), and $v(n)$ is the filtered sample of the signal-independent interference (such as noise, jammers, co-channel interference, etc.). Let $M = Q + N - 1$ denote the number of received samples. Collecting all the observation samples $\{r(n)\}_{n=0}^{M-1}$ in the observation vector $\boldsymbol{r} = [r(0), \ldots, r(M-1)]^T \in \mathbb{C}^M$, we have

$$\boldsymbol{r} = \alpha \cdot \boldsymbol{T}\boldsymbol{s} + \boldsymbol{C}\boldsymbol{s} + \boldsymbol{v}, \tag{2}$$

with $\boldsymbol{T} = \sum_{n=0}^{Q-1} t(n)\boldsymbol{J}_n$ and $\boldsymbol{C} = \sum_{n=-N+1}^{M-1} c(n)\boldsymbol{J}_n$ being the TIR and the CIR matrices, respectively; and the matrix $\boldsymbol{J}_n$ denotes the $M \times N$-dimensional shift matrix given by

$$\boldsymbol{J}_n(\ell_1, \ell_2) = \delta_{\ell_1, \ell_2},\ \ell_1 \in \{1, \ldots, M\}, \ell_2 \in \{1, \ldots, N\}. \tag{3}$$

Herein, the vector $\boldsymbol{v} = [v(0), \ldots, v(M-1)]^T$ represents the interference vector. We assume that $\boldsymbol{v}$ is a (circularly symmetric complex) Gaussian vector with zero-mean and the covariance matrix $\boldsymbol{R}_v = \mathbb{E}[\boldsymbol{v}\boldsymbol{v}^\dagger]$. Let the clutter vector $\boldsymbol{c}$ be defined as $\boldsymbol{c} = [c(-N+1), \ldots, c(M-1)]^T \in \mathbb{C}^P$ with $P = M + N - 1$. In the following, we model $\boldsymbol{c}$ as a (circularly symmetric complex) zero-mean Gaussian random vector with covariance matrix $\boldsymbol{R}_c = \mathbb{E}[\boldsymbol{c}\boldsymbol{c}^\dagger]$, that is, $\boldsymbol{c} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{R}_c)$ [16]. Note that clutter and interference covariance matrices are assumed a priori known exploiting cognitive paradigms that resort to geographical, meteorological, and previous scans information [3, 5, 17]. Finally, the vectors $\boldsymbol{v}$ and $\boldsymbol{c}$ are supposed statistically independent.

2.3. Detection Problem

Considering the signal model (2), we are interested to establish whether the received signal contains the extended target return. The problem can be formulated as the following binary hypothesis test

$$\begin{cases} \mathcal{H}_0: & \boldsymbol{r} = \boldsymbol{C}\boldsymbol{s} + \boldsymbol{v} \\ \mathcal{H}_1: & \boldsymbol{r} = \alpha \cdot \boldsymbol{T}\boldsymbol{s} + \boldsymbol{C}\boldsymbol{s} + \boldsymbol{v} \end{cases}. \tag{4}$$

Due to the Gaussian distributions of the interference and the clutter vectors, the statistical distributions of $\boldsymbol{r}$ (for a prefixed transmit code vector $\boldsymbol{s}$ and a given factor α), is

$$\boldsymbol{r}|\alpha \sim \begin{cases} \mathcal{H}_0: & \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{R}_s) \\ \mathcal{H}_1: & \mathcal{N}_{\mathbb{C}}(\alpha \cdot \boldsymbol{T}\boldsymbol{s}, \boldsymbol{R}_s) \end{cases}, \tag{5}$$

with $\boldsymbol{R}_s \triangleq \mathbb{E}[\boldsymbol{C}\boldsymbol{s}\boldsymbol{s}^H\boldsymbol{C}^H] + \boldsymbol{R}_v$. Note that

$$\mathbb{E}[\boldsymbol{C}\boldsymbol{s}\boldsymbol{s}^H\boldsymbol{C}^H] = \mathbb{E}[\boldsymbol{S}\boldsymbol{c}\boldsymbol{c}^H\boldsymbol{S}^H] = \boldsymbol{S}\mathbb{E}[\boldsymbol{c}\boldsymbol{c}^H]\boldsymbol{S}^H = \boldsymbol{S}\boldsymbol{R}_c\boldsymbol{S}^H$$

where $\boldsymbol{S} = \sum_{n=0}^{N-1} s(n)\bar{\boldsymbol{J}}_{N-n-1}^T$, with $\bar{\boldsymbol{J}}_n$ being the $P \times M$-dimensional shift matrix defined similar to (3). The GLRT detector associated with (4), which coincides with the optimum test (according to the Neyman-Pearson criterion, if the phase of α is uniformly distributed in $[-\pi, \pi[$), is given by [18]:

$$|\boldsymbol{w}^H\boldsymbol{r}|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta, \tag{6}$$

with η being the detection threshold set according to a desired level of probability of false alarm (P_{fa}) and $\boldsymbol{w} = \boldsymbol{R}_s^{-1}\boldsymbol{T}\boldsymbol{s}$ denotes the receive filter associated with the above GLRT detector. In light of the definition of $\boldsymbol{w}$, the SINR of the receive filter output can be expressed as

$$\text{SINR}(\boldsymbol{s}) = \rho^2 = \boldsymbol{s}^H\boldsymbol{T}^H\boldsymbol{R}_s^{-1}\boldsymbol{T}\boldsymbol{s} = \boldsymbol{w}^H\boldsymbol{R}_s^{-1}\boldsymbol{w}. \tag{7}$$

The detection probability P_d for the non-fluctuating target and given value P_{fa} can be expressed as [19]:

$$P_d = Q\left(\sqrt{2}|\alpha|\rho, \sqrt{-2\ln P_{fa}}\right), \tag{8}$$

where $Q(\cdot,\cdot)$ denotes the Marcum Q function of order 1. Note that for a given value of P_{fa}, the probability of detection P_d is a monotonically increasing function of the SINR (i.e., ρ^2). Consequently, in the following we deal with the waveform design via maximizing the SINR in (7).

3. TRANSMIT CODE DESIGN

The performance of the GLRT detector (6) is a monotonically increasing function of the SINR. Therefore, the transmit code design problem can be formulated as the following constrained optimization problem

$$\max_{\boldsymbol{s} \in \mathcal{C}} \quad \boldsymbol{s}^H\boldsymbol{T}^H(\boldsymbol{S}\boldsymbol{R}_c\boldsymbol{S}^H + \boldsymbol{R}_v)^{-1}\boldsymbol{T}\boldsymbol{s} \tag{9}$$

where $\mathcal{C}$ denotes the desired constrained set (to be discussed in Section 3.1). The problem (9) is non-convex and we devise a novel cyclic method to tackle it. First consider the following proposition which gives an equivalent for the above problem.

Proposition 1. *Let $\boldsymbol{y} \in \mathbb{C}^{M+1}$ be an auxiliary vector. Then, (9) can be equivalently cast as the following minimization*

$$\begin{aligned} \min_{\boldsymbol{y}, \boldsymbol{s} \in \mathcal{C}} \quad & \boldsymbol{y}^H \boldsymbol{R} \boldsymbol{y}, \\ \text{s.t.} \quad & \boldsymbol{y}^H \boldsymbol{u} = 1, \end{aligned} \tag{10}$$

where $\boldsymbol{u} = [1 \;\; \mathbf{0}_{M\times 1}]^T$, $\boldsymbol{R} \triangleq \begin{bmatrix} \theta & \boldsymbol{s}^H \boldsymbol{T}^H \\ \boldsymbol{T}\boldsymbol{s} & \boldsymbol{R}_v + \boldsymbol{S}\boldsymbol{R}_c\boldsymbol{S}^H \end{bmatrix}$, and θ is sufficiently large such that $\boldsymbol{R} \succ \mathbf{0}$.

The problem (10) is still non-convex and the idea is to cyclically solve it with respect to (w.r.t.) $\boldsymbol{y}$ and $\boldsymbol{s}$. For fixed $\boldsymbol{s}$, the solution to the problem (10) is given by [20]

$$\boldsymbol{y} = \frac{\boldsymbol{R}^{-1}\boldsymbol{u}}{\boldsymbol{u}^H \boldsymbol{R}^{-1} \boldsymbol{u}}. \tag{11}$$

For fixed $\boldsymbol{y}$, the optimization in (10) boils down to the following problem

$$\min_{\boldsymbol{s} \in \mathcal{C}} \quad \boldsymbol{y}^H \boldsymbol{R} \boldsymbol{y}. \tag{12}$$

Next, we explicitly express the objective function of the problem (12) w.r.t. $\boldsymbol{s}$. Let $\boldsymbol{y} = [y_1 \;\; \boldsymbol{y}_2^T]^T$ and observe that the terms associated with the sought code $\boldsymbol{s}$ in the objective of (12) can be written as

$$\operatorname{tr}\{\boldsymbol{S}\boldsymbol{R}_c\boldsymbol{S}^H \boldsymbol{y}_2 \boldsymbol{y}_2^H\} + 2\Re(y_1 \boldsymbol{y}_2^H \boldsymbol{T}\boldsymbol{s}). \tag{13}$$

Note that the first term of (13) implicitly depends on $\boldsymbol{s}$. To obtain an explicit expression w.r.t. $\boldsymbol{s}$, we consider the identity $\boldsymbol{S}^H \boldsymbol{y}_2 = \boldsymbol{Y}_2 \boldsymbol{s}^*$ with

$$\boldsymbol{Y}_2 = \begin{bmatrix} 0 & & y_2(1) & \dots & y_2(M) \\ & \iddots & \iddots & \iddots & \\ y_2(1) & \dots & y_2(M) & & 0 \end{bmatrix}_{N \times P}. \tag{14}$$

Consequently, it can be straightforwardly verified that

$$\begin{aligned} \operatorname{tr}\{\boldsymbol{S}\boldsymbol{R}_c\boldsymbol{S}^H \boldsymbol{y}_2 \boldsymbol{y}_2^H\} &= \boldsymbol{y}_2^H \boldsymbol{S}\boldsymbol{R}_c\boldsymbol{S}^H \boldsymbol{y}_2 = \boldsymbol{s}^T \boldsymbol{Y}_2^H \boldsymbol{R}_c \boldsymbol{Y}_2 \boldsymbol{s}^* \\ &= \boldsymbol{s}^H \boldsymbol{Y}_2^T \boldsymbol{R}_c^* \boldsymbol{Y}_2^* \boldsymbol{s}, \end{aligned} \tag{15}$$

where the last equation holds due to the fact that $\boldsymbol{Y}_2^H \boldsymbol{R}_c \boldsymbol{Y}_2 \succeq \mathbf{0}$. Finally, the optimization problem (10) for fixed $\boldsymbol{y}$ is equivalent to

$$\min_{\boldsymbol{s} \in \mathcal{C}} \quad \boldsymbol{s}^H \boldsymbol{Q} \boldsymbol{s} + 2\Re(\boldsymbol{b}^H \boldsymbol{s}), \tag{16}$$

where $\boldsymbol{Q} = \boldsymbol{Y}_2^T \boldsymbol{R}_c^* \boldsymbol{Y}_2^*$ and $\boldsymbol{b}^H = y_1 \boldsymbol{y}_2^H \boldsymbol{T}$.

Note that, the cyclic minimization of (10) leads to an iterative maximization of the SINR (see [21] for a similar proof). This increasing property along with the fact that the SINR is upper bounded by

$$\text{SINR}(\boldsymbol{s}) \leq \|\boldsymbol{s}\|^2 \lambda_{\max}\left(\boldsymbol{T}^H \boldsymbol{R}_v^{-1} \boldsymbol{T}\right), \tag{17}$$

result in the convergence of the values of the SINR.

3.1. PAR Constrained Problem

Let us now focus on problem (9) when $\mathcal{C}$ accounts for both an energy and a PAR constraint given by

$$\text{PAR}(\boldsymbol{s}) = \frac{\max_{n=0,\cdots,N-1} |s(n)|^2}{\frac{1}{N}\|\boldsymbol{s}\|^2} \leq \zeta, \tag{18}$$

where ζ denotes the desired PAR level. In such case, we have the following version of (16)

$$\begin{aligned} \min_{\boldsymbol{s}} \quad & \boldsymbol{s}^H \boldsymbol{Q} \boldsymbol{s} + 2\Re(\boldsymbol{b}^H \boldsymbol{s}), \\ \text{s.t.} \quad & \|\boldsymbol{s}\|^2 = N, \max_{n=0,\cdots,N-1} |s(n)|^2 \leq \zeta. \end{aligned} \tag{19}$$

The above problem, which is a quadratically constrained quadratic program (QCQP), is NP-hard in general but *good* solutions to it can be obtained via employing the proposed method in [22]. Observe that the problem in (19) can be equivalently cast as

$$\begin{aligned} \min_{\boldsymbol{s}} \quad & \bar{\boldsymbol{s}}^H \boldsymbol{K} \, \bar{\boldsymbol{s}}, \\ \text{s.t.} \quad & \|\boldsymbol{s}\|^2 = N, \max_{n=0,\cdots,N-1} |s(n)|^2 \leq \zeta, \end{aligned} \tag{20}$$

where $\bar{\boldsymbol{s}} = [\boldsymbol{s}^T \;\; 1]^T$ and $\boldsymbol{K} = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{b} \\ \boldsymbol{b}^H & 0 \end{bmatrix}$. For any $\mu > \lambda_{\max}(\boldsymbol{K})$, we can reformulate the above problem as

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & \bar{\boldsymbol{s}}^H \boldsymbol{H} \, \bar{\boldsymbol{s}}, \\ \text{s.t.} \quad & \|\boldsymbol{s}\|^2 = N, \max_{n=0,\cdots,N-1} |s(n)|^2 \leq \zeta, \end{aligned} \tag{21}$$

with $\boldsymbol{H} = \mu \boldsymbol{I}_{N+1} - \boldsymbol{K}$. The problem in (21) can be tackled via the discussed method in [23]. More precisely, the code vector $\boldsymbol{s}$ of the $(k+1)^{th}$ iteration (denoted by $\boldsymbol{s}^{(k+1)}$) can be obtained from $\boldsymbol{s}^{(k)}$, by solving the optimization problem

$$\begin{aligned} \min_{\boldsymbol{s}^{(k+1)}} \quad & \|\boldsymbol{s}^{(k+1)} - \hat{\boldsymbol{s}}^{(k)}\|, \\ \text{s.t.} \quad & \|\boldsymbol{s}^{(k+1)}\|^2 = N, \max_{n=0,\cdots,N-1} |s^{(k+1)}(n)|^2 \leq \zeta, \end{aligned} \tag{22}$$

where $\hat{\boldsymbol{s}}^{(k)}$ represents the vector containing the first N entries of $\boldsymbol{H} \, \bar{\boldsymbol{s}}^{(k)}$. The optimization problem (22) is a "Nearest-Vector" problem with PAR constraint. Such PAR constrained problems can be dealt with using a recursive algorithm proposed in [24]. **Algorithm 1** summarizes the design procedure.

4. NUMERICAL EXAMPLES

In this section, we provide some numerical examples to assess the effectiveness of the devised algorithm. We mainly focus on the detection performance of the system and use the receiver operating characteristic (ROC) of the detector associated with the optimized SINR for the analysis. Besides, we

Algorithm 1 : Code Optimization Algorithm

1: Set $k = 0$, $\boldsymbol{s}^{(0)} = \boldsymbol{s}_0$;
2: Construct matrix $\boldsymbol{R}$ according to the Proposition 1;
3: Construct $\boldsymbol{y}$ and $\boldsymbol{Y}_2$, according to (11) and (14), respectively;
4: Construct $\boldsymbol{Q}$ and $\boldsymbol{b}$ using $\boldsymbol{y}$ and $\boldsymbol{Y}_2$, and then obtain $\boldsymbol{H}$;
5: Solve the problem (22) finding a solution $\boldsymbol{s}^{(k)}$;
6: Set $k = k + 1$ and repeat steps 2-5 until $|\text{SINR}(\boldsymbol{s}^{(k)}) - \text{SINR}(\boldsymbol{s}^{(k-1)})| \leq \xi$ for a given $\xi > 0$.

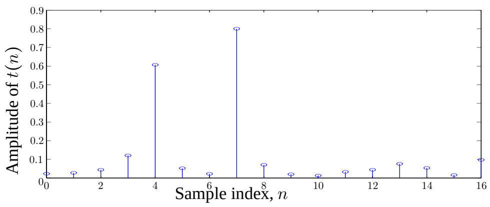

Fig. 1: Amplitude of the TIR of a MiG-21 fighter.

study the impact of the PAR constraint on the SINR and the detector performance.

Throughout the simulations, we model the TIR employing the radar backscattering data of a MiG-21 fighter (see [25] for details), and set the TIR support interval length to $Q = 17$. The sequence amplitude is shown in Fig. 1. We assume a white interference with variance $\sigma_v^2 = 1$, i.e., $\boldsymbol{R}_v = \boldsymbol{I}$. As to the clutter, we consider a homogeneous clutter environment sharing an exponentially shaped covariance matrix, where the correlation between the clutter scatterers at n^{th} and $(n')^{th}$ range bins, is given by

$$\mathbb{E}\left[c(n)c^*(n')\right] = \sigma_c^2 \rho_c^{-|n-n'|}, \tag{23}$$

for $n, n' \in \{-N+1, \ldots, M-1\}$, with $\sigma_c^2 = \mathbb{E}[|c(n)|^2] = 1$ and parameter $\rho_c = 0.9$ [16]. As to the transmit code length, we set $N = 20$. For the comparison purposes, we also report the SINR and the ROC of the linear frequency modulated (LFM) waveform, given by $\boldsymbol{s}_0(n) = e^{j\pi \frac{n^2}{2N}}$, for $n = 0, \ldots, N-1$. Finally, regarding the stop criterion of the devised methods, we set $\xi = 10^{-4}$.

In Fig. 2, the SINR behavior is plotted versus the iteration number, for the unconstrained ($\zeta = N$, i.e., with just energy constraint) and the constant-modulus ($\zeta = 1$) code designs. As expected, the achieved SINR curves have monotonically increasing behaviors and converge.

The ROCs associated with the optimized transmit codes of Fig. 2 are depicted in Fig. 3. Here, we assume a reflection factor with the amplitude $|\alpha|^2 = 2$. Note that both figures include the upper bound on the SINR values obtained in [11]. This upper bound has been derived in frequency domain for the unconstrained design. The achieved SINR values associated with the synthesized (time-domain) code, obtained in [11], have also been considered (labeled with achieved SINR in the figures). It can be seen that the system employing our devised method significantly outperforms the system with the

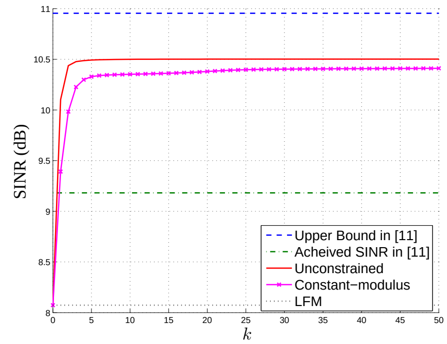

Fig. 2: The values of SINR versus iteration number.

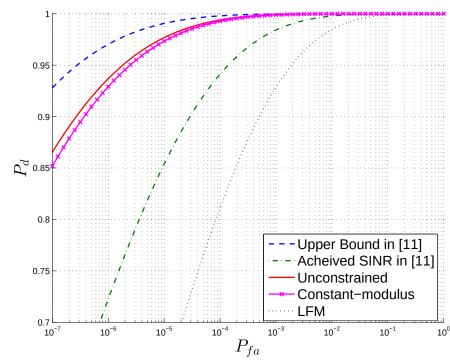

Fig. 3: ROC curves associated with Fig. 2.

synthesized waveform in [11], as well as the conventional LFM. This is due to the fact that our method doesn't suffer from the synthesis loss. Other observations related to the figures reveal that the constant-modulus case enjoys SINR and P_d values closed to those of the unconstrained design. This clearly suggests to exploit constant-modulus waveforms which trades off practical advantages of constant-modulus with a small SINR and detection loss.

5. CONCLUSION

The transmit code design has been considered to optimize the detection probability of an extended target in the presence of signal-dependent interference. The target response has been modeled with deterministic TIR and an unknown reflection factor. The performance of the GLRT detector has been determined by the received SINR. Hence, the design problem has been cast considering the SINR as its figure of merit. Moreover, a PAR constraint has been imposed to the transmit waveform so as to ensure its practical implementation. We have devised a novel method, based on the cyclic minimization technique, to tackle the highly non-convex design problem. Numerical examples have been provided to show the effectiveness of the proposed method. Specifically, this preliminary study has highlighted significant improvements w.r.t. the design methodology in [11] as well as the LFM.

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