A BERNOULLI FILTER APPROACH TO DETECTION AND ESTIMATION OF HIDDEN MARKOV MODELS USING CLUTTERED OBSERVATION SEQUENCES

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ABSTRACT

Hidden Markov Models (HMMs) are powerful statistical techniques with many applications, and in this paper they are used for modeling asymmetric threats. The observations generated by such HMMs are generally cluttered with observations that are not related to the HMM. In this paper a Bernoulli filter is proposed, which processes cluttered observations and is capable of detecting if there is an HMM present, and if so, estimate the state of the HMM. Results show that the proposed filter is capable of detecting and estimating an HMM except in circumstances where the probability of observing the HMM is lower than the probability of receiving a clutter observation.

***Index Terms*—** Hidden Markov model, detection, estimation, random finite sets, Bernoulli filter.

1. INTRODUCTION

The term *asymmetric threat* refers to tactics employed by, e.g., terrorist groups to carry out attacks on a superior opponent, while trying to avoid direct confrontation. Terrorist groups are elusive, secretive, amorphously structured decentralized entities that often appear unconnected. Analysis of prior terrorist attacks suggests that a high magnitude terrorist attack requires certain enabling events to take place.

In this paper terrorist activites are modeled using Hidden Markov Models (HMMs). In previous work HMMs have been shown to provide powerful statistical techniques, and they have been applied to various problems such as speech recognition, DNA sequence analysis, robot control, fault diagnosis, and signal detection, to name a few. Excellent tutorials on HMMs can be found in [1, 2]. The applicability of HMMs for terrorist activity modeling and other national security applications has been illustrated in previous work, see e.g. [3, 4, 5, 6, 7, 8]. For example, Coffman and Marcus use HMMs to identify groups with suspicious behaviour [4], and Schrodt use HMMs for pattern recognition of international crises [3].

A number of different terrorist plan HMMs are proposed in [5, 6, 7, 8], including models for a truck bombing, a plane hijacking, and production of weapons grade material. These HMMs include multiple paths from plan initiation to plan completion, following the intuition that there are multiple ways to, e.g., hijack a plane. An empirical HMM can be constructed using available prior information, or with the help from experienced intelligence analysts [5]. For example, the HMM for *development of a nuclear weapons program* (DNWP) in [7] is gleaned using the open sources [9, 10, 11, 12, 13].

The basic motivation of modeling terrorist activities via HMMs is twofold. Firstly, carrying out a terrorist activity requires planning and preparations, following steps that form a pattern. This pattern of actions can be modeled using a Markov chain. Secondly, the terrorists leave detectable clues about these enabling events in the observation space. The clues are not direct observations of the planning and preparations, but are rather related to them, meaning that the states in the Markov model are hidden. For example, an observation of a purchase of chemicals could be indicative of intentions to produce a chemical weapon. However, a purchase of chemicals could very well be a benign event, which motivates inclusion of a model of observations that are unrelated to the HMM. Following the target tracking literature, see e.g. [14], such observations are here designated as clutter observations.

The problem considered in this paper is to process a sequence of observations and detect if there is a terrorist activity being planned and organized, and if so, what stage of planning the activity is in. In the next section, we give a formal problem definition.

2. PROBLEM DEFINITION

Let $\mathbf{s}_k \in \mathcal{S}$ denote the HMM state at time t_k, where $\mathcal{S}$ is a discrete state space with N_s states, $\mathcal{S} = \{S_1, S_2, \ldots, S_{N_s}\}$. Further, let $\mathbf{t}_k \in \mathcal{T} = \{0, 1\}$ denote the transition state, defined as $\mathbf{t}_k = 1$ if $\mathbf{s}_k \neq \mathbf{s}_{k-1}$ and $\mathbf{t}_k = 0$ otherwise. The transition state is important because in the variant of HMMs used here the observations become available upon state transitions. Let $\zeta_k = (\mathbf{s}_k, \mathbf{t}_k)$ denote the joint variable. For the joint transition probability $\pi(\zeta_k|\zeta_{k-1}) = \pi(\mathbf{s}_k, \mathbf{t}_k|\mathbf{s}_{k-1}, \mathbf{t}_{k-1})$ the following holds

$$\pi(\zeta_k|\zeta_{k-1}) = \pi(\mathbf{t}_k|\mathbf{s}_k, \mathbf{s}_{k-1}, \mathbf{t}_{k-1})\pi(\mathbf{s}_k|\mathbf{s}_{k-1}). \quad (1)$$

Supported by NPS via ONR N00244-14-1-0033, by ONR under N000014-13-1-0231 and by ARO W991NF-10-1-0369.

Proceedings of ICASSP, April 2015, Brisbane, Australia.

The HMM state transitions follow a first order Markov chain with transition probability $\pi(\mathbf{s}_k|\mathbf{s}_{k-1})$. For the transition state the transition matrix is

$$\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix} \text{ if } \mathbf{s}_k \neq \mathbf{s}_{k-1}, \quad \begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix} \text{ otherwise.} \tag{2}$$

The observations $\mathbf{z}_k \in \mathcal{Z}$ are discrete random variables, where $\mathcal{Z}$ is a discrete state space with N_z states, $\mathcal{Z} = \{Z_1, Z_2, \ldots, Z_{N_z}\}$. With a state dependent probability of detection

$$p_{\mathrm{D}}(\zeta_k) = \begin{cases} p_{\mathrm{D}}(\mathbf{s}_k) \in (0,1) & \text{if } \mathbf{t}_k = 1, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

the HMM generates an observation $\mathbf{z}_k$. The observation process is defined by the likelihood $g_s(\mathbf{z}_k|\zeta_k) = g_s(\mathbf{z}_k|\mathbf{s}_k)$. There are also clutter observations (false alarms) superimposed on the true HMM observations. In each time-step, with probability $0 < p_{\mathrm{FA}} < 1$ a clutter observation is generated as a random sample from a process with probability mass function (pmf) $g_{\mathrm{FA}}(\mathbf{z}_k)$. Thus, at each time step there are 0, 1 or 2 observations.

Now, consider a sequence of time steps t_1 to t_N. For some of the time steps there are observations, for others there are not (denoted $\emptyset$), e.g.:

$$\begin{array}{lccccccc} \text{Time:} & \ldots & t_{k-2} & t_{k-1} & t_k & t_{k+1} & t_{k+2} & \ldots \\ \text{Obs.:} & \ldots & \mathbf{z}_{k-2} & \emptyset & \emptyset & \mathbf{z}_{k+1} & \emptyset & \ldots \end{array} \tag{4}$$

The problem considered in this paper is, given a time sequence of observations, to determine if there are any time steps for which an HMM is causing the observations (detection), and if so estimate what state $\mathbf{s}_k$ the HMM is in at those time steps (estimation).

The scope of the paper is limited by the assumption that there is at most a single HMM whose transition probability $\pi(\cdot|\cdot)$ and likelihood $h(\cdot|\cdot)$ are known. Note however that no assumptions are made regarding the HMM's existence, nor regarding measurement origin (HMM or clutter).

3. RANDOM FINITE SET MODELING

Single target detection and state estimation using cluttered observations is well studied in the target tracking literature. In this work we will use Finite Set Statistics (FISST) and Random Finite Set (RFS) theory to model the problem, specifically the so called Bernoulli RFSs. A tutorial of random set methods is given in [15], with in-depth descriptions of FISST and RFS found in [16]. A tutorial introduction to Bernoulli filters is given in [17]. In previous work these methods have typically been applied to problems where both the state and the observations are continuous random variables, in contrast to the work here where the states and observations are discrete.

An RFS is a random variable whose realizations are sets with a finite cardinality (number of elements). The cardinality, and each element, are all random variables. Specifically, a Bernoulli RFS $\mathbf{X}$ is either an empty set, with probability $1-q$, or has a single element, with probability q. In case there is an element $\mathbf{x}$, it is distributed over the state space $\mathcal{X}$ according to the probability mass function (pmf) $P(\mathbf{x})$. The FISST probability density function (pdf) of $\mathbf{X}$ is

$$f(\mathbf{X}) = \begin{cases} 1-q, & \text{if } \mathbf{X} = \emptyset, \\ q \cdot P(\mathbf{x}), & \text{if } \mathbf{X} = \{\mathbf{x}\}\,. \end{cases} \tag{5}$$

The state space for $\mathbf{X}$ is $\emptyset \cup \sigma(\mathcal{X})$, where $\sigma(\mathcal{X})$ is the set of all singletons $\{\mathbf{x}\}$ such that $\mathbf{x} \in \mathcal{X}$. A singleton is a set with cardinality one. For a Bernoulli pdf a set integral is defined as follows [16],

$$\int f(\mathbf{X})\delta\mathbf{X} = f(\emptyset) + \int f(\{\mathbf{x}\})\mathrm{d}\mathbf{x} \tag{6a}$$

$$= 1 - q + \int qP(\mathbf{x})\mathrm{d}\mathbf{x} = 1, \tag{6b}$$

and it follows that $f(\mathbf{X})$ as defined in (5) is indeed a proper pdf. Note that integrals over the discrete random variable $\mathbf{x}$ are sums, e.g.

$$\int m(\mathbf{x})P(\mathbf{x})\mathrm{d}\mathbf{x} = \sum_{X \in \mathcal{X}} m(\mathbf{x} = X)P(\mathbf{x} = X) \tag{7}$$

for a function $m(\mathbf{x})$, however for brevity we will use the integral notation rather than the sum notation.

3.1. HMM state model

The joint HMM state ζ_k at time t_k is modeled as a Bernoulli RFS $\mathbf{S}_k$. The state space is $\emptyset \cup \sigma(\mathcal{S} \times \mathcal{T})$, where $\sigma(\mathcal{S} \times \mathcal{T})$ is the set of all singletons $\{\mathbf{s}, \mathbf{t}\}$ such that $\mathbf{s} \in \mathcal{S}$ and $\mathbf{t} \in \mathcal{T}$. The binary random variable $\varepsilon_k \in \{0, 1\}$ models the existence of the HMM: if $\varepsilon_k = 1$ the HMM exists at time t_k.

3.2. Dynamics model

The dynamics of HMM existence ε_k are modeled as a first order Markov chain with transition probability matrix

$$P^{\varepsilon}_{k|k-1} = \begin{bmatrix}(1-p_b) & p_b \\ (1-p_s) & p_s\end{bmatrix}. \tag{8}$$

The probability $p_b = P(\varepsilon_k = 1|\varepsilon_{k-1} = 0)$ models the probability of HMM birth, i.e. the probability that at time t_k a plan is initiated. The probability $p_s = P(\varepsilon_k = 1|\varepsilon_{k-1} = 1)$ is the probability of HMM survival, i.e. the probability that an initiated plan is not cancelled. If an HMM is intiated at time t_k the initial pmf is denoted $P^b_{k|k-1}(\zeta)$.

The dynamic model of the RFS $\mathbf{S}$ is a Markov process with transition density $P^{\mathbf{S}}_{k|k-1}(\mathbf{S}|\mathbf{S}')$,

$$P^{\mathbf{S}}_{k|k-1}(\mathbf{S}|\emptyset) = \begin{cases} 1-p_b, & \text{if } \mathbf{S} = \emptyset, \\ p_b \cdot P^b_{k|k-1}(\zeta), & \text{if } \mathbf{S} = \{\zeta\}\,, \end{cases} \tag{9a}$$

$$P^{\mathbf{S}}_{k|k-1}(\mathbf{S}|\{\zeta'\}) = \begin{cases} 1-p_s, & \text{if } \mathbf{S} = \emptyset, \\ p_s \cdot \pi(\zeta|\zeta'), & \text{if } \mathbf{S} = \{\zeta\}\,. \end{cases} \tag{9b}$$

3.3. Observations

Let $\mathbf{Z}_k$ be the RFS observation at time t_k. Let $\mathbf{Z}^k$ denote all such observation from time t_1 to t_k, $\mathbf{Z}^k = \{\mathbf{Z}_1, \mathbf{Z}_2, \ldots, \mathbf{Z}_k\}$. If $\varepsilon_k = 0$ then $\mathbf{Z}_k = \mathbf{C}_k$ and if $\varepsilon_k = 1$ then $\mathbf{Z}_k$ is the union of two independent RFS,

$$\mathbf{Z}_k = \mathbf{W}_k \cup \mathbf{C}_k, \tag{10}$$

where $\mathbf{W}_k$ is HMM generated observations and $\mathbf{C}_k$ is clutter observations. The clutter observations are modeled as a Bernoulli RFS with FISST pdf

$$\kappa(\mathbf{Z}) = \begin{cases} 1 - p_{\mathrm{FA}}, & \text{if } \mathbf{Z} = \emptyset, \\ p_{\mathrm{FA}} \cdot g_{\mathrm{FA}}(\mathbf{z}), & \text{if } \mathbf{Z} = \{\mathbf{z}\}\,. \end{cases} \tag{11}$$

At timesteps for which an HMM exists ($\varepsilon_k = 1$, $\mathbf{S}_k = \{\zeta_k\}$), the HMM generated observations are modeled as a Bernoulli RFS with FISST pdf

$$\eta(\mathbf{Z}|\{\zeta\}) = \begin{cases} 1 - p_{\mathrm{D}}(\zeta), & \text{if } \mathbf{Z} = \emptyset, \\ p_{\mathrm{D}}(\zeta) \cdot g_s(\mathbf{z}|\zeta), & \text{if } \mathbf{Z} = \{\mathbf{z}\}\,. \end{cases} \tag{12}$$

The observation likelihood function is denoted $\varphi(\mathbf{Z}|\mathbf{S})$ and has two forms: one for $\mathbf{S} = \emptyset$ and one for $\mathbf{S} = \{\zeta\}$. In the former case we have the FISST pdf $\varphi(\mathbf{Z}|\emptyset) = \kappa(\mathbf{Z})$, and in the latter case the FISST pdf is

$$\varphi(\mathbf{Z}|\{\mathbf{s}\}) = \sum_{\mathbf{W} \subseteq \mathbf{Z}} \eta(\mathbf{W}|\{\mathbf{s}\})\kappa(\mathbf{Z} \backslash \mathbf{W}), \tag{13}$$

where $\backslash$ denotes set difference. For both the clutter observations and the HMM observations the set can be either empty or singletons, and thus the union (10) can have zero, one or two elements. The summation then has three different cases

$$\varphi(\mathbf{Z}|\{\zeta\}) = \tag{14}$$
$$\begin{cases} \eta(\emptyset|\{\zeta\})\kappa(\emptyset) & \text{if } \mathbf{Z} = \emptyset, \\ \eta(\mathbf{z}|\{\zeta\})\kappa(\emptyset) + \eta(\emptyset|\{\zeta\})\kappa(\mathbf{z}) & \text{if } \mathbf{Z} = \{\mathbf{z}\}, \\ \eta(\mathbf{z}^1|\{\zeta\})\kappa(\mathbf{z}^2) + \eta(\mathbf{z}^2|\{\zeta\})\kappa(\mathbf{z}^1) & \text{if } \mathbf{Z} = \{\mathbf{z}^1, \mathbf{z}^2\}. \end{cases}$$

4. BERNOULLI FILTER

In this section, using the models defined above, we propose a filter within the RFS framework such that a joint estimate of the probability of HMM existence and of the HMM state distribution (pmf) are obtained. The derivations are omitted due to page length restrictions.

Assume that the posterior FISST pdf $f_{k-1|k-1}(\mathbf{S}_{k-1}|\mathbf{Z}^{k-1})$ is known. The posterior FISST pdf at time step t_{k-1} for the Bernoulli RFS $\mathbf{S}_{k-1}$ is specified by the posterior probability of HMM existence and the posterior pmf of the joint HMM state of ζ_{k-1},

$$q_{k-1|k-1} = P(|\mathbf{S}_{k-1}| = 1|\mathbf{Z}^{k-1}), \tag{15a}$$
$$P_{k-1|k-1}(\zeta) = P(\zeta_{k-1}|\mathbf{Z}^{k-1}). \tag{15b}$$

The Bernoulli filter (BF) propagates both quantities over time using a prediction equation

$$f_{k|k-1}(\mathbf{S}_k|\mathbf{Z}^{k-1}) = P^{\mathbf{S}}_{k|k-1}(\mathbf{S}_k|\emptyset) f_{k-1|k-1}(\emptyset|\mathbf{Z}^{k-1}) + \int P^{\mathbf{S}}_{k|k-1}(\mathbf{S}_k|\{\zeta\}) f_{k-1|k-1}(\{\zeta\}|\mathbf{Z}^{k-1})\mathrm{d}\zeta \tag{16}$$

and a correction equation

$$f_{k|k}(\mathbf{S}_k|\mathbf{Z}^k) = \frac{\varphi(\mathbf{Z}_k|\mathbf{S}_k) f_{k|k-1}(\mathbf{S}_k|\mathbf{Z}^{k-1})}{f(\mathbf{Z}_k|\mathbf{Z}^{k-1})}, \tag{17}$$

where the FISST likelihood $f(\mathbf{Z}_k|\mathbf{Z}^{k-1})$ is defined as

$$f(\mathbf{Z}_k|\mathbf{Z}^{k-1}) = \varphi(\mathbf{Z}_k|\emptyset) f_{k|k-1}(\emptyset|\mathbf{Z}^{k-1}) + \int \varphi(\mathbf{Z}_k|\{\zeta\}) f_{k|k-1}(\{\zeta\}|\mathbf{Z}^{k-1})\mathrm{d}\zeta. \tag{18}$$

The predicted probability of HMM existence and the predicted joint HMM state pmf are

$$q_{k|k-1} = p_b(1 - q_{k-1|k-1}) + p_s q_{k-1|k-1}, \tag{19a}$$
$$P_{k|k-1}(\zeta) = \frac{p_b(1 - q_{k-1|k-1})}{q_{k|k-1}} P^b_{k|k-1}(\zeta_k) + \frac{p_s q_{k-1|k-1} \int \pi(\zeta_k|\zeta) P_{k-1|k-1}(\zeta)\mathrm{d}\zeta}{q_{k|k-1}}. \tag{19b}$$

The correction has three different cases: for $\mathbf{Z}_k = \emptyset$ we have

$$q_{k|k} = \frac{1 - \Delta^1_{k|k-1}}{1 - q_{k|k-1}\Delta^1_{k|k-1}} q_{k|k-1}, \tag{20a}$$
$$P_{k|k}(\zeta) = \frac{1 - p_{\mathrm{D}}(\zeta)}{1 - \Delta^1_{k|k-1}} P_{k|k-1}(\zeta), \tag{20b}$$
$$\Delta^1_{k|k-1} = \int p_D(\zeta) P_{k|k-1}(\zeta)\mathrm{d}\zeta, \tag{20c}$$

for $\mathbf{Z}_k = \{\mathbf{z}\}$ we have

$$q_{k|k} = \frac{1 - \Delta_{k|k-1}}{1 - q_{k|k-1}\Delta_{k|k-1}} q_{k|k-1}, \tag{20d}$$
$$P_{k|k}(\zeta) = \frac{1 - p_{\mathrm{D}}(\zeta)}{1 - \Delta_{k|k-1}} P_{k|k-1}(\zeta) + \frac{1 - p_{\mathrm{FA}}}{p_{\mathrm{FA}} g_{\mathrm{FA}}(\mathbf{z})} \frac{p_{\mathrm{D}}(\zeta) g_s(\mathbf{z}|\zeta)}{1 - \Delta_{k|k-1}} P_{k|k-1}(\zeta), \tag{20e}$$
$$\Delta_{k|k-1} = \Delta^1_{k|k-1} - \Delta^2_{k|k-1}, \tag{20f}$$
$$\Delta^2_{k|k-1} = \frac{1 - p_{\mathrm{FA}}}{p_{\mathrm{FA}} g_{\mathrm{FA}}(\mathbf{z})} G^s_{k|k-1}(\mathbf{z}), \tag{20g}$$
$$G^s_{k|k-1}(\mathbf{z}) = \int p_{\mathrm{D}}(\zeta) g_s(\mathbf{z}|\zeta) P_{k|k-1}(\zeta)\mathrm{d}\zeta, \tag{20h}$$

and for $\mathbf{Z}_k = \{\mathbf{z}^1, \mathbf{z}^2\}$ we have $q_{k|k} = 1$ and

$$P_{k|k}(\zeta) = \frac{g_{\mathrm{FA}}(\mathbf{z}^1) p_{\mathrm{D}}(\zeta) g_s(\mathbf{z}^2|\zeta) P_{k|k-1}(\zeta)}{g_{\mathrm{FA}}(\mathbf{z}^1) G^s_{k|k-1}(\mathbf{z}^2) + g_{\mathrm{FA}}(\mathbf{z}^2) G^s_{k|k-1}(\mathbf{z}^1)} + \frac{g_{\mathrm{FA}}(\mathbf{z}^2) p_{\mathrm{D}}(\zeta) g_s(\mathbf{z}^1|\zeta) P_{k|k-1}(\zeta)}{g_{\mathrm{FA}}(\mathbf{z}^1) G^s_{k|k-1}(\mathbf{z}^2) + g_{\mathrm{FA}}(\mathbf{z}^2) G^s_{k|k-1}(\mathbf{z}^1)}. \tag{20i}$$

$p_D \backslash p_{FA}$	0.75	0.50	0.25	0.01
0.25	10.3	10.6	9.7	10.5
0.50	10.1	10.4	10.1	10.2
0.75	9.8	10.3	10.3	10.2
0.99	10.4	10.5	9.9	9.1

Table 1. HMM existence: false alarm rates [%]

$p_D \backslash p_{FA}$	0.75	0.50	0.25	0.01
0.25	86.9	64	58.8	82.4
0.50	58.2	76.1	84	93.6
0.75	83.9	93.5	96.8	97.9
0.99	99.2	99.8	99.9	99.9

Table 2. HMM existence: detection rates [%]

$p_D \backslash p_{FA}$	0.75	0.50	0.25	0.01
0.25	6.9	10.7	20.5	41.4
0.50	38	50.6	55.1	62.6
0.75	66.6	73.6	75.2	76.7
0.99	94.9	97	97.6	98.3

Table 3. HMM state estimation: % time steps where $\hat{\mathbf{s}}_{k|k} = \mathbf{s}_k$

5. SIMULATION RESULTS

Intelligence observation data of the kind considered here is inherently secret, and for this reason results for real observation data records are unavailable, and could not be published if they were. Instead we present results for simulated data. Multiple different HMMs have been simulated, see [5, 6, 7, 8] for details. The results for the different HMMs are comparable. Here we will highlight results from a representative HMM that has 27 states and models the *production of weapons grade material* (PWGM) [8]. The birth and survival probabilities were set to $p_b = 0.01$ and $p_s = 0.99$. We have tested different probabilities of detection $p_D(\zeta) = p_D \in \{0.25, 0.50, 0.75, 0.99\}$ and probabilities of false alarm $p_{FA} \in \{0.01, 0.25, 0.50, 0.75\}$.

For each p_D, p_{FA} pair, the proposed BF was evaluated as follows. First we simulated 10^5 clutter observations and determined which existence probability thresholds $\tau \in [0, 1]$ that gave 10% empirical false alarm rates. Next the PWGM-HMM was simulated 1000 times; in each simulation HMM birth time, state transitions, HMM observations, and clutter observations were all randomly simulated. In each time step t_k, if $q_{k|k} > \tau$ a maximum a posteriori (MAP) HMM state estimate was computed,

$$\hat{\mathbf{s}}_{k|k} = \arg\max_{\mathbf{s} \in \mathcal{S}} P_{k|k}(\mathbf{s}), \qquad (21)$$

where $P_{k|k}(\mathbf{s}) = \int P_{k|k}(\mathbf{s}, \mathbf{t}) \mathrm{d}\mathbf{t}$ is the marginal posterior distribution. We did not evaluate estimates of the transition state $\mathbf{t}_k$, because knowing the HMM state $\mathbf{s}_k$ is more important than knowing whether or not the HMM just transitioned to that state.

In Table 1 empirical false alarm rates are given, i.e. for $\varepsilon_k = 0$ the % time steps t_k for which $q_{k|k} > \tau$. In Table 2 empirical detection rates are given, i.e. for $\varepsilon_k = 1$ the % time steps t_k for which $q_{k|k} > \tau$. In Table 3 we give, for $\varepsilon_k = 1$ and $q_{k|k} > \tau$, the % time steps for which $\hat{\mathbf{s}}_{k|k} = \mathbf{s}_k$. By varying the existence probability threshold τ between 0 and 1 receiver operating characteristic (ROC) curves are obtained, see Figure 1. Table 1 confirm that existence probability threshold indeed can be computed using sequences of clutter observations. As expected, it is more difficult to estimate the existence of an HMM when p_D is lower and p_{FA} is higher, this can be seen in both Table 2 and Figure 1. A comparison shows that the rates in Table 3 are lower than the rates in Table 2. This is a result of missed detections: a missed detection makes it more difficult for the BF to estimate the state transition, and subsequently the estimated state $\hat{\mathbf{s}}_{k|k}$ is incorrect for a couple of time steps following the missed detection.

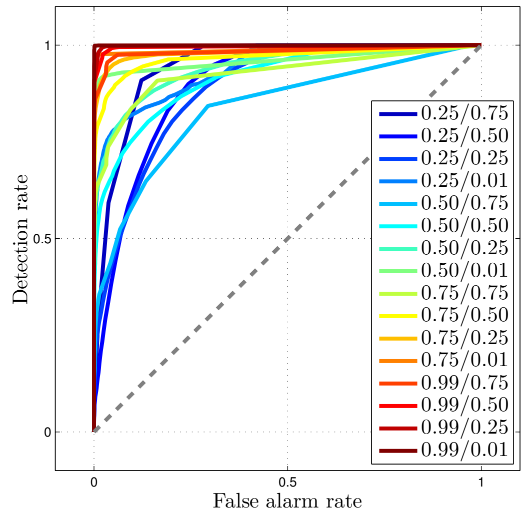

Fig. 1. Receiver Operating Characteristics (legend: p_D/p_{FA})

In a MATLAB implementation run on a desktop computer with two 2.66 GHz processors and 4 GB RAM, the median time for a single iteration (prediction and correction) is 0.5ms, indicating that the proposed BF is capable of real-time performance.

6. CONCLUSIONS AND FUTURE WORK

The proposed Bernoulli filter approach to joint detection and estimation of HMMs is shown to give good results except in very adverse conditions (low probability of detection and high probability of false alarm). In future work we also intend to perform an analysis of the detectability of HMMs whose observations are submerged in clutter observations – what are the bounds on the error probabilities, and how do these relate to the properties of the underlying HMM? These questions are important for determining under which circumstances one can expect the problem to be solvable with reasonable performance.

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