

# MODEL-BASED PARAMETERS ESTIMATION OF NON-STATIONARY SIGNALS USING TIME WARPING AND A MEASURE OF SPECTRAL CONCENTRATION

Andrei Anghel<sup>†\*</sup> Gabriel Vasile<sup>†</sup> Cornel Ioana<sup>†</sup> Remus Cacoveanu<sup>\*</sup> Silviu Ciochina<sup>\*</sup>

<sup>†</sup> Grenoble INP / CNRS, Grenoble-Image-sPeech-Signal-Automatics Lab (GIPSA-lab)  
11 rue des Mathematiques, Grenoble Campus, 38402 Saint Martin d'Herès Cedex, FRANCE

<sup>\*</sup> University Politehnica of Bucharest, 1-3 Iuliu Maniu, 061071 Bucharest, ROMANIA

## ABSTRACT

This paper proposes a parameters estimation algorithm for signals composed of multiple non-stationary components having the same basis modulation function which is described by an a priori known model and depends on a few unknown parameters. The procedure is based on time warping the signal in turn with every basis function resulted from different model parameters combinations and evaluating the concentration of the warped signal spectrum. The estimated parameters of the model are the ones which provide the best spectral concentration. Onwards, the amplitude, phase and modulation rate for each component are determined from the signal warped with the optimal basis function. The algorithm is tested with simulations and real data consisting of de-chirped radar signals and acoustic signals with harmonic components from underwater mammals.

**Index Terms**— Non-stationary signals; Time-frequency analysis; Warping; Signal Representation.

## 1. INTRODUCTION

In many signal processing applications the analyzed signals are non-stationary in the sense that the frequency changes over time and are comprised of multiple components each having its particular instantaneous frequency law (IFL). Most of the times the phase function of an analytic non-stationary component can be described by a parametric model. If there is no a priori information of the signal's nature, a widely employed model is the polynomial phase signal (PPS) model based on the Weierstrass approximation theorem. Typical parameters estimation methods for PPSs with multiple components are based on the high-order ambiguity function (HAF) [1, 2, 3, 4] or on nonlinear least squares approaches [5]. These algorithms have to deal with specific problems of multi-component signals: the interaction between the components which may lead to cross-terms and the inability to successively demodulate and separate a certain component in the presence of others.

When the time-frequency shape of the components forming the signal is known, the parameters estimation can be accomplished using a matched signal transform (MST) [6, 7] which localizes signals with a certain nonlinear characteristic basis function at their frequency modulation (FM) rate in the same manner as the Fourier transform localizes sinusoids at their frequencies. Moreover, such a transform avoids the cross-terms problem because it includes an amplitude modulation with the basis function's IFL which makes the components orthonormal [8]. Various classical transforms are actually MSTs such as the Mellin transform [9, 10], the  $k$ -th power transform [8] or the exponential transform [11].

As a trade-off between the PPS estimation techniques and the MST approach, this paper addresses the problem of estimating the parameters of a non-stationary signal comprised of multiple components with the same time-frequency shape when the basis modulation function is only partially known and depends on a few parameters which also have to be estimated. The proposed approach is based on time warping the given signal with different parametric functions and identifying for which parameters the warped signal has the best spectral concentration. The cost function used for evaluating the spectral concentration is derived from the measures designed to evaluate time-frequency distributions concentration [12].

Notice that the method proposed in this paper can be viewed as a generalization of some chirp rate estimation methods based on various transforms: the fan-chirp transform [13], the adaptive harmonic fractional Fourier transform [14] or the multiangle centered discrete fractional Fourier transform [15]. More specifically, in comparison with the previously mentioned works, the basis modulation function can be any kind of monotonic one-to-one function depending on certain parameters (not only a second order polynomial depending on the chirp rate).

This paper is organized as follows. Section II describes the proposed parameters estimation method in three parts. The time warping approach is discussed first. Afterwards we introduce the spectral concentration measure followed by the discrete signal model and Cramér-Rao bound computation. In Section III the algorithm is applied on synthetic signals and real data. The conclusions are stated in Section IV.

## 2. ESTIMATION METHOD

### 2.1. Time warping

We consider a signal consisting of a sum of non-stationary components each having the same time-frequency shape described by a monotonic one-to-one function of time  $\theta(t)$  (a basis function) defined on the interval  $[0, T]$ . Such a signal with  $K$  components can be written as

$$s(t) = \sum_{k=1}^K a_k \exp \{j(\varphi_k + \alpha_k \theta(t))\}, \quad (1)$$

where  $a_k$ ,  $\varphi_k$  and  $\alpha_k$  are respectively the amplitude, phase and modulation rate of component  $k$ . If the signal in (1) is viewed in a warped time axis  $\theta = \theta(t)$  it will appear as a signal composed of a sum of complex sinusoids

$$s(\theta) = \sum_{k=1}^K a_k \exp \{j(\varphi_k + \alpha_k \theta)\}. \quad (2)$$

The Fourier transform of (2)

$$S(\alpha) = \int_{\theta(0)}^{\theta(T)} s(\theta) \exp(-j2\pi\alpha\theta) d\theta. \quad (3)$$

will give peaks at the modulation rates  $\alpha_k$  of the  $K$  components. The transform in (3) can also be computed in terms of the initial time axis as

$$S(\alpha) = \int_0^T |\theta'(t)| s(t) \exp(-j2\pi\alpha\theta(t)) dt. \quad (4)$$

Notice that (4) is actually a modified form of the MST applied for the basis function  $\theta(t)$ . Additionally, the squared magnitude of the MST is a maximum likelihood (ML) estimator for the modulation rate of signals with a certain characteristic function in the same way as the periodogram is a ML estimator for the frequency of sinusoidal signals [7].

In this context, the goal of this work is to extend the MST (based on time warping) for parameters estimation of signals having the form in (1) when the basis function  $\theta(t)$  is partially known and depends on a few parameters which also have to be determined. Thus, we introduce the notation  $\theta_{\mathbf{p}}(t)$  for a basis function depending on the vector  $\mathbf{p} = [p_1, p_2, \dots, p_L]^T$  which contains the model parameters.

## 2.2. Spectral concentration as a cost function

A criteria for evaluating the quality of a time-frequency representation is the concentration. A better concentration in the time-frequency plane means that the signal's energy is focused in a smaller region and consequently any detection or estimation performed in the time-frequency plane is expected to be more reliable [12]. Several concentration measures have been proposed in literature which are based on distribution norms [16, 17, 18] or have been derived from a classical definition of a time-limited signal's duration [19]. Each of these concentration measures can be applied only over frequency to obtain a measure of the spectral concentration.

Typically, a measure of concentration is used to determine an optimum parameter in a time-frequency representation (e.g. the window length for a spectrogram). However, such a measure can also be used to estimate the unknown parameters of the basis function  $\theta_{\mathbf{p}}(t)$ . The idea is to warp the signal in (1) with different test functions obtained for various values assigned to the parameters. When the test function matches the real basis function, the warped signal will essentially be a sum of complex sinusoids and its spectrum will have the highest degree of concentration. Consequently, a concentration measure applied to the warped signal's spectrum  $S_{\mathbf{p}}(\alpha)$  will reach its optimum value when the warping is done with the optimal basis function. So the measure of concentration can be viewed as a cost function which has to be minimized with respect to some model parameters describing  $\theta_{\mathbf{p}}(t)$ . After the optimal model parameters are determined, the amplitude, phase and modulation rate of each component can be extracted from the optimal power spectrum  $|S_{opt}(\alpha)|^2$  obtained from the signal  $s(t)$  warped with the best-matched basis function.

We have chosen for the proposed estimation method the spectral concentration measure from [19] because in the optimization process it searches for a compromise such that all components are well concentrated and does not favor peaky components over others. This

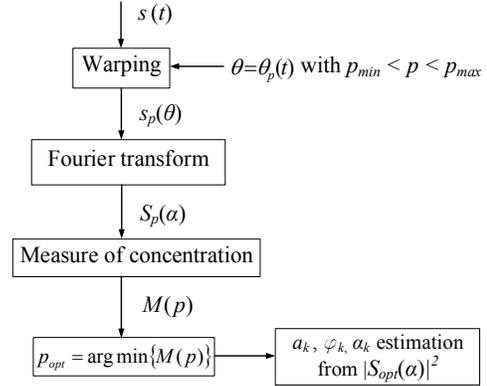


Fig. 1. Estimation algorithm diagram for the single parameter case.

measure is defined as

$$M(\mathbf{p}) = \left( \int_{-\alpha_M}^{\alpha_M} |S_{\mathbf{p}}(\alpha)| d\alpha \right)^2 / \left( \int_{-\alpha_M}^{\alpha_M} |S_{\mathbf{p}}(\alpha)|^2 d\alpha \right), \quad (5)$$

where  $[-\alpha_M, \alpha_M]$  is the support of the warped signal's spectrum. A diagram of the proposed algorithm in the case when the model has only one parameter  $p$  is shown in Fig. 1. Because the method requires a grid search which can get computationally demanding for a large number of parameters, we limit the examples to basis functions with one or two parameters. This drawback could be addressed by employing adaptive techniques used for finding the optimum parameters for time-frequency representations [12], but this is out of the scope of this paper.

## 2.3. Discrete signal model and the Cramér-Rao bound

In the discrete signal model we add noise and consider the basis function dependent on the parameters vector  $\mathbf{p}$ . So the discrete form  $s[n]$  of the signal in (1) uniformly sampled at  $N$  time instants  $t_0, t_1, \dots, t_{N-1}$  embedded in a white circular Gaussian noise  $w[n]$  with variance  $\sigma^2$  is expressed as

$$x[n] = \sum_{k=1}^K s_k[n] + w[n] \quad (6)$$

where

$$s_k[n] = a_k \exp \{j [\varphi_k + \alpha_k \theta_{\mathbf{p}}(t_n)]\}. \quad (7)$$

We introduce the following notations:  $\mathbf{a} = [a_1, a_2, \dots, a_K]^T$ ,  $\boldsymbol{\varphi} = [\varphi_1, \varphi_2, \dots, \varphi_K]^T$ ,  $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_K]^T$ ,  $\mathbf{s}_k = [s_k[0], s_k[1], \dots, s_k[N-1]]^T$  and  $\mathbf{w} = [w[0], w[1], \dots, w[N-1]]^T$ . The signal in (6) can be rewritten in vectorial forms as

$$\mathbf{x} = \mathbf{s} + \mathbf{w} = \sum_{k=1}^K \mathbf{s}_k + \mathbf{w} \quad (8)$$

and the set of unknown parameters that have to be estimated is  $\boldsymbol{\vartheta} = (\mathbf{p}^T, \mathbf{a}^T, \boldsymbol{\varphi}^T, \boldsymbol{\alpha}^T)^T$ .

Note that in a warped time axis  $\theta$ , the samples of  $x[n]$  are related to the time instants  $\theta_n = \theta(t_n)$  which leads to a non-uniformly sampled signal. Consequently the computation of the Fourier transform

of  $x[n]$  in the  $\theta$  time axis can be efficiently implemented by a re-sampling of the initial signal (to obtain a uniformly sampled signal) followed by a Fast Fourier Transform (FFT). So an MST could be performed directly by applying (4), but the advantage of the time warping approach is the computation efficiency.

The Cramér-Rao (CRB) bound for the parameters that have to be estimated can be computed as follows. From [20] the elements of the Fisher's information matrix are

$$I_{ij} = \frac{2}{\sigma^2} \text{Re} \left\{ \frac{\partial \mathbf{s}^H}{\partial \theta_i} \frac{\partial \mathbf{s}}{\partial \theta_j} \right\}. \quad (9)$$

It can be shown that each vector from this expression can take the following four forms depending with respect to which parameter is the derivative taken:

$$\begin{aligned} \frac{\partial \mathbf{s}}{\partial p_l} &= \{[s_1 \ s_2 \ \dots \ s_K] \alpha\} \circ j \left[ \frac{\partial \theta_{\mathbf{p}}(t_0)}{\partial p_l}, \frac{\partial \theta_{\mathbf{p}}(t_1)}{\partial p_l}, \dots, \frac{\partial \theta_{\mathbf{p}}(t_{N-1})}{\partial p_l} \right]^T \\ \frac{\partial \mathbf{s}}{\partial a_k} &= \mathbf{s}_k \\ \frac{\partial \mathbf{s}}{\partial \phi_k} &= j \mathbf{s}_k \\ \frac{\partial \mathbf{s}}{\partial \alpha_k} &= j \mathbf{s}_k \circ [\theta_{\mathbf{p}}(t_0), \theta_{\mathbf{p}}(t_1), \dots, \theta_{\mathbf{p}}(t_{N-1})]^T \end{aligned} \quad (10)$$

where  $\circ$  denotes the element-wise product. After computing the information matrix  $I$  the Cramér-Rao bounds on the variances of the estimates are expressed as  $\text{diag}(I^{-1})$ . Notice that due to the presence of multiple components the bounds will depend on the parameters' values and don't have a straight forward analytic expression, but can be numerically computed. A similar situation was reported in [5] for multi-component PPSs.

### 3. RESULTS

In this section the proposed estimation algorithm is applied first on a synthetic signal and then in two different practical contexts - as a nonlinearity correction algorithm for frequency modulated continuous wave (FMCW) radars and as kernel in a time-frequency tracking procedure for signals with harmonics.

#### 3.1. Simulations

We consider a signal with three components whose parameters are  $\mathbf{a} = (1, 1, 1)$ ,  $\boldsymbol{\varphi} = (0, \pi/2, \pi)$ ,  $\boldsymbol{\alpha} = (25, 50, 100)$ . The basis function depends on a parameter  $p$  and has the form

$$\theta_p(t) = t + t_r \left( \frac{t}{t_r} \right)^p, \quad (11)$$

where  $t_r$  is considered equal to one and was added only for units of measurement consistency. The true value for  $p$  is set to 3 and the search interval is considered [2, 4]. The signal's duration is 1 s and the number of samples is 2000. The estimation algorithm was applied for a signal-to-noise ratio (SNR) of 20 dB and the resulting measure of concentration versus  $p$  is shown in Fig. 2. The spectrograms of the initial signal and of the signal warped with the optimal function are shown in Figs. 3(a) and 3(b), respectively.

To evaluate the performance of the method regarding the estimation of the model parameters, the mean error and variance of the proposed estimator for parameter  $p$  were obtained with Monte Carlo simulations on 1000 realizations for each SNR value. Additionally, the CRB was numerically computed. The results are presented in Fig. 4.

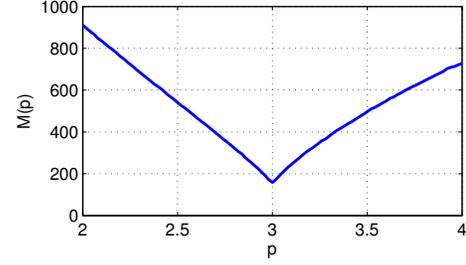


Fig. 2. Measure of concentration versus the model parameter  $p$ .

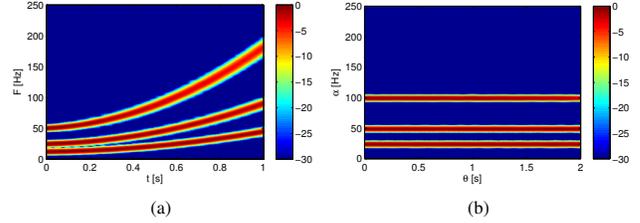


Fig. 3. Spectrograms of the simulated synthetic signal: (a) in the original time axis and (b) in the warped time axis for the optimal basis function.

#### 3.2. FMCW radar nonlinearity correction

A typical issue of an FMCW radar is that the voltage controlled oscillator (VCO) adds a certain degree of nonlinearity which leads to a deteriorated resolution by spreading the targets' energy through different frequencies. For many VCOs the frequency-voltage characteristic can be approximated by a third order polynomial expression. Under this assumption it can be shown [21] that in the case of a linear tuning voltage the beat signal for  $K$  targets resulted from mixing the received signal with the local oscillator is expressed in a sweep period  $T$  as

$$s_b(t) = \sum_{k=1}^K a_k \exp \left\{ j \left[ \varphi_k + \frac{2r_k}{c_0} \beta_0 (t + p_1 t^2 + p_2 t^3) \right] \right\}, \quad (12)$$

where  $c_0$  is the speed of light,  $\beta_0$  is the chirp rate in the origin,  $r_k$  is the range of target  $k$  and  $p_{1,2}$  are the coefficients describing the nonlinearity. Notice that the beat signal can be brought to the form in (1) by taking the basis function

$$\theta_{p_1, p_2}(t) = t + p_1 t^2 + p_2 t^3. \quad (13)$$

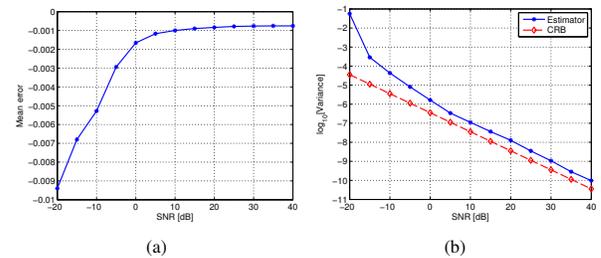
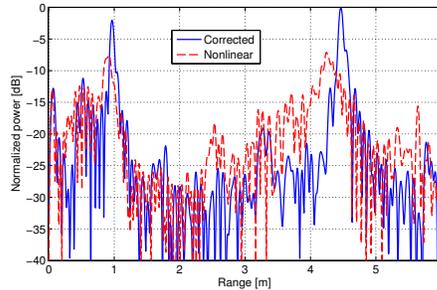


Fig. 4. Mean error (a) and variance (b) of the estimator for parameter  $p$  obtained with Monte Carlo simulations on 1000 realizations.

By applying the proposed algorithm on the beat signal, the nonlinearity coefficients are determined and the range profile (computed as the Fourier transform of the signal warped with the optimal basis function) gets corrected of nonlinearities. Fig. 5 shows the range profiles obtained with the short-range X-band radar platform presented in [22] for a scene with two targets (placed approximatively at 1 m and 4.5 m from the radar, respectively) before and after the nonlinearity correction. For each range profile, a Blackman window was applied before the FFT. In the initial range profile, the targets' energies are spread in frequency while in the corrected one the targets appear as clear peaks. Note that this nonlinearity correction



**Fig. 5.** Range profiles for a short-range X-band FMCW radar before and after nonlinearity correction with the concentration measure method.

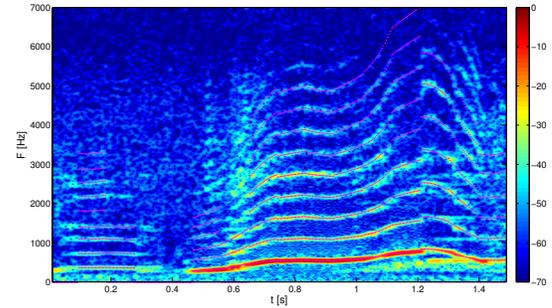
algorithm based on the measure of concentration can be applied disregarding the number of targets and can be viewed as an upgrade of the HAF-based algorithm presented in [23] where a reference response was needed. For comparison, in Table 1 we present the half power lobe width (at 3 GHz bandwidth) obtained with the method proposed in this paper for the target placed at 4.5 m in comparison with the lobe width obtained with the HAF-based correction method when the two targets are in turn used as reference responses. When the correction is applied to a quite different range interval than that of the considered reference target (e.g., to the target placed at 4.5 m when the nonlinearity parameters are estimated using the target placed at 1 m) the HAF-based correction is outperformed by the concentration measure approach. Conversely, when the nonlinearity estimation and correction are done for the same target, the two methods have similar performances.

**Table 1.** Nonlinearity Correction Algorithms: -3 dB lobe widths comparison for the target placed at 4.5 m from the radar.

Algorithm	Lobe width (cm)
Concentration measure	10.70
HAF, reference at 4.5 m	10.87
HAF, reference at 1 m	12.17

### 3.3. Time-Frequency tracking

The parameters estimation algorithm proposed in this paper can be easily applied for time-frequency tracking of non-stationary signals composed of multiple harmonics. We emphasize this by analyzing a



**Fig. 6.** Spectrogram of a humpback whale vocalizations superimposed with the identified time-frequency trajectories.

signal consisting in vocalizations of humpback whales which contain a fundamental component and several harmonics. The signal is divided in several non-overlapping windows and for each of them the estimation algorithm is applied using the basis function given in (13) which implies that for each window, the IFL of every component will be approximated by a second-order polynomial expression. Because the components that have to be tracked are in harmonic relation, the signal obtained in every window has the form in (1) with the particularity that the modulation rate  $\alpha_k$  will be an integer multiple of the fundamental's modulation rate  $\alpha_1$ . By locally warping the signal with the optimal basis function all the non-stationary harmonics become sinusoids in harmonic relation. The estimated modulation rates are the frequencies of these sinusoids in the warped time axis. The IFLs in each window can be easily computed using the estimated basis function and the modulation rates.

In the conducted analysis we considered only the signals having in addition to the fundamental component at least the next two consecutive harmonics. The time-frequency trajectories obtained by joining the estimated IFLs are shown superimposed on the spectrogram of the analyzed signal in Fig. 6. This tracking method simultaneously finds the harmonics in a given window and can be applied in extraction and classification of time-frequency contours [24]. However, the procedure is limited to signals composed from a fundamental component and a number of harmonics in comparison to other multi-component time-frequency tracking approaches [4, 25].

## 4. CONCLUSIONS

In this paper we have proposed a parameters estimation technique designed for signals consisting of multiple components modulated with the same basis function which depends on a few parameters. The algorithm is based on warping the signal according to different modulation functions obtained for various parameters and selecting as the best-matched function the one that delivers the highest spectral concentration of the warped signal.

The proposed approach is conceptually situated between the matched signal transform and the typical polynomial phase estimation algorithms. Therefore, on one hand has the advantage that can naturally deal with multiple components, but on the other hand requires that all the components should obey an a priori known model. In future work, a more extensive comparison between the proposed method and other multi-component estimation algorithms will be performed in terms of theoretical performances and application oriented aspects.

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