ESTIMATION OF RAPIDLY VARYING SEA CLUTTER USING NEAREST KRONECKER PRODUCT APPROXIMATION

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ABSTRACT

In this paper, we propose a method to estimate the space-time covariance matrix of rapidly varying sea clutter. The method first develops a dynamic state space representation for the covariance matrix and then approximates the covariance using the nearest Kronecker product to reduce computational complexity. Particle filtering is then applied to estimate the dynamic elements of the covariance matrix. We validate the nearest Kronecker product approximation using real sea clutter radar measurements. We further demonstrate the use of the estimated space-time covariance matrix in the track-before-detect filter to track a low observable target in sea clutter.

I. INTRODUCTION

Tracking low observable targets in maritime applications is a difficult problem due to severe degradation caused by sea wave reflections. Low signal-to-clutter ratio (SCR) conditions result as the radar cross section of sea swells is substantial and fast moving waves cause large Doppler shifts. Various target detection methods have been investigated to increase SCR based on modeling the statistical properties of real sea clutter. In particular, it was shown that sea clutter amplitude follows a non-Gaussian distribution [1]–[4]. The compound Gaussian (CG) model [5] is well-established for characterizing sea clutter; it models small scale structures on the sea surface with short decorrelation time as complex Gaussian distributed speckle. The speckle is modulated by slow-varying texture, a component associated with long sea waves and swell structure, that decorrelates much slower than speckle. Using the CG model, many adaptive detection approaches have been proposed that estimate the covariance matrix of sea clutter using sample covariance, assuming independence between neighboring range bins [6]–[8]. However, as sea clutter has been shown to be highly correlated between neighboring range bins [5], space-time correlation properties of sea clutter need to be considered.

The estimation of the space-time covariance matrix (ST-CM) of dynamic sea clutter using multiple particle filtering (PF) [9] was proposed and validated in [10]. This approach is not practically feasible as it estimates all ST-CM elements; the number of elements exponentially increases as the number of range bins and/or number of pulses used for coherence processing increases. In addition, the ST-CM positive definiteness is not guaranteed due to independent multiple PF, and it assumes knowledge of a noisy covariance matrix, estimated by averaging a large number of measurements.

In this paper, we propose estimating the sea clutter ST-CM using the nearest Kronecker product approximation (NKPA) with PF. Using the NKPA drastically reduces the number of elements to be estimated and does not require a large number of measurements. The likelihood function in the PF updates the particle weights using a significantly lower number of fast time measurements. Finally, the proposed method is guaranteed to yield a positive definite estimate for the covariance matrix. The sea clutter covariance estimation is applied to a target tracking problem under low SCR.

This paper is organized as follows. In Section II, we propose an NKPA based state space covariance estimation approach and demonstrate its PF implementation in Section III. In Section IV, we demonstrate the estimation in a low SCR tracking application. Simulations to demonstrate the validity of the sea clutter covariance estimation approach are provided in Section V.

II. SEA CLUTTER MODEL

A. Measurement Model

We consider a pulse Doppler radar operating at F_{PRF} Hz pulse repetition frequency (PRF) and transmitting N_p pulses per dwell in rapidly varying sea clutter. The same transmit signal is used within each dwell, given by a linear frequency-modulated (LFM) chirp $s[n]$, $n=0,\ldots,N_s-1$, with bandwidth $\mathcal{B}_s$ Hz and pulse width N_s samples. At the receiver, we assume that the region of measurement selection for track updates or validation gate at the kth dwell consists of M_k range bins, m_k is the first range bin. The noisy observation signal $y(n/f_s,p)$ from the pth pulse at the kth dwell is sampled at f_s Hz to obtain

$$y_k[n,p] = \sum_{m=n-N_s+1}^{n} a_k[m,p]s[n-m] + u_k[n,p]\,, \tag{1}$$

where $n=m_k,\ldots,m_k+M_k+N_s-1$, $p=0,\ldots,N_p-1$, $u_k[n,p]$ is assumed to be zero-mean, white Gaussian observation noise at the kth dwell, $a_k[n,p]=\xi_k[n,p]\,\exp(j2\pi\nu_k\,p/F_{\text{PRF}})$, and $\xi_k[n,p]$ is the complex reflectivity of the aggregated scatterers associated with the nth range bin with Doppler shift ν_k. At the kth dwell, considering all N_p pulses and M_k+2N_s-1 range bins, the overall scatterer contribution is represented by the $((M_k+2N_s-1)\times N_p)$ reflection matrix $\mathbf{A}_k$ whose (n,p)th element is $a_k[n,p]$.

If a target is present, the signal in (1) includes both the target and clutter. If β_k is the target reflectivity at range bin m_t, which is assumed unknown [11], with Doppler shift ν, and we denote $\beta_{k,n}=\beta_k\,\exp(j2\pi\nu p/F_{\text{PRF}})$ then the received measurement is

$$y_k[n,p] = \beta_{k,n}s[n-m_t] + \sum_{m=n-N_s+1}^{n} a_k[m,p]s[n-m] + u_k[n,p].$$

Note that the range r_k and range rate $\dot{r}_k$ of the target at the kth dwell are given by $r_k=m_t v_c/(2f_s)$ and $\dot{r}_k=\nu\,v_c/(2f_c)$, where v_c is the velocity of propagation and f_c Hz is the carrier frequency. After matched filtering, the resulting measurement is given by

$$z_k[n,p] = \sum_{m=n}^{n+N_s-1} y_k[n,p]\,s_p^*[n-m] \tag{2}$$

for $n = m_k, \ldots, m_k + M_k - 1$, $p = 0, \ldots, N_p - 1$. Here, we use $y_k[n, p]$ as defined in (1) and assume that only clutter is present. The overall clutter measurements at the kth dwell can be represented by the $(M_k \times N_p)$ matrix $\mathbf{Z}_k$.

B. Measurement State Space Model

As sea clutter is dynamically varying, its state transitions between adjacent range bins depending on the sea waves relative velocity with respect to the radar. In [10], the spectral component of the reflectivity matrix was dynamically modeled to show its transition between adjacent bins using the Doppler shifts. Using a similar approach, we model the matched filter output using the $(M_k \times N_p)$ matrix $\mathbf{B}_k = \mathbf{Z}_k \mathbf{D}$, where $\mathbf{D}$ is an $(N_p \times N_p)$ discrete Fourier transform matrix. The elements of $\mathbf{D}$ are such that the first $(N_p - 1)/2$ columns list the negative Doppler shifts, the middle column is the zero Doppler shift, and the remaining columns list the positive Doppler shifts. In this model, most of the sea clutter components can be shown to concentrate around the middle column under calm sea state conditions but move away from the middle under turbulent conditions.

We represent the clutter state transition in vector form by stacking the columns of $\mathbf{B}_k$ from left to right to form the $(M_k N_p \times 1)$ vector $\mathbf{b}_k = \text{vec}(\mathbf{B}_k)$. We similarly represent the matched filter output at the kth dwell as the vector $\mathbf{z}_k = \text{vec}(\mathbf{Z}_k)$. The relation between these two vectors can be shown to be $\mathbf{b}_k = (\mathbf{D}^H \otimes \mathbf{I}_{M_k})\, \mathbf{z}_k$, where H denotes Hermitian transpose and $\mathbf{I}_{M_k}$ is the $(M_k \times M_k)$ identity matrix. The Kronecker product (KP) operator $\otimes$ computes the KP on the $(N_p \times N_p)$ matrix $\mathbf{D}^H$ and the $(M_k \times M_k)$ matrix $\mathbf{I}_{M_k}$ to form an $(M_k N_p \times M_k N_p)$ block matrix. The clutter state transition can be modeled using the state equation

$$\mathbf{b}_{k+1} = \mathbf{F}\mathbf{b}_k + \mathbf{v}_{k+1}\,, \tag{3}$$

where $\mathbf{v}_{k+1}$ is the modeling random error process assumed zero-mean complex Gaussian with covariance $\mathbf{V}_{k+1}$. The $(M_k N_p \times M_k N_p)$ state transition matrix $\mathbf{F}$ (defined in Equation (9) in [10]) represents the scattering movement between dwells and populates the range-Doppler bins moving into the validation gate. It represents the transition of a fast moving clutter between from range bins n and $n + m$ if the reflector is moving away from the radar and between range bins n and $n - m$ if the reflector is moving towards the radar; the value of m is determined by the Doppler shift.

C. Clutter Covariance Matrix State Space Model

In order to estimate the covariance matrix $\mathbf{\Sigma}_{\mathbf{z}_k}$ of $\mathbf{z}_k$ in Equation (2), we use the relationship between $\mathbf{z}_k$ and $\mathbf{b}_k$ to relate their corresponding covariance matrices $\mathbf{\Sigma}_{\mathbf{z}_k}$ and $\mathbf{\Sigma}_{\mathbf{b}_k}$. From (3), the covariance matrix of $\mathbf{b}_{k+1}$ can be written as

$$\mathbf{\Sigma}_{\mathbf{b}_{k+1}} = \mathbf{F}^H\, \mathbf{\Sigma}_{\mathbf{b}_k}\, \mathbf{F} + \mathbf{G}_{k+1} \tag{4}$$

where $\mathbf{G}_{k+1}$ is assumed Wishart distributed with parameters $\mathbf{V}_{k+1}$ and $M_k N_p$ degrees of freedom. Since $\mathbf{b}_k = (\mathbf{D}^H \otimes \mathbf{I}_{M_k})\, \mathbf{z}_k$,

$$\mathbf{\Sigma}_{\mathbf{b}_k} = (\mathbf{D}^H \otimes \mathbf{I}_{M_k}) \mathbf{\Sigma}_{\mathbf{z}_k} (\mathbf{D} \otimes \mathbf{I}_{M_k})\,. \tag{5}$$

The covariance $\mathbf{\Sigma}_{\mathbf{z}_k}$ estimate can be obtained by inverting (5). Replacing (5) in (4), we obtain

$$\mathbf{\Sigma}_{\mathbf{b}_{k+1}} = \mathbf{F}^H (\mathbf{D}^H \otimes \mathbf{I}_{M_k})\, \mathbf{\Sigma}_{\mathbf{z}_k} (\mathbf{D} \otimes \mathbf{I}_{M_k})\, \mathbf{F} + \mathbf{G}_{k+1}\,. \tag{6}$$

This covariance state space model is similar to the one in [10]. Thus, as the size of the covariance matrix grows exponentially with N_p and M_k, the estimation of the covariance in (6) becomes very computationally intensive.

D. Covariance Nearest Kronecker Product Approximation

Considering the formulation of the $(M_k \times N_p)$ measurement matrix $\mathbf{Z}_k$ in Section A, we can view the rows and columns of the matrix as an $(M_k \times 1)$ temporal vector $\mathbf{q}_k$ and an $(N_p \times 1)$ spatial vector, $\mathbf{c}_k$, respectively. In particular, assuming that the temporal and spatial vectors have the same distribution for all range bins and pulses, respectively, then we can model the measurement matrix as the KP on the two random vectors. Specifically,

$$\mathbf{Z}_k = \mathbf{q}_k^H \otimes \mathbf{c}_k\,. \tag{7}$$

This assumption is consistent with the popular compound-Gaussian sea clutter model in which the slow varying texture component can be viewed as a spatial process and the fast varying speckle component can be viewed as a temporal process. Using KP properties [12], the covariance of $\mathbf{Z}_k$ can be written as

$$\mathbf{\Sigma}_{\mathbf{z}_k} = \mathbf{\Sigma}_{\mathbf{q}_k} \otimes \mathbf{\Sigma}_{\mathbf{c}_k}\,. \tag{8}$$

The state space model in (6) can then be given by

$$\mathbf{\Sigma}_{\mathbf{b}_{k+1}} = \mathbf{F}^H (\mathbf{D}^H \mathbf{\Sigma}_{\mathbf{q}_k} \mathbf{D} \otimes \mathbf{\Sigma}_{\mathbf{c}_k})\, \mathbf{F} + \mathbf{G}_{k+1}\,. \tag{9}$$

In order to maintain the KP form in (8) at the $(k+1)$th dwell transition, we impose the following covariance constraint

$$\mathbf{\Sigma}_{\mathbf{z}_{k+1}} = \arg \min_{\mathbf{\Sigma}_{\mathbf{q}_{k+1}}, \mathbf{\Sigma}_{\mathbf{c}_{k+1}}} \| \hat{\mathbf{\Sigma}}_{\mathbf{z}_{k+1}} - \mathbf{\Sigma}_{\mathbf{q}_{k+1}} \otimes \mathbf{\Sigma}_{\mathbf{c}_{k+1}} \|_{\mathbb{F}} \tag{10}$$

where $\| \cdot \|_{\mathbb{F}}$ is the Frobenius matrix norm. The covariance matrix $\hat{\mathbf{\Sigma}}_{\mathbf{z}_{k+1}}$ is a function of $\mathbf{\Sigma}_{\mathbf{q}_k}$ and $\mathbf{\Sigma}_{\mathbf{c}_k}$ at the kth dwell, and it is obtained by substituting (9) in (5). The minimization problem in (10) corresponds to a nearest KP approximation (NKPA) problem [13]; the solution is the cross product of the singular vector corresponding to the maximum singular value of the permuted version of $\hat{\mathbf{\Sigma}}_{\mathbf{z}_{k+1}}$. Solving the minimization in (10) using the NKPA results in a drastic reduction in computational complexity when estimating the measurement covariance matrix. Specifically, the NKPA reduces the number of matrix elements to be estimated from $(N_p M_k (N_p M_k + 1)/2)$ to $[N_p(N_p + 1) + M_k(M_k + 1)]/2$. For example, if $N_p = 10$ pulses and $M_k = 10$ range bins, the element estimation reduction is from 5050 to 110 elements.

III. PARTICLE FILTER IMPLEMENTATION

As the state model in (9) is not linear, we use PF to estimate the covariance matrix elements [14]. The PF represents the spatial and the temporal covariance matrices by a set of particles and corresponding weights. Given the initial particle states, the predicted matrices at dwell $k + 1$ are obtained using the state model. The predicted particles are updated using the clutter measurement likelihood function. Specifically, if we denote the particle covariance matrices as $\{[\mathbf{\Sigma}_{\mathbf{q}_k}^{(i)}, \mathbf{\Sigma}_{\mathbf{c}_k}^{(i)}], w^{(i)}\}$, where $w^{(i)}$ is the weight for the ith particle, then assuming that the clutter measurement is complex Gaussian, the log likelihood function is

$$l\Big(\mathbf{z}_k |\, \mathbf{\Sigma}_{\mathbf{q}_k}^{(i)}, \mathbf{\Sigma}_{\mathbf{c}_k}^{(i)}\Big) = -N_p \log(|\mathbf{\Sigma}_{\mathbf{c}_k}^{(i)}|) - M_k \log(|\mathbf{\Sigma}_{\mathbf{q}_k}^{(i)}|)$$
$$-\text{tr}\left\{ \hat{\mathbf{\Sigma}}_k \left[(\mathbf{\Sigma}_{\mathbf{q}_k}^{(i)})^{-1} \otimes (\mathbf{\Sigma}_{\mathbf{c}_k}^{(i)})^{-1} \right] \right\}$$

where $|\mathbf{\Sigma}|$, $\mathbf{\Sigma}^{-1}$, and $\text{tr}(\mathbf{\Sigma})$ are the determinant, inverse and trace of $\mathbf{\Sigma}$, respectively, and the sample covariance matrix is obtained as

$$\hat{\mathbf{\Sigma}}_k = \frac{1}{N_T} \sum_{\ell = k - N_T - 1}^{k} \mathbf{z}_\ell \mathbf{z}_\ell^H\,.$$

Here, we assume that the clutter statistics do not drastically change while the N_T (past and present) dwell measurements are obtained. Since the clutter can be fast varying, the number of samples used for the sample covariance estimate is usually much smaller than the vector dimension, i.e., $N_T \ll N_p M_k$. If we do not use the NKPA on the covariance matrix, the maximum likelihood estimate (MLE) is the sample covariance matrix, which will only be positive definite if $N_T \geq N_p M_k$. However, by assuming the NKPA, the MLE in KP form is positive definite as long as $N_T \geq [\max\{(M_k/N_p), (N_p/M_k)\} + 1]$ [15], [16]. Moreover, the likelihood computation is further simplified since we compute the inverse and determinant of two matrices of low dimension instead of one matrix of higher dimension.

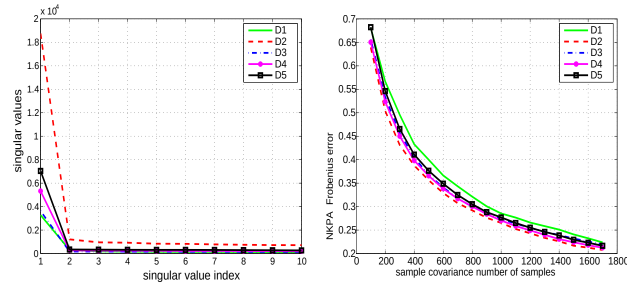
Assuming that the initial covariance matrices $\mathbf{\Sigma}_{\mathbf{q}_k}$, $\mathbf{\Sigma}_{\mathbf{c}_k}$ and the modeling error matrix $\mathbf{G}_{k+1}$ are positive definite, then $\mathbf{\Sigma}_{\mathbf{q}_{k+1}}$ in (9) is also positive definite. Using (5), $\hat{\mathbf{\Sigma}}_{\mathbf{z}_{k+1}}$ in (10) is also positive definite. For a symmetric positive definite matrix, the solution to the NKPA also results in symmetric positive definite matrices [13]. Therefore, $\mathbf{\Sigma}_{\mathbf{q}_{k+1}}$ and $\mathbf{\Sigma}_{\mathbf{c}_{k+1}}$ in (10) are positive definite. Since all the particles correspond to positive definite matrices, the updated particles are also positive definite. This ensures that the proposed covariance matrix estimate is always positive definite.

IV. TRACK-BEFORE-DETECT IN SEA CLUTTER

In this section, we use the proposed NKPA-based covariance matrix estimation to track a low observable target in the presence of sea clutter. We consider a target moving in a two-dimensional (2-D) plane with state vector $\mathbf{x}_k = [x_k\, \dot{x}_k\, y_k\, \dot{y}_k]^{\mathrm{T}}$, where (x_k, y_k) and $(\dot{x}_k, \dot{y}_k)$ are the 2-D Cartesian coordinates of the target position and velocity, respectively, at the kth dwell. The target state is modeled as $\mathbf{x}_k = \mathbf{H}(\mathbf{x}_{k-1}) + \mathbf{w}_k$, where $\mathbf{H}$ is a state transition function and $\mathbf{w}_k$ is the modeling error.

The single target recursive track-before-detect (TBD) algorithm in [17] is modified for use with our measurement model. Using this algorithm, a target leaving the field-of-view (FOV) and a target already in the FOV are represented by two sets of particles. The posterior probability density of the target is obtained as a weighted combination of the particles from both sets. The algorithm can also provide an analytical expression for estimating the target existence probability. The measurement component associated with the target is present in the neighborhood of the range bin under test (in which the target is present). This component is governed by the correlation properties of the transmitted signal [18]. Thus, detection and tracking must be performed using all the neighborhood range bins, including the range bin under test. Specifically, for a target present at range bin m_t, the measurement data is extracted from the measurement matrix $\mathbf{Z}_k$ (that contains both the target and clutter) as $\mathbf{z}_{k,m_t} = \mathrm{vec}(\mathbf{Z}_k[m_t - N_h : m_t + N_h, 0 : N_p - 1])$, where N_h is the number of neighborhood bins. The covariance matrix $\mathbf{\Sigma}_{k,m_t}$ that corresponds to this vector is a principal sub-matrix of the full covariance matrix estimated in Section D. This sub-matrix is also positive definite since any principal sub-matrix of a positive definite matrix is also positive definite [19].

In an actual tracking application, estimating the clutter covariance matrix is a challenging problem as the measurements include the target component. In practice, the clutter is assumed homogeneous so that the clutter covariance can be estimated using range bins in the neighborhood of the range bin under test. If the clutter is heterogeneous, then this assumption can lead to poor detection performance. Here, we exploit the state space clutter model to predict the clutter covariance matrix $\mathbf{\Sigma}_{k,m_t}$ from the previous clutter covariance matrix estimate $\mathbf{\Sigma}_{\mathbf{z}_{k-k_0}}$. Specifically,



(a) (b)

Fig. 1. (a) Singular value of permuted covariance matrix using real sea clutter. (b) NKPA error for data sets D1-D5.

we assume that the probability that the target is still present in range bin m_t at the $(k - k_0)$th dwell is very low. Thus, the sub-matrix $\mathbf{\Sigma}_{k,m_t}$ can be extracted from the predicted covariance matrix obtained as,

$$\mathbf{\Sigma}_{\mathbf{z}_{k-k_0+1}} = (\mathbf{D}^{-H} \otimes \mathbf{I}_{M_k})\mathbf{F}^{H}(\mathbf{D}^{H}\mathbf{\Sigma}_{\mathbf{q}_{k-k_0}}\mathbf{D} \otimes \mathbf{\Sigma}_{\mathbf{c}_{k-k_0}})\mathbf{F}(\mathbf{D}^{-1} \otimes \mathbf{I}_{M_k})$$

$$\mathbf{\Sigma}_{\mathbf{z}_{k-k_0+1}} = \mathrm{NKPA}[\mathbf{\Sigma}_{\mathbf{z}_{k-k_0+1}}]$$

$$\vdots$$

$$\mathbf{\Sigma}_{\mathbf{z}_k} = (\mathbf{D}^{-H} \otimes \mathbf{I}_{M_k})\mathbf{F}^{H}(\mathbf{D}^{H}\mathbf{\Sigma}_{\mathbf{q}_{k-1}}\mathbf{D} \otimes \mathbf{\Sigma}_{\mathbf{c}_{k-1}})\mathbf{F}(\mathbf{D}^{-1} \otimes \mathbf{I}_{M_k})$$

$$\mathbf{\Sigma}_{\mathbf{z}_k} = \mathrm{NKPA}[\mathbf{\Sigma}_{\mathbf{z}_k}]$$

where NKPA$[\mathbf{A}]$ is the NKPA of a matrix $\mathbf{A}$. The likelihood function for a signal embedded in complex Gaussian clutter can then be derived as

$$l(\mathbf{z}_k|\mathbf{x}_k) = \exp\left(\mathrm{Re}\{\mathbf{r}_{k,m_t}^{H}\mathbf{\Sigma}_{k,m_t}^{-1}\mathbf{z}_{k,m_t}\}^2/\mathbf{r}_{k,m_t}^{H}\mathbf{\Sigma}_{k,m_t}^{-1}\mathbf{r}_{k,m_t}\right).$$

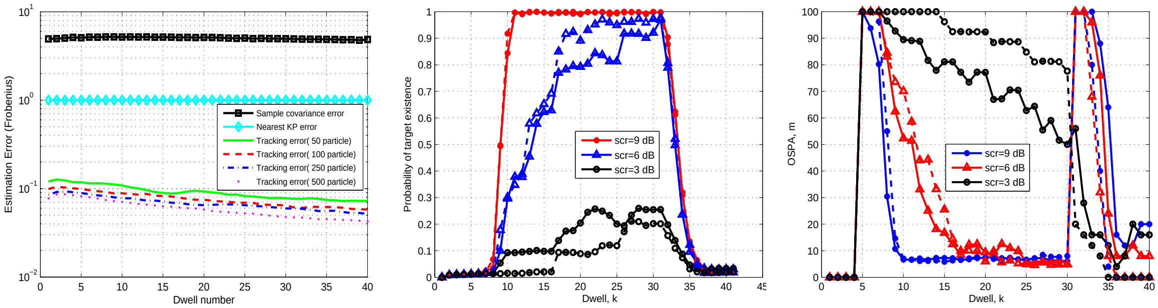
If we define the cross-correlation of the transmitted signal as

$$r_s[n] = \sum_{m=0}^{N_s-1} s[m]s^*[m-n]\,,$$

then $\mathbf{r}_{k,m_t}$ is obtained by vectorizing the matrix formed by stacking the vectors $[r_s[m]\, r_s[m]e^{j2\pi\nu/F_{\mathrm{PRF}}} \ldots r_s[m]e^{j2\pi\nu(N_p-1)/F_{\mathrm{PRF}}}]^{\mathrm{T}}$, $m = -N_h, \ldots, N_h$.

V. SIMULATIONS

Real Sea Clutter Covariance Estimation. We first investigated the validity of the NKPA using real clutter data from the DSTO INGARA radar sea clutter database [4]. The clutter data was obtained using the following radar parameter: 96 MHz signal bandwidth, 8 μs pulse width, 9.375 GHz carrier frequency, 500 Hz PRF, 1.5 m range resolution and vertical-transmit, vertical-receive polarization. The wind speed was at 10-12 knots, resulting in a 2-3 sea state. As the true covariance matrix was not available, the NKPA was validated using the sample covariance matrix, obtained by averaging across multiple dwells and calculated by constructing a measurement dwell with $N_p = 10$ pulses and $M_k = 10$ range bins. Figure 1(a) shows the singular values of the permuted sample covariance matrix computed by averaging over 1,700 dwells (3.4 s) from 5 different data sets. The first singular value was the most dominant one for all 5 sets, thus most of the energy could be compacted by a single NKPA. Figure 1(b) shows the normalized Frobenius norm error between the sample covariance and the NKPA



(a) (b) (c)

Fig. 2. (a) Covariance matrix estimation (Frobenius) error (b) Probability of target existence and (c) tracking error for varying SCR: actual and NKPA (solid), NKPA only (dash).

as a function of sample size. As it can be seen, the error decreases as the number of samples increases implying, that as the sample covariance matrix asymptotically approaches the true covariance matrix the approximation error is also reducing. The approximation error was around 0.2 for all data sets, indicating that NKPA is a reasonable approximation to use.

PF-based Clutter Covariance Estimation. We demonstrated the PF implementation of the covariance estimation using an LFM signal with $\mathcal{B}_s = 15$ MHz bandwidth, $f_c = 9.375$ GHz carrier frequency, $F_{\text{PRF}} = 500$ Hz PRF, $N_p = 11$ pulses per dwell, [8000 8300] m validation gate range, 10 m range resolution, 30 range bins and 60 rpm beam scan rate. The initial covariance matrix $\mathbf{\Sigma}_0^z$ was obtain from compound Gaussian distributed clutter whose speckle and texture correlation was based on real clutter from the Osborne Head Gunnery Range (OHGR) IPIX radar [20], [21]. The speckle samples were drawn from a circularly symmetric complex Gaussian distribution, and the texture components were distributed based on a gamma distribution. The sample covariance matrix was calculated from 3,300 independent dwell measurements. The fast time clutter measurement was obtained by drawing samples from a complex Gaussian distribution with the covariance matrix derived at each dwell. We compared the mean-squared error (MSE) between the true and estimated temporal and spatial covariance matrices using a varying number of (50, 100, 250, and 500) particles in Figure 2(a), averaged over 25 Monte Carlo simulations. Also shown is the tracking MSE for the sample covariance matrix and its corresponding NKPA. The sample covariance matrix is obtained by averaging the measurement from five dwells. The tracking MSE for the sample covariance matrix is much higher; the MSE is somewhat reduced when the NKPA of the sample covariance matrix is used. The tracking MSE of our proposed estimation approach outperformed the other two methods. The improved performance of the covariance matrix estimation is due to exploiting the underlying physical model of the sea structure using the transition matrix $\mathbf{F}$. Note that the tracking MSE can also be reduced by increasing the number of particles.

Tracking Application. We applied the clutter estimation approach to a target tracking problem with similar parameters in the previous simulation. We compared the performance of the algorithm to track a low observable target moving at constant velocity under varying SCR values. The target is assumed to leave and enter the FOV at dwells 5 and 30, respectively. The initial position and velocity for the target were set to (5825.7, 5825.7) m and (-5.4, -5.4) m/s, respectively. PF used 500 particles when the target survived and 2500 particles when the target entered the FOV. The tracking error is quantified using the OSPA metric with parameters $c = 100$ and $p = 2$ [22], averaged over 25 Monte-Carlo simulations. The tracking performance was analyzed under two conditions: (i) the measurement was generated as in (5) and the covariance was estimated using (9) and (10); (ii) both the measurement and the covariance followed the NKPA in (9) and (10). Figures 2(b) and 2(c) show the probability of target existence and the tracking error for different SCRs. The latency in detecting a target increased as the SCR decreased. Similarly, there was delay in detecting a target leaving the FOV. The probability of detection was very low at 3 dB SCR and the tracking error was high. As the probability of detection was in general low, the probability of detecting a target leaving the FOV at 3 dB was also low, as evident by lower OSPA values during dwells 30-35. At 6 dB SCR, the probability of detection increased when the true model does not follow the NKPA; however, this did not result in increased tracking performance because of higher OPSA. In general, the tracking performance improved when the true and assumed models followed the NKPA. Nevertheless, the performance did not degrade significantly when the assumed but not the true model followed the NKPA. This result is relevant to real target tracking applications since, even if the actual covariance does not completely follow the KP structure, we can apply the NKPA without significantly affecting the tracking performance.

VI. CONCLUSION

We impose the KP assumption on the underlying physical model to efficiently track the space-time covariance matrix. The KP assumption enables the estimation of rapidly varying sea clutter by using the sample covariance matrix that is estimated from very few dwells. Given an accurate state space model representation of sea clutter, the estimation error using the state space model with the KP assumption is lower than the NKPA of the sample covariance matrix. The proposed state space model based method is implemented using particle filtering. The NKPA is validated using real sea clutter data, and we applied the estimate to track a lower observable target in sea clutter. More real sea clutter data should be analyzed further to accurately generalize our proposed model.

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