

# LOCALIZATION OF A MOVING NON-COOPERATIVE RF TARGET IN NLOS ENVIRONMENT USING RSS AND AOA MEASUREMENTS

Chi Cheng, Wuhua Hu, Wee Peng Tay

School of Electrical and Electronic Engineering  
Nanyang Technological University, Singapore  
Email: {chengchi, hwh, wptay}@ntu.edu.sg

## ABSTRACT

We propose an alternating optimization algorithm for localizing a mobile non-cooperative target using a wireless sensor network. We consider the scenario where sensors receive single-bounce non-line-of-sight signals from the moving target. Each sensor is able to measure the target signal's angle-of-arrival and received signal strength. The transmit powers of the non-cooperative target at different locations are unknown, and estimated jointly with its locations and the orientations of the scatterers off which the target signals are reflected before reaching the sensors. We formulate the problem as a non-convex least squares problem, and then transform and approximate it into a form that is solvable by an alternating algorithm. We show that our algorithm converges, and simulation results demonstrate that our algorithm is able to localize the target with good accuracy.

*Index Terms*— Localization, wireless sensor network, alternating optimization, RSS, AOA

## 1. INTRODUCTION

Wireless sensor networks (WSNs) have wide applications in control, tracking, and monitoring. Location information of the sensors in a WSN and other targets monitored by it plays an important role in many applications like location-based services [1], and intrusion tracking [2]. The widespread implementation of different wireless technologies has also made wireless localization a service that can be available anytime and anywhere [3].

In this paper, we consider the problem of localizing a mobile non-cooperative RF target using a WSN, where the sensors in the WSN have known locations, and the RF target can be another sensor or a target whose position is unknown and must be estimated. The localization of the RF target can be done via noisy measurements of received signal strength (RSS), signal angle-of-arrival (AOA), time-of-arrival (TOA) and time-difference-of-arrival (TDOA). The techniques we can use are often limited by the actual application. When the source waveform signature is known, it is possible to use techniques like TOA to localize the target. However, in the case of non-cooperative target sources, one must resort to other techniques like TDOA, AOA or RSS. To use TDOA based techniques however, the sensors in the WSN need to be synchronized to within a few tens of nanoseconds. This is because of the high propagation speed of wireless signals, equivalent to the speed of light, which results in small timing errors being translated into large distance errors. Implementation of such high accuracy synchronization for the sensors in the WSN is however a highly non-trivial challenge [4, 5]. In applications where a more simplistic implementation is desired, measurements like RSS and AOA, which do not require sensors to be synchronized, can be utilized.

To perform RSS based localization, since the transmit power is not available for a non-cooperative target, we have to jointly estimate it together with the target location. RSS based sensor localization with unknown transmit power is discussed in [6]. In that paper, a semi-definite relaxation technique [7] is introduced to find the sub-optimal solution of the maximum likelihood estimator. However, when the RSS and AOA measurement equations are combined, it is not easy to directly relax the problem to a semi-definite programming (SDP) problem. Hybrid localization methods using TDOA with AOA can be found in [8, 9]. In [10], two novel hybrid RSS and AOA emitter location estimators are proposed. However, the focus of that paper is to investigate how additional RSS sensors can improve the performance of traditional AOA localization. A related work is [11], in which the authors try to fuse AOA and RSS measurements to improve localization accuracy. They use a set of linear equations to approximate both AOA and RSS measurement equations, and then solve the linear system using weighted least square methods. However, the linearization approach only works when the noise is sufficiently low.

All these techniques [6–11] have been investigated in the context of line-of-sight (LOS) scenarios, where a LOS exists between the target and the different sensing nodes. In an urban environment, typical signal paths from a target source to a sensor is non-line-of-sight (NLOS). To the best of our knowledge, there are no works that have addressed the problem of RF source localization in a NLOS environment using RSS and AOA measurements at multiple sensors. This is because the hybrid equations are highly non-linear and non-convex, and any linearization approach would fail when the noise correlates with both AOA measurements and the unknown scatterer orientations. In this work, we consider the problem of target localization when we have predominantly NLOS signals from a *mobile* target, where the NLOS signal paths are single-bounce paths between the target and sensors. As the target is non-cooperative, we consider using only RSS and AOA measurements of the target signal. To overcome the problem of handling NLOS signals, we propose an optimization approach to *jointly* estimate orientations of the scatterers off which the target signals are reflected before reaching the sensors, and the target's locations and transmission powers. We investigate the use of an alternating optimization method to solve the optimization problem iteratively.

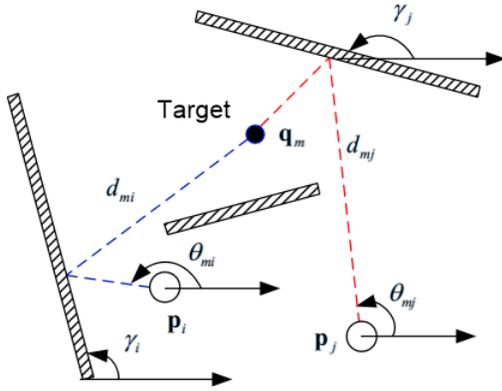
The rest of the paper is organized as follows. In Section 2, we present the target and measurement model, and formulate our localization problem. We briefly describe how to transform, approximate and then solve the optimization problem with our proposed localization algorithm in Section 3. Then in Section 4, we verify the performance of the algorithm using Monte Carlo simulations. Finally, we present our conclusions in Section 5.

## 2. PROBLEM FORMULATION

We consider a network of  $N$  static sensors localizing a mobile RF target. The target emits a signal and the signal reaches each sensor through either a single-bounce NLOS signal path or an LOS signal path, as illustrated in Figure 1. The positions and orientations of the scatterers are unknown a priori. Our analysis in the following shows that it is impossible to localize the target at any given instant if we use sensors' measurements only for that instant. This motivates us to use sensors' measurements at multiple instants to infer the locations of the target at these instants, simultaneously. This leads to a novel localization model of a mobile target under the NLOS environments, as described in detail below.

For each sensor  $n \in \mathcal{N} \triangleq \{1, 2, \dots, N\}$ , and each time instant  $m \in \mathcal{M} \triangleq \{1, 2, \dots, M\}$ , let  $d_{mn}$  be the length of the signal path from the target at the  $m$ -th position to the sensor,  $\theta_{mn}$  be the AOA measurement of the received signal at the sensor, and  $\gamma_n$  be the orientation of the scatterer from which the signal path from the target  $m$  to sensor  $n$  bounces off. As shown in Figure 1, all angles are measured with respect to a commonly agreed fixed horizontal direction. If the signal follows a LOS path, we take the scatterer orientation to be the same as the AOA  $\theta_{mn}$ .

We assume that: i) the  $M$  target positions are different from each other; and ii) for each sensor  $n$ , the target's transmitted signals at these  $M$  positions bounce off the same scatterer before they reach the sensor, i.e.,  $\gamma_n$  is independent of the time instant  $m$ . The latter assumption is true if the target moves within a small area during the period of observation by the sensors.



**Fig. 1.** An example for the one-bounce reflection path from the target at the  $m$ -th position to sensor  $i$  and sensor  $j$  [4].

We have the following relationship, as shown in [4]:

$$d_{mn} = \mathbf{g}^T(\theta_{mn}, \gamma_n)(\mathbf{p}_n - \mathbf{q}_m) \quad (1)$$

where  $\mathbf{p}_n$  is the sensor location,  $\mathbf{q}_m$  is the target location at time instant  $m$ , and  $\mathbf{g}(\theta_{mn}, \gamma_n)$  is given by

$$\mathbf{g}(\theta_{mn}, \gamma_n) = \frac{1}{\cos(\theta_{mn} - \gamma_n)} \begin{bmatrix} \cos \gamma_n \\ \sin \gamma_n \end{bmatrix}.$$

At the same time, we can model the received power (in dBm)  $P_{mn}$ , under log-normal shadowing as

$$P_{mn} = P_{m0} - 10\beta \log_{10} \frac{d_{mn}}{d_{m0}} + n_{mn}, \quad (2)$$

where  $P_{m0}$  (in dBm) is the reference power at distance  $d_{m0}$  from the target (which depends on the transmit power),  $\beta$  is the path loss exponent, and  $n_{mn}$  is the shadowing term, which has an unknown distribution. Without loss of generality, we assume  $d_{m0} = 1$  for each position of the target.

By eliminating the intermediate variable  $d_{mk}$ , the two equations (1) and (2) can be combined into

$$P_{mn} = P_{m0} - 10\beta \log_{10} \mathbf{g}^T(\theta_{mn}, \gamma_n)(\mathbf{p}_n - \mathbf{q}_m) + n_{mn}. \quad (3)$$

We then obtain a non-linear least squares estimation problem defined as:

$$\min_{P_{m0}, \mathbf{q}_m, \gamma_n} \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} (P_{mn} - P_{m0} + 10\beta \log_{10} \mathbf{g}^T_{mn}(\mathbf{p}_n - \mathbf{q}_m))^2, \quad (4)$$

where  $0 \leq \gamma_n < 2\pi$ , and  $\mathbf{g}_{mn} \triangleq \mathbf{g}(\theta_{mn}, \gamma_n)$ . In the formulation (4), if we know a priori that the target's transmit power does not change during the observation period, then  $P_{m0}$  is the same for all the  $M$  different locations. In this case, the number of optimization variables in (4) is reduced, leading to a higher estimation accuracy for the target locations.

In a 2D plane, there are  $3M + N$  decision variables in total:  $2M$  unknown target location variables,  $M$  unknown source transmission powers, and  $N$  unknown reflector angles. The total number of measurement equations is  $MN$  if all sensors receive signals from the target at the  $M$  locations. To make the problem well-posed (i.e., have a unique solution), we should have  $MN \geq 3M + N$ . The feasible values for  $(M, N)$  can be checked to be given by the following set:

$$\Omega = \{(M, N) : M = 2, N \geq 6\} \cup \{(M, N) : M = 3, N \geq 5\} \\ \cup \{(M, N) : M \geq 4, N \geq 4\}.$$

If we assume  $P_{m0} = P_0$  for all  $m \in \{0, 1, \dots, M\}$ , then the problem is well-posed if the pair  $(M, N)$  satisfies  $MN \geq 2M + N + 1$ , i.e.,  $(M, N)$  belongs to the feasible set defined below:

$$\Omega' = \{(M, N) : M = 2, N \geq 5\} \cup \{(M, N) : M = 3, N \geq 4\} \\ \cup \{(M, N) : M \geq 4, N \geq 3\}.$$

Comparing  $\Omega'$  with  $\Omega$ , we observe that the minimum number of sensors required is one less when the target transmit power does not change, given measurements of the target at the same number of locations. This may also imply that the localization accuracy is higher if the same number of sensors are used. Moreover, in either case it can be seen that the measurement of one target at a single location ( $M = 1$ ) is insufficient for performing localization, regardless of the number of sensors in use. Indeed this insight has motivated our problem formulation to use measurements of a single target at multiple locations.

The estimation problem in (4) is non-convex. In the following section, we develop an our alternating algorithm for solving this problem.

**Remark 1.** *The localization problem described in (4) is still applicable if there are more than one static or moving targets emitting signals. In that case, the  $M$  locations in (4) correspond to distinct locations of the  $M$  targets at certain time instants. The problem is well-posed if the pair  $(M, N)$  belongs to the feasible set  $\Omega$  (or  $\Omega'$  if all targets have the same transmit power). In another case, if there are multiple targets but only one sensor, then the problem remains well-posed when the sensor moves and performs measurements at  $N$  different locations while the  $M$  targets remain static during the whole measurement period.*

### 3. ALTERNATING LOCALIZATION ALGORITHM

The optimization problem (4) is highly nonlinear and non-convex due to the logarithm term in the summand, which involves trigonometric functions of the unknown scatterers' orientations and the unknown locations of the target. This makes it very challenging to solve the problem efficiently. In this section, we transform and approximate the problem into a form that is solvable by an alternating optimization algorithm, which is guaranteed to converge. Some open issues related to the algorithm will also be discussed.

Motivated by the technique proposed in [6], we rewrite the measurement equation (3) and approximate the right hand side with Taylor series expansion to the first order, obtaining

$$10^{\frac{P_{mn}}{5\beta}} \cdot \mathbf{g}_{mn}^T \mathbf{H}_{mn} \mathbf{g}_{mn} = 10^{\frac{P_{m0}}{5\beta}} \cdot 10^{\frac{n_{mn}}{5\beta}} \approx 10^{\frac{P_{m0}}{5\beta}} \left( 1 + \frac{\ln 10}{5\beta} n_{mn} \right), \quad (5)$$

where  $\mathbf{H}_{mn} \triangleq (\mathbf{p}_n - \mathbf{q}_m)(\mathbf{p}_n - \mathbf{q}_m)^T$ , and the higher order noise terms  $o\left(\left(\frac{n_{mk}}{5\beta}\right)^2\right)$  are omitted in the approximation.

Let  $\tilde{P}_{mn} \triangleq 10^{\frac{P_{mn}}{5\beta}}$  and  $\tilde{P}_{m0} \triangleq 10^{\frac{P_{m0}}{5\beta}}$ . Then, from (5) we define our objective function as

$$J = \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} \left( \tilde{P}_{mn} \mathbf{g}_{mn}^T \mathbf{H}_{mn} \mathbf{g}_{mn} - \tilde{P}_{m0} \right)^2.$$

We introduce an auxiliary variable  $\mathbf{R}_m = \mathbf{q}_m \mathbf{q}_m^T$ , and relax this non-convex equality constraint to  $\mathbf{R}_m \geq \mathbf{q}_m \mathbf{q}_m^T$ . Then, our optimization problem becomes

$$\begin{aligned} \min_{\tilde{P}_{m0}, \mathbf{q}_m, \mathbf{R}_m, \gamma_n} \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} \left( \tilde{P}_{mn} \mathbf{g}_{mn}^T \mathbf{H}_{mn} \mathbf{g}_{mn} - \tilde{P}_{m0} \right)^2 \quad (6) \\ \text{subject to } \mathbf{H}_{mn} = \mathbf{p}_n \mathbf{p}_n^T - \mathbf{q}_m \mathbf{p}_n^T - \mathbf{p}_n \mathbf{q}_m^T + \mathbf{R}_m, \\ \mathbf{q}_m \mathbf{q}_m^T - \mathbf{R}_m \leq 0, \end{aligned}$$

for all  $n \in \mathcal{N}$  and  $m \in \mathcal{M}$ . The first constraint is linear in the decision variables  $\mathbf{q}_m$  and  $\mathbf{R}_m$ , and the second constraint  $\mathbf{q}_m \mathbf{q}_m^T - \mathbf{R}_m \leq 0$  is equivalent to

$$\begin{bmatrix} \mathbf{R}_m & \mathbf{q}_m \\ \mathbf{q}_m^T & 1 \end{bmatrix} \succeq 0,$$

which is a convex semi-definite constraint. Though the objective function is still nonlinear and non-convex, it becomes relatively simpler since it no longer contains any logarithmic functions and can be optimized via an alternating optimization approach.

To solve the optimization problem (6), we partition the unknown variables into two groups as  $\{\tilde{P}_{m0}, \mathbf{q}_m, \mathbf{R}_m\}$  and  $\{\gamma_n\}$ . We can observe that, if values of  $\{\gamma_n\}$  are given, the objective function of (6) becomes a summation of quadratic functions of terms that are linear in the unknowns  $\{\mathbf{q}_m\}$ ; consequently our problem becomes a convex optimization problem. On the other hand, if values of  $\{\tilde{P}_{m0}, \mathbf{q}_m, \mathbf{R}_m\}$  are given, our problem can be decomposed into  $N$  simpler problems each with unique global optimum

$$\min_{\gamma_n} J_n \quad \text{for } n \in \{0, 1, \dots, N\} \quad (7)$$

where  $J_n = \sum_{m \in \mathcal{M}} \left( \tilde{P}_{mn} \mathbf{g}_{mn}^T \mathbf{H}_{mn} \mathbf{g}_{mn} - \tilde{P}_{m0} \right)^2$ . Therefore, we can solve the problem (6) by alternately solving these two sub-problems. Our proposed procedure for estimating target positions,

scatterer orientations and target transmit powers is presented in Algorithm 1. The objective function is guaranteed not to increase in each iteration, and since it is non-negative, the algorithm converges [12].

**Remark 2.** *The relaxation of the non-convex equality constraint introduced above is frequently used in the literature as a way to obtain a convex approximation of the constraint [6, 13, 14]. In our simulations we find that in many cases it is tight and does not bias the solution obviously if there are sufficient sensors to perform the localization. In cases when there are limited number of sensors, the relaxation does not work well, and new ways to handle the aforementioned non-convex equality constraint need to be explored.*

**Remark 3.** *As the problem (6) is nonlinear and may have multiple local minima, Algorithm 1 may converge only to a local optimal solution of (6). For it to converge to a solution that is close to a global optimal solution, we need to have good initial estimates of the scatterers' orientations as inputs to the algorithm. This is possible in applications where a priori knowledge of the surrounding environment is available. For example, based on this knowledge, we can specify feasible ranges for the scatterers' orientations as constraints in (6), and choose our initial guesses in Algorithm 1 from these ranges.*

---

#### Algorithm 1 Alternating localization algorithm

---

Initialization:  $\gamma_n \leftarrow \gamma_n^0$ ;  $J^{(-1)} \leftarrow \infty$ ;  $i \leftarrow 0$ ;

$J^0 \leftarrow 0$ ;  $\epsilon \leftarrow$  a small positive scalar;

**while**  $|J^i - J^{i-1}| \geq \epsilon$  **do**

$i \leftarrow i + 1$ ;  $J^{i-1} \leftarrow J^i$ ;  $J^i \leftarrow 0$ ;

$\{\tilde{P}_{m0}, \mathbf{q}_m, \mathbf{R}_m\} \leftarrow \arg \min_{\tilde{P}_{m0}, \mathbf{q}_m, \mathbf{R}_m} J$  given  $\{\gamma_n\}$

**for**  $n = 1 : N$  **do**

$\gamma_n \leftarrow \arg \min_{\gamma_n} J_n$  given  $\{\tilde{P}_{m0}, \mathbf{q}_m, \mathbf{R}_m\}$ ;

$J_n^{new} \leftarrow J_n$  given  $\{\tilde{P}_{m0}, \mathbf{q}_m, \mathbf{R}_m, \gamma_n\}$ ;

$J^i \leftarrow J^i + J_n^{new}$ ;

**end for**

**end while**

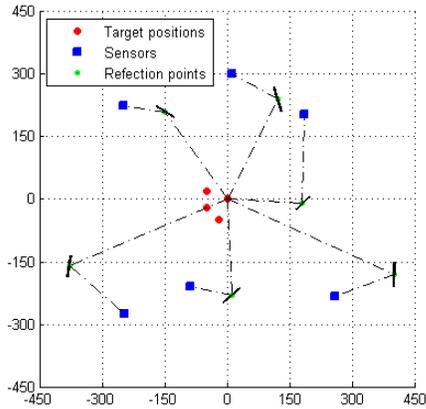
---

### 4. SIMULATION RESULTS

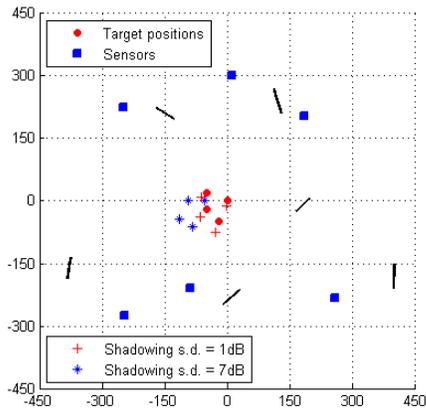
In this section, we present simulation results to demonstrate the performance of our proposed alternating optimization algorithm. In the simulation scenario, 6 sensor nodes are placed around a mobile target, and the target moves to 4 different locations. NLOS signals from the target reflects off 6 scatterers, one corresponding to each sensor. We assume that sensors know the scatterers have an unknown orientation in the range of  $[\gamma^0 - 7^\circ, \gamma^0 + 7^\circ]$ , where  $\gamma^0$  is the true scatterer orientation. As an example, we show the signal paths from the target located at (0, 0) to all sensors using dotted lines in Figure 2.

For our MATLAB simulation, convex optimization toolbox CVX [15] is used for solving the SDP. The value of the path loss exponent  $\beta$  is known and set to 4. For every experiment scenario, we run simulations for 300 times.

We assume that the AOA noise follows a Gaussian distribution. In the first simulation experiment, we fix the standard deviation (s.d.)



**Fig. 2.** Configuration of sensors and target locations. The dotted lines indicate the signal paths from targets to sensors.



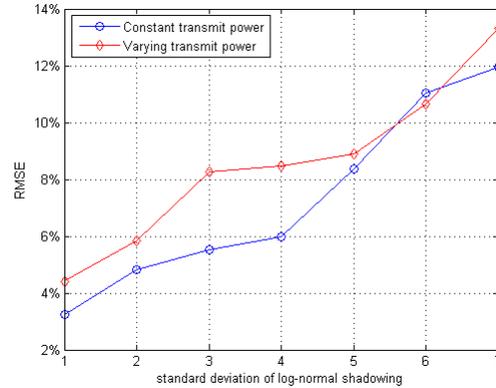
**Fig. 3.** Estimated target locations with shadowing noises at 1dB and 7dB.

of the AOA noise to be  $2^{\circ}$ <sup>1</sup> and vary the s.d. of noise shadowing from 1dB to 7dB. We plot the estimated positions for the cases where the shadowing noise s.d. is 1 dB and 7 dB in Figure 3. It can be seen that the estimated target locations are close to the true locations in general even when the noise is as high as 7 dB. The root mean square error (RMSE) of the estimated target positions versus shadowing noise s.d. is shown in Figure 4. The RMSE is expressed as a percentage of the average signal path length from the target to the sensors, which in our simulation example is 448 m.

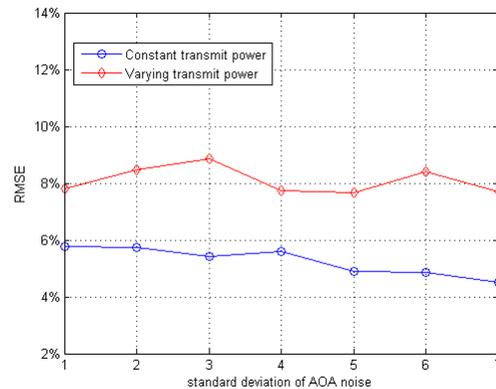
In Figure 5, we set the s.d. of noise shadowing to be 2dB and vary the s.d. of AOA noise from  $1^{\circ}$  to  $7^{\circ}$ . The irregularity of the result is due to the relaxation in equation (3). The noise term  $n_{mn}$  in equation (3) contains AOA noise that correlates with AOA measurements and scatterer orientations, which results in biases in the estimated locations. Part of our future work involves finding ways to mitigate these biases.

<sup>1</sup>If the AOA noise is assumed to have a uniform distribution as in [4], this roughly corresponds to a support of  $[-4.2^{\circ}, 4.2^{\circ}]$  assuming that the Gaussian AOA noise falls within two s.d. of the mean 95% of the time.

In both the above simulation scenarios, we also compare the accuracy of localizing the target when its transmit power varies across positions. As expected, there is some performance gain when the target transmit power is constant.



**Fig. 4.** RMSE of the estimated target positions versus shadowing noise.



**Fig. 5.** RMSE of the estimated target positions versus AOA noise.

## 5. CONCLUSIONS

In this work, we discuss how to use RSS and AOA measurements for localization of a mobile non-cooperative target in an NLOS environment. We formulate the problem using measurements of the target's signals at multiple positions, and present an alternating optimization method to jointly estimate the target positions, angles of the scatterers, as well as the transmit powers of the target. Simulation results suggest that the proposed algorithm has good performance. Part of our future research work is finding an efficient way to initialize the orientations of the scatterers as required in the proposed algorithm when there is little prior knowledge about their values, and a robust way to find an approximate solution that is close to the global optimal solution of the original non-convex problem. Moreover, other measurement metrics like Doppler estimation [16] can also be incorporated in our algorithm in future work.

## 6. REFERENCES

- [1] H. Harroud, A. Berrado, M. Boulmalf, and A. Karmouch, "Location-based services provisioning using wsn," in *Microwave Symposium (MMS), 2009 Mediterranean*, Nov 2009, pp. 1–5.
- [2] Y. Wang, X. Wang, B. Xie, D. Wang, and D. P. Agrawal, "Intrusion detection in homogeneous and heterogeneous wireless sensor networks," *IEEE Trans. Mobile Comput.*, vol. 7, no. 6, pp. 698–711, 2008.
- [3] D. Agrawal and Q.-A. Zeng, *Introduction to wireless and mobile systems*. Cengage Learning, 2010.
- [4] W. Xu, F. Quitin, M. Leng, W. P. Tay, and S. G. Razul, "Distributed localization of a non-cooperative RF target in NLOS environments," in *Int. Conf. on Information Fusion*, 2014.
- [5] O. Jean and A. Weiss, "Passive localization and synchronization using arbitrary signals," *IEEE Trans. Signal Process.*, vol. 62, no. 8, pp. 2143–2150, April 2014.
- [6] R. Vaghefi, M. Gholami, R. Buehrer, and E. Strom, "Cooperative received signal strength-based sensor localization with unknown transmit powers," *IEEE Trans. Signal Process.*, vol. 61, no. 6, pp. 1389–1403, March 2013.
- [7] Z.-Q. Luo, W.-K. Ma, A.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.
- [8] L. Cong and W. Zhuang, "Hybrid TDOA/AOA mobile user location for wideband CDMA cellular systems," *IEEE Trans. Wireless Commun.*, vol. 1, no. 3, pp. 439–447, 2002.
- [9] A. Broumandan, T. Lin, J. Nielsen, and G. Lachapelle, "Practical results of hybrid AOA/TDOA geo-location estimation in CDMA wireless networks," in *Vehicular Technology Conference, 2008. IEEE 68th*. IEEE, 2008, pp. 1–5.
- [10] S. Wang, B. R. Jackson, and R. Inkol, "Hybrid RSS/AOA emitter location estimation based on least squares and maximum likelihood criteria," in *Communications, 2012 26th Biennial Symposium on*. IEEE, 2012, pp. 24–29.
- [11] Y. Chan, F. Chan, W. Read, B. Jackson, and B. Lee, "Hybrid localization of an emitter by combining angle-of-arrival and received signal strength measurements," in *Electrical and Computer Engineering (CCECE), 2014 IEEE 27th Canadian Conference on*, May 2014, pp. 1–5.
- [12] J. C. Bezdek and R. J. Hathaway, "Some notes on alternating optimization," in *Advances in Soft Computing*. Springer, 2002, pp. 288–300.
- [13] S. Burer and A. Saxena, "The MILP road to MIQCP," in *Mixed Integer Nonlinear Programming*. Springer, 2012, pp. 373–405.
- [14] A. Qualizza, P. Belotti, and F. Margot, "Linear programming relaxations of quadratically constrained quadratic programs," in *Mixed Integer Nonlinear Programming*. Springer, 2012, pp. 407–426.
- [15] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," Mar. 2014. [Online]. Available: <http://cvxr.com/cvx>
- [16] X. Jiang, W.-J. Zeng, and X.-L. Li, "Time delay and doppler estimation for wideband acoustic signals in multipath environments," *The Journal of the Acoustical Society of America*, vol. 130, no. 2, pp. 850–857, 2011.