MOBILE ADAPTIVE NETWORKS FOR PURSUING MULTIPLE TARGETS

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ABSTRACT

We examine the design of self-organizing mobile adaptive networks with multiple targets in which the network nodes form distinct clusters to learn about and purse multiple targets, all while moving in a cohesive collision-free manner. We build upon previous distributed diffusion-based adaptive learning networks that focused on a single target to examine the case with multiple targets in which the nodes do not know the number of targets, and exchange local information with their neighbors in their learning objectives. In particular, we design a method allowing the nodes to switch the target they are tracking thereby engendering the formation of distinct stable learning groups that can split up and purse their distinct targets over time. We provide analytical mean stability and steady state mean-square deviation results along with simulations that demonstrate the efficacy of the proposed method.

Index terms— adaptive networks, diffusion adaptation, self-organization, distributed signal processing, mobility

1. INTRODUCTION

Signal processing research has increasingly focused on the problems associated with networks of nodes sensing and learning about an environment, and engaging in collective decision making, all in a distributed manner. This thrust has stemmed from a desire to understand the natural world, and to engineer autonomous systems. In particular, researchers want to understand the swarm intelligence inherent in the natural world, such as schools of fish finding food sources [1], birds flocking [2], and honey bees finding a new home [3]. In addition, engineers have attempted to design mobile networks of autonomous agents without global control that can collectively complete tasks in complex environments. Some approaches have relied upon consensus algorithms to achieve agreement among nodes [4], [5], [6], while others have pursued the diffusion approach based on adaptive filtering principles [7], [8], [9].

In [7], the authors proposed mobile adaptive networks that can pursue a single target. In particular, the nodes exchange limited information with their neighbors, and use distributed adaptive learning to collectively learn and move towards the target location while avoiding collision among the nodes. In [8], in a two target network, nodes agree on one common target through a collaborative decision making process involving the inner products between a node's update vector for its target estimate with its neighbors' update vectors, assuming that the targets have a greater than $\frac{\pi}{2}$ separation. The nodes only move after an agreement is reached. In [9], N non-mobile network nodes learn T objectives with $T < N$, down-weighting and subsequently cutting their network connections to emphasize neighbors with the same learning objectives. While providing valuable insight, these papers have not examined the problem of mobile adaptive networks with multiple targets.

In this paper, we focus on designing mobile adaptive networks with N nodes and T static targets in which the nodes collectively form distinct clusters to learn about and pursue distinct targets, all while moving. In particular, we assume that each node is initially assigned a random target, and that the nodes do not know the total number of targets in their environment. Similar to [7], each node exchanges information with its neighbors. However, the nodes do not know which target their neighbors are sensing, and in fact, there is no mechanism for naming or numbering the targets. We design a method for allowing each node to switch its sensing direction based on local information, thereby allowing a node to pursue a different target. In addition, we focus our design on reducing the number of switches a node makes to engender the formation of stable network groups that can split to learn about and pursue their distinct targets. We provide a performance analysis and simulation results that demonstrate the efficacy of our algorithm.

The remaining sections are as follows. In Section 2, we review mobile adaptive network diffusion. In Section 3, we formulate the target switching mechanism. In Section 5, we derive the algorithm mean stability and steady state mean-square deviation. Simulation results are illustrated in Section 6 and Section 7 concludes.

2. MOBILE ADAPTIVE NETWORK

Consider a collection of N mobile nodes and T static targets randomly distributed over a plane. Node k tracks the target located at $w^o_{n(k)}$ where $n(k) \in \{1, 2, ...T\}$ in a global coordinate system. Each node initially tracks a random target. Assume the network has no isolated nodes. Each node k finds its neighbors within a range R radius in each time i. Let N_k be the set of neighbors of node k and $n_k = |N_k|$ and constrained to $n_k \leq n_{max}$. At time i, node k is located at $x_{k,i}$ and it estimates its distance, $d_k(i)$, to $w^o_{n(k)}$ [7]

$$d^o_k(i) = u^o_{k,i}(w^o_{n(k)} - x_{k,i}) \tag{1}$$

where ^{o} denotes the optimal or true value. $u^o_{k,i}$ is the unit sensing direction row vector pointing to $w^o_{n(k)}$. The objective of the network is to estimate $w^o_{n(k)}$ for all $k \in \{1, 2, ...N\}$ by minimizing

$$J^{glob}(w_{n(1)}, ...w_{n(N)}) = \sum_{k=1}^{N} \mathbb{E}|d_k(i) - u_{k,i}(w_{n(k)} - x_{k,i})|^2. \tag{2}$$

Each node k exchanges the measurement data $\{u_{k,i}, x_{k,i}\}$ with its neighbors and solves (2) distributively as in [7]. In particular, each node k computes,

$$\psi_{k,i} = w_{k,i-1} + \mu_k \sum_{l \in N_k} b^w_{l,k}(q_{k,i} - w_{k,i-1}) \tag{3}$$

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$$w_{k,i} = \sum_{l \in N_k} a^w_{l,k} \psi_{l,i} \tag{4}$$

where μ_k is the learning step size and $b^w_{l,k}$ and $a^w_{l,k}$ are the non-negative coefficients. $\psi_{k,i}$ and $w_{k,i}$ are node k's intermediate and local estimates of $w^o_{n(k)}$. Note that $q_{k,i} = x_{k,i} + d_k(i)u^T_{k,i}$ is the noisy measurement of the location of $w^o_{n(k)}$ at time i. From (4), if node k's neighbors are tracking different targets $(w_{n(l)} \neq w_{n(k)})$ $l \in N_k$, then $w_{k,i}$ will converge to a linear combination of the different targets. As in [9], we need to set the combination weights $a^w_{l,k}$ to emphasize the estimates of the neighbors that are learning about the same target. Therefore, in (3) and (4), we set $b^w_{l,k} = \delta_{lk}$ in terms of the Kronecker delta function and design $a^w_{l,k}$ as,

$$a^w_{l,k}(i) = \frac{\|w_{k,i-1} - \psi_{l,i}\|^{-2}}{\sum_{n \in N_k} \|w_{k,i-1} - \psi_{n,i}\|^{-2}} \tag{5}$$

$$a^w_{l,k}(i) \geq 0, \quad \sum_{l \in N_k} a^w_{l,k}(i) = 1, \quad a^w_{l,k}(i) = 0 \text{ if } l \notin N_k.$$

Node k updates its location as $x_{k,i} = x_{k,i-1} + \Delta t v_{k,i}$. For coherent collision-free group motion, each node calculates $v_{k,i}$ [7],

$$\begin{aligned} v_{k,i} = {} & \lambda \frac{w_{k,i-1} - x_{k,i-1}}{\|w_{k,i-1} - x_{k,i-1}\|} + \beta v^g_{k,i-1} \\ & + \gamma \sum_{l \in N_k \setminus \{k\}} (\|x_{l,i} - x_{k,i}\| - r) \frac{x_{l,i} - x_{k,i}}{\|x_{l,i} - x_{k,i}\|} \end{aligned} \tag{6}$$

where λ, β and γ are non-negative velocity update coefficients. The first term in (6) is node k's direction vector to $w^o_{n(k)}$. Note that $v^g_{k,i}$ is node k's estimate of the group velocity which is designed to allow for coherent motion. The last term in (6) is a combination of the internal attractive and repulsive forces between neighboring nodes, while r is the minimum distance between two nodes. Each node estimates $v^g_{k,i}$ distributively as in [7],

$$\Phi_{k,i} = v^g_{k,i-1} + \nu_k (v_{k,i} - v^g_{k,i-1}) \tag{7}$$

$$v^g_{k,i} = \sum_{l \in N_k} a^v_{l,k} \Phi_{l,i} \tag{8}$$

where ν_k is a step size and $a^v_{l,k}$ is a non-negative weight.

3. TARGET SWITCHING MECHANISM

To design a mobile adaptive network that pursues multiple targets that are not necessarily separated by an angle of $\frac{\pi}{2}$, we need to develop methods for nodes to detect that they are not aligned with their neighbors' learning objectives, and to subsequently switch their sensing direction and therefore the target they are pursuing. Moreover, we need to ensure that the nodes can split into stable subgroups that can pursue distinct targets. To begin with, we look at the iterative local cost of node k as in [10],

$$\begin{aligned} J^{loc}(w_{k,i-1}) &= \mathbb{E}|q_{k,i} - w_{k,i-1}|^2 \\ &= \mathbb{E}|w^o_n(k) + \boldsymbol{\nu_{k,i}}_{k,i} - w_{k,i-1}|^2 \end{aligned} \tag{9}$$

where from [7], $q_{k,i} = w^o_{n(k)} + \boldsymbol{\nu_{k,i}}$. Let $\boldsymbol{\nu}_{k,i}$ be a zero mean white random vector with variance proportional to the distance between $x_{k,i}$ and $w^o_{n(k)}$as in [7]

$$\begin{aligned} \sigma^2_{\boldsymbol{\nu}_k,i} &= \kappa \|w^o_{n(k)} - x_{k,i}\|^2 \\ &= \kappa \|w^o_{n(k)} - x_{k,i-1} - \Delta t v_{k,i}\|^2. \end{aligned} \tag{10}$$

Eq. (9) contains the learning error $E\|w^o_{n(k)} - w_{k,i-1}\|^2$ and the measurement error $\sigma^2_{\boldsymbol{\nu}_k,i}$. Both errors are expected to decrease as node k gets closer to its target [7]. In (6), if $v_{k,i}$ is dominated by a $v^g_{k,i}$ that is different from node k's sensing direction, then $x_{k,i}$ will not reach $w^o_{n(k)}$. Then $\sigma^2_{\boldsymbol{\nu}_k,i}$ is non-decreasing as $i \to \infty$, and

$$J^{loc}(w_{k,i}) \geq J^{loc}(w_{k,i-1}). \tag{11}$$

Since in this case node k will never reach its target $w^o_{n(k)}$, then node k needs to switch its target to become aligned with its neighbors that are pursuing a different target. How can we detect this condition while being conservative in switching to avoid nodes rapidly switching their sensing direction? We focus on the angle between node k's sensing direction $u_{k,i}$, and its velocity $v_{k,i}$, detecting the persistent increase in this angle. We use $c_k(i)$, the cosine of the angle between $u_{k,i}$ and $v_{k,i}$ due to its low computational cost and the value limited range of $[-1, 1]$. Note that

$$c_k(i) = \frac{u_{k,i} v_{k,i}}{\|v_{k,i}\|}, \tag{12}$$

and $c_k(i)$ can be smoothed through a first-order filter,

$$c_k(i) = c_k(i-1) + \mu_c [\frac{u_{k,i}.v_{k,i}}{\|v_{k,i}\|} - c_k(i-1)]. \tag{13}$$

When $c_k(i)$ is decreasing, the node is moving in a different direction from its target and the local cost becomes a non-decreasing function. The higher the value of $c_k(i)$, the more confidence the node has that it is moving towards $w^o_{n(k)}$. Therefore, node k decides to get a new sensing direction vector $u^{new}_{k,i}$ if $c_k(i) \leq \eta$. When switching to a new target, we want to minimize the direction difference between $u^{new}_{k,i}$ and $v_{k,i}$, i.e.,

$$\min_{(u^{new}_{k,i})^T} \|(u^{new}_{k,i})^T - v_{k,i}\|. \tag{14}$$

From (6), (7) and (8), we have

$$v^g_{k,i} \approx \sum_{l \in N_k} a^v_{l,k} \frac{w_{l,i-1} - x_{l,i-1}}{\|w_{l,i-1} - x_{l,i-1}\|} \approx \sum_{l \in N_k} a^v_{l,k} u^T_{l,i}, \tag{15}$$

$$\sum_{l \in N_k} a^v_{l,k} \gamma \sum_{l \in N_k \setminus \{k\}} (\|x_{l,i} - x_{k,i}\| - r) \frac{x_{l,i} - x_{k,i}}{\|x_{l,i} - x_{k,i}\|} \approx 0$$

Assuming $v^g_{k,i} \approx v^g_{k,i-1}$, from (6) and (15) we solve (14),

$$u^{new}_{k,i} = \sum_{l \in N_k \setminus \{k\}} s_{l,k} u_{l,i} \tag{16}$$

where $s_{l,k}$ is a non-negative coefficient. Node k sets $w_{k,i} = 0$ when it gets the new sensing direction. In target switching, node k uses the new sensing direction $u^{new}_{k,i}$ to find a new target $w_{n(k)}$ within the angle θ. As in [7] and [8], we ignore occlusion. The change in the sensing direction is restricted to cases when $d_k(i) > \epsilon$. This requirement is needed because when the nodes are very close to a target (or targets), the value of $c_k(i)$ can rapidly change due to the changes in $v_{k,i}$ in (6) (e.g., node repulsive force for collision avoidance).

Now let us consider how to design the weights $s_{l,k}$ that determine the new sensing direction $u^{new}_{k,i}$. Node k needs to emphasize the sensing directions $u_{l,i}$ for $l \in N_k$ that are aligned with the group velocity, i.e.,

$$\min_{s_{l,k}(i)} \| \sum_{l \in N_k \setminus \{k\}} s_{l,k}(i) u^T_{l,i} - v^g_{k,i} \| \tag{17}$$

$$s_{l,k}(i) \geq 0, \quad \sum_{l \in N_k \setminus \{k\}} s_{l,k}(i) = 1 \quad s_{l,k}(i) = 0 \text{ for } l \notin N_k.$$

We get the solution for $s_{l,k}$ using [11],

$$s_{l,k}(i) = \frac{\|u_{l,i}^T - v_{k,i}^g\|^{-1}}{\sum_{n\in N_k\setminus\{k\}}\|u_{n,i}^T - v_{k,i}^g\|^{-1}}, \tag{18}$$

with more weight to nodes $l \in N_k$ whose sensing direction $u_{l,i}$, is closer to $v_{k,i}^g$. If $s_{l,k}(i) < \zeta$, we set $s_{l,k}(i) = 0$ to de-emphasize the neighbors that have a different sensing direction.

4. PERFORMANCE ANALYSIS

Let us assume that each node's final target $z_{n(k)} \in \{w_1, w_2, ..., w_T\}$ is known. Let $w_{n_i(k)}^o$ be the location of the target that node k tracking at time i. We introduce two error vectors at node k as in [8],

$$\tilde{w}_{k,i} = z_{n(k)}^o - w_{k,i} \quad \tilde{z}_{k,i}^o = z_{n(k)}^o - w_{n_i(k)}^o \tag{19}$$

where $\tilde{w}_{k,i}$ is node k's estimation error to its final target and $\tilde{z}_{k,i}^o$ is the error difference between node k's final and current targets at time i. The error recursion for estimation in (3) and (4) is

$$\begin{aligned}\tilde{w}_{k,i} = &\sum_{l\in N_k} a_{l,k}^w(i)(1-\mu_k)\tilde{w}_{l,i-1} + \sum_{l\in N_k} a_{l,k}^w(i)\mu_k\tilde{z}_{l,i}^o \\ &- \sum_{l\in N_k} a_{l,k}^w(i)\mu_k\boldsymbol{\nu}_{l,i} + z_{n(k)}^o - \sum_{l\in N_k} a_{l,k}^w(i) z_{n(l)}^o.\end{aligned} \tag{20}$$

Then, we can write the above recursion in the state space form as

$$\tilde{w}_i = \mathcal{A}_i^T(I_{NM} - \mathcal{M})\tilde{w}_{i-1} + \mathcal{A}_i^T\mathcal{M}\tilde{z}_i - \mathcal{A}_i^T\mathcal{M}g_i + b_i \tag{21}$$

where the block vectors and matrices are defined as,
$\tilde{w}_i = col\{\tilde{w}_{1,i}, \tilde{w}_{2,i}, ..., \tilde{w}_{N,i}\}$
$\tilde{z}_i = col\{\tilde{z}_{1,i}^o, \tilde{z}_{2,i}^o, ..., \tilde{z}_{N,i}^o\}$
$\mathcal{M} = diag\{\mu_1 I_M, \mu_1 I_M, ..., \mu_N I_M\}$
$g_i = col\{\boldsymbol{\nu}_{1,i}, \boldsymbol{\nu}_{2,i}, ..., \boldsymbol{\nu}_{N,i}\}$
$b_i = col\{b_{1,i}, b_{1,i}, ..., b_{N,i}\}$
where $b_{k,i} = z_{n(k)}^o - \sum_{l\in N_k} a_{lk}^w(i) z_{n(l)}^o$
$\mathcal{A}_i = A_i \otimes I_M$
Where A_i is $N \times N$ matrix with individual non-negative real entries $\{a_{l,k}\}$ and $\otimes$ is the Kronecker product of two matrices.

4.1. Mean Stability

Taking the expectation of both side of (21),

$$\mathbb{E}[\tilde{w}_i] = \mathcal{A}_i^T(I_{NM} - \mathcal{M})\mathbb{E}[\tilde{w}_{i-1}] + \mathcal{A}_i^T\mathcal{M}\tilde{z}_i + b_i. \tag{22}$$

Since we assume static targets, $\mathbb{E}[\tilde{z}_i] = \tilde{z}_i$ and $\mathbb{E}[g_i] = 0$. Let $\mathcal{B}_i = \mathcal{A}_i^T(I_{NM} - \mathcal{M})$. We need the following conditions for the mean convergence in (22),

$$\|\mathcal{B}_i\|_1 < 1 \text{ and } \mathcal{A}_i^T\mathcal{M}\tilde{z}_i + b_i = 0 \tag{23}$$

where $\|.\|_1$ is the L_1 norm of a matrix. Since $\mathcal{A}_i$ is a left stochastic matrix, we can write the first condition in (23) as $|1-\mu_k| < 1$. Then we can select the step size of $0 < \mu_k < 2$ for all k. The second term in (23) ($\mathcal{A}_i^T\mathcal{M}\tilde{z}_i + b_i$) becomes zero when nodes form distinct groups pursuing particular targets ($z_{n(k)}^o = z_{n(l)}^o$ for all $l \in N_k$), then $\tilde{z}_i = 0$. Then (22) converges. However, if node k has neighbors that have different targets ($z_{n(k)}^o \neq z_{n(l)}^o$ for $l \in N_k$), then node k's estimate will converge to a convex combination of target locations [8],

$$w_{k,i} = \sum_{l\in N_k} a_{l,k}^w(i) z_{n(l)}^o \text{ as } i \to \infty. \tag{24}$$

Algorithm 1 Mobile Adaptive Network with Target Switching

Assign a random target to each node, $w_{n-1(k)}^o$ where $n_k \in \{1, 2, ...T\}$ and initialize $w_{k,-1} = 0$, and $c_{k,-1} = 1$ for all k.
for $i \geq 0$ and $k = 1$ to N **do**
1) Each node has the local data $\{d_k(i), u_{k,i}, v_{k,i}, x_{k,i}\}$.
2) Find the neighbors N_k within R and $|N_k| \leq n_{max}$
3) Exchange $\{u_{k,i}, v_{k,i}, x_{k,i}\}$ with node l where $l \in N_k$
4) Find the noisy location of target, $q_{k,i} = x_{k,i} + d_k(i)u_{k,i}^T$
5) Perform the local adaptation steps,

$$\psi_{k,i} = w_{k,i-1} + \mu_k(q_{k,i} - w_{k,i-1})$$
$$\Phi_{k,i} = v_{k,i-1}^g + \nu_k(v_{k,i} - v_{k,i-1}^g)$$

6) Exchange $\{\psi_{k,i}, \Phi_{k,i}\}$ with node l where $l \in N_k$
7)
if $(\|w_{k,i-1} - \psi_{l,i}\| < \delta)$ **then**

$$a_{l,k}^w(i) = \frac{(\|w_{k,i-1} - \psi_{l,i}\|)^{-2}}{\sum_{n\in N_k}(\|w_{k,i-1} - \psi_{n,i}\|)^{-2}}$$

end if
8) Perform the combination steps,

$$w_{k,i} = \sum_{l\in N_k} a_{l,k}^w(i)\psi_{l,i}$$
$$v_{k,i}^g = \sum_{l\in N_k} a_{l,k}^v\Phi_{l,i}$$

9) Find the node velocity,

$$\begin{aligned}v_{k,i+1} = &\lambda\frac{w_{k,i} - x_{k,i}}{\|w_{k,i} - x_{k,i}\|} + \beta v_{k,i}^g \\ &+ \gamma\sum_{l\in N_k\setminus\{k\}}(\|x_{l,i} - x_{k,i}\| - r)\frac{x_{l,i} - x_{k,i}}{\|x_{l,i} - x_{k,i}\|}.\end{aligned}$$

10)
if $d_k(i) > \epsilon$ **then**

$$c_k(i) = c_k(i-1) + \mu_c[\frac{u_{k,i}v_{k,i}}{\|v_{k,i}^g\|} - c_k(i-1)]$$

if $c_k(i) \leq \eta$ **then**
i) Compute the weighting combiner

$$s_{l,k}(i) = \frac{\|u_{l,i}^T - v_{k,i}^g\|^{-1}}{\sum_{n\in N_k\setminus\{k\}}\|u_{n,i}^T - v_{k,i}^g\|^{-1}}$$

if $s_{l,k}(i) < \zeta$, set $s_{l,k}(i) = 0$.
ii) Find the new sensing direction,

$$u_{k,i}^{new} = \sum_{l\in N_k\setminus\{k\}} s_{l,k}(i)u_{l,i}$$

Set $w_{k,i} = 0$ and $c_k(i) = 1$.
end if
end if
11) Update the node location, $x_{k,i+1} = x_{k,i} + \Delta t\, v_{k,i+1}$
end for

4.2. Mean Square Performance

Let Σ_i be an NM x NM positive semi-definite Hermitian matrix. Using the energy conservation arguments in [10], the weighted norm of $\tilde{w}_i$ can be obtained as,

$$\mathbb{E}\|\tilde{w}_i\|^2_{\Sigma_i} = \mathbb{E}\|\tilde{w}_{i-1}\|^2_{\Sigma_{i-1}} + \text{Tr}(\mathcal{Y}_i \Sigma_i) \quad (25)$$

where, $\Sigma_{i-1} = \mathcal{B}_i^* \Sigma_i \mathcal{B}_i$
$\mathcal{B}_i = \mathcal{A}_i^T (I - \mathcal{M})$
$\mathcal{Z} = \mathbb{E}[\tilde{z}_i \tilde{z}_i^*]$
$\mathcal{G} = \mathbb{E}[g_i g_i^*]$

$$\mathcal{Y}_i = \mathcal{B}_i \mathbb{E}\tilde{w}_{i-1} b_i^* + \mathcal{A}_i^T \mathcal{M}\mathcal{Z}\mathcal{M}\mathcal{A}_i + \mathcal{A}_i^T \mathcal{M}\tilde{z}_i b_i^* \\ - \mathcal{A}_i^T \mathcal{M}\mathcal{G}\mathcal{M}\mathcal{A}_i + b_i(\mathbb{E}\tilde{w}_{i-1})^* \mathcal{B}_i^* + b_i \tilde{z}_i^* \mathcal{M}\mathcal{A}_i + b_i b_i^*. \quad (26)$$

Since the recursion Σ_{i-1} runs backwards, we can rewrite (25) as,

$$\mathbb{E}\|\tilde{w}_i\|^2_{\Sigma_i} = \mathbb{E}\|\tilde{w}_{-1}\|^2_{\Sigma_{-1}} + \sum_{j=0}^{i} \text{Tr}(\mathcal{Y}_j \Sigma_j) \quad (27)$$

where,
$\Sigma_j = \mathcal{B}_{j+1}^* \mathcal{B}_{j+2}^* ... \mathcal{B}_i^* \Sigma_i \mathcal{B}_i .. \mathcal{B}_{j+2} \mathcal{B}_{j+1}$ for $-1 \leq j \leq i-1$.
From [12] and [9], we can write,

$$\|\Sigma_j\|_* \leq c^2 [1 - \mu_k]^{2(i-j)} . \|\Sigma_i\|_* \quad (28)$$

where $\|.\|_*$ is the nuclear norm. For any Hermitian and positive definite matrix, the nuclear norm is equivalent to the trace of the matrix. Since $\mathcal{Y}_i$ in (26) is uniformly bounded, $\xi = \sup_{j \geq 0} \|\mathcal{Y}_j^{1/2}\|_*^2 < \infty$ and from [9], we can conclude the error recursion (21) is mean-square stable with,

$$\lim_{i\to\infty} \mathbb{E}\|\tilde{w}_i\|^2_{\Sigma_i} = \frac{c^2 \xi \|\Sigma_\infty\|_*}{[1 - (1-\mu)^2]}. \quad (29)$$

Select $\Sigma_\infty = \frac{I_{NM}}{N}$ and define the steady state mean-square deviation of the network as

$$\text{MSD} = \lim_{i\to\infty} \frac{1}{N} \sum_{k=1}^{N} \mathbb{E}\|\tilde{w}_{k,i}\|_2^2$$

$$\approx \frac{1}{N} \sum_{j=0}^{N} \text{Tr}(\mathcal{B}_i .... \mathcal{B}_{j+1} \mathcal{Y}_i \mathcal{B}_{j+1}^* .... \mathcal{B}_i^*). \quad (30)$$

5. SIMULATION RESULTS

We simulate the proposed algorithm over a 3 target network with 50 nodes shown in Figure 1. Each node is initially assigned a random target. We select $\mu_k = \nu_k = 0.05$ for all k. $(\delta, \alpha, \beta, \gamma, a_{l,k}^v)$ are set to $(0.001, 0.5, 0.5, \frac{1}{n_k - 1}$ and $\frac{1}{n_k})$. The distance threshold for switching is set to $\epsilon = 7$. The new sensing direction range θ is $45°$ and $\zeta = 0.1$. Each node selects N_k within the radius $R = 7$ with $n_{max} = 7$. $r = 2$ and $\kappa = 0.01$. The nodes apply Algorithm 1 with the value of $\eta = -0.75$. The simulation results are shown in Figure 1. In Figure 1 (c), the network has split into 3 subgroups through target switching, with each subgroup pursuing a different target. In Figure 1 (d), the nodes reach their targets. We tested the proposed algorithm over other scenarios with different target configurations and numbers of targets and observed similar network convergence. Due to space limitations, we cannot illustrate those results here. In Figure 2, we plot the network MSD using the traditional ATC diffusion algorithm [7] with no switching along with the results using Algorithm 1 with different η values . These results are averaged over 10 experiments with the same initial state of $w_{k,-1}$ for all k. With different η values, the network forms different subgroups. When the value of η is higher (i.e., $\eta = 0$), the network converges quickly but the nodes have multiple switches between targets. The nodes reduce their numbers of switches with

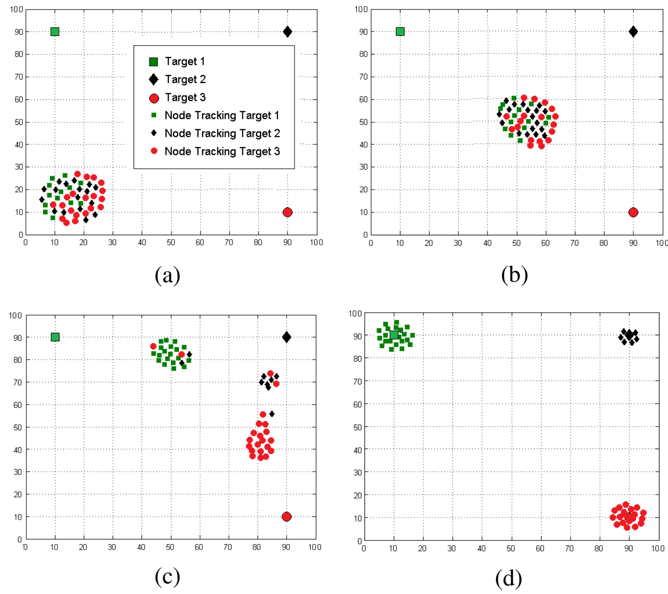

Figure 1: Mobile network maneuvers with 3 targets for iteration i (a) $i = 0$ (b) $i = 100$ (c) $i = 200$ (d) $i = 300$

lower η values (i.e., $\eta = -0.5$ and $\eta = -0.75$). Also in Figure 2, we see that the empirical MSD results closely match the analytical MSD results from Eq. (30) in steady state.

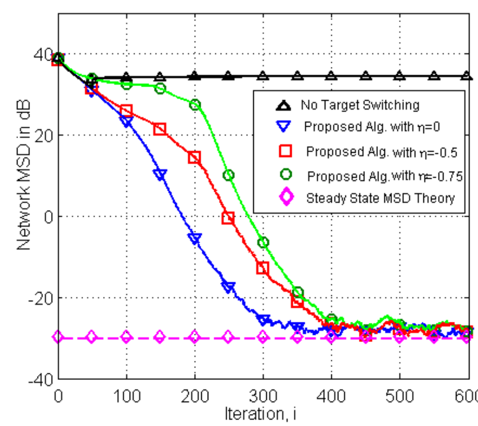

Figure 2: Comparison of network transient MSD from simulations along with the theoretical steady-state MSD from (30). The transient MSD is illustrated for the case of no target switching along with 3 cases of target switching for different η values.

6. CONCLUSION

We designed self-organizing mobile adaptive networks based on adaptive diffusion learning that can pursue multiple targets through the formation of distinct clusters of nodes that focus on distinct targets. Simulation results demonstrate the efficacy of the algorithm while the performance analysis focused on the steady-state mean stability and mean-square deviation. Future work will focus on the dynamics of cluster formation.

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