GNSS SPOOFING DETECTION USING MULTIPLE MOBILE COTS RECEIVERS

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ABSTRACT

In this paper we deal with spoofing detection in GNSS receivers. We derive the optimal genie detector when the true positions are perfectly known, and the observation errors are Gaussian, as a benchmark for other detectors. The system model considers three dimensional positions, and includes correlated errors. In addition, we propose several detectors that do not need any position knowledge, that outperform recently proposed detectors in many interesting cases.

Index Terms— GNSS, GPS, spoofing, jamming, detection

1. INTRODUCTION

The vulnerability of global navigation satellite systems (GNSS) towards jamming and spoofing has been known for many years. These issues were highlighted in the so called Volpe report [1] in 2001, and much effort has been put in the research on spoofing [2–4] and anti-spoofing techniques [5–16] in the last decade. The recent achievements on GPS spoofing techniques [2–4] have further raised these issues, and shown that it is a real threat.

A spoofing attack aims at deceiving its target GNSS receiver to reporting malicious spoofer-manipulated positioning and timing information. It has been shown that this can be done in practice in a controlled manner [3]. Meaconing is a simpler form of spoofing, defined in the Volpe report [1] as *the reception, delay, and rebroadcast of radionavigation signals to confuse a navigation system or user*. The position and timing information cannot be manipulated with the same sense of control in a meaconing attack, but its simplicity makes it a serious threat.

Several spoofing detectors have been proposed based on cross checks with other sensors, for example intertial measurement units (IMU) [2] or cross-correlation with a secure GNSS receiver [9, 10, 17]. Other techniques have been proposed based on the detection of different types of anomalies in the correlator outputs caused by spoofing [11–14]. This kind of detectors can be implemented in a single receiver, but one drawback is that the distortions in the correlator outputs caused by a spoofer are hard to discriminate from distortions caused by multipath fading [12]. Attempts to circumvent this problem have been made by combining several measures, for example correlator distortions and received power, to make a joint decision [13, 14]. Exploiting signal anomalies to detect a spoofer is in general a very challenging task. An intelligent enough spoofer could, at least theoretically, emulate the authentic GNSS signals, effectively making the spoofer signal impossible to detect.

For that reason, much work on spoofing detection has focused on array processing using multiple antennas or a single moving antenna [5–7]. A lot of research has been performed in recent years on signal detection using multiple antennas (cf. [18] and the references therein). Most of these papers focus on applications other than spoofing detection, although the same techniques are applicable.

Multiple receivers were used for detection on a higher level in [15, 16]. That is, detection was performed based on the position solutions from multiple receivers, rather than on the received sampled data itself. The main advantage of this kind of methods is that the they can be implemented by using commercial off-the-shelf (COTS) GNSS receivers, where the actual sampled data is not available to the user. In the papers [15, 16], the locations of the receivers relative to each other were assumed to be known, and the position errors were assumed to be Gaussian. The optimal detector was derived in [15] assuming known two dimensional positions and uncorrelated noise. A detector was also proposed for the case when the spoofed position and the true receiver positions were unknown (but known relative to each other). That work was further extended in [16] to include correlation between errors in east and north directions, allow for multiple samples instead of a single snapshot, and adapt the proposed detector to use three-dimensional position data.

In this work, we deal with a similar problem as [15, 16]. However, we model the positions in three dimensions directly, and allow the position errors to be correlated not only between different directions but also between receivers. This is commonly the case in practice, since receivers in the vicinity of one another experience similar fading effects (e.g. shadowing). Hence, the proposed model also includes receivers with different error characteristics. We derive the optimal detector under these assumptions. In addition, the model and proposed detectors are straightforward to extend to an arbitrary number of dimensions, which allows for inclusion of additional metrics such as pseudo-ranges or SNR estimates that are often available from standard COTS GPS receivers. We also propose detectors that do not require any knowledge of the true spoofed or receiver positions, based on the position deviations between the receivers. This allows for mobile receivers, without requiring any knowledge of their relative positions, provided that they operate in the vicinity of one another. The position deviations are exploited through estimated distances between the receivers, and through properties of the singular values of the observation matrix. This problem has applications, for example, for reliable first responder and soldier positioning. The detection performance of the proposed methods is analyzed numerically, and compared with similar state-of-the-art methods.

2. SYSTEM MODEL

The system model and assumptions will be described in the following. There are K cooperating GNSS receivers, each one delivering its calculated position solution to a fusion center. The receivers are assumed to be COTS receivers, so that only high level data such as the position solution is available. We wish to determine whether

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the GNSS receivers are being spoofed or not. Suppose that all of the available satellite signals are being recreated and transmitted by the spoofer, as in a meaconing attack. If the receivers are being spoofed, by tracking all satellite signals from the spoofer, their error-free position solutions would be equal. However, if they are not being spoofed, their position solutions depend on their actual, distinct, positions. Hence, we wish to discriminate between the hypotheses

$$\begin{aligned} H_0 &: \mathbf{x}_k = \mathbf{p}_k + \mathbf{e}_k,\ k = 1, \ldots, K, \\ H_1 &: \mathbf{x}_k = \mathbf{s} + \mathbf{e}_k,\ k = 1, \ldots, K, \end{aligned} \tag{1}$$

where $\mathbf{p}_k$ is the true position of receive antenna k, $\mathbf{s}$ is the true spoofed position, and $\mathbf{e}_k$ is noise. The position vectors $\mathbf{p}_k \triangleq [p_{k,1}, p_{k,2}, p_{k,3}]^T$ and $\mathbf{s} \triangleq [s_1, s_2, s_3]^T$ represent the north, east and altitude components respectively of the positions. Equation (1) can be equivalently written in matrix form as

$$\begin{aligned} H_0 &: \mathbf{X} = \mathbf{P} + \mathbf{E}, \\ H_1 &: \mathbf{X} = \mathbf{S} + \mathbf{E}, \end{aligned} \tag{2}$$

$$\text{where} \quad \mathbf{X} \triangleq \begin{bmatrix} \mathbf{x}_1^T \\ \vdots \\ \mathbf{x}_K^T \end{bmatrix}, \ \mathbf{P} \triangleq \begin{bmatrix} \mathbf{p}_1^T \\ \vdots \\ \mathbf{p}_K^T \end{bmatrix} \text{ and } \mathbf{S} \triangleq \begin{bmatrix} \mathbf{s}^T \\ \vdots \\ \mathbf{s}^T \end{bmatrix}. \tag{3}$$

Assume that the noise is zero mean Gaussian (as was also done in [15, 16]). Moreover, let $\boldsymbol{e} \triangleq \mathrm{vec}(\mathbf{E})$, and assume that $\boldsymbol{\Omega}$ is the covariance matrix of $\boldsymbol{e}$. That is, the position components are allowed to be correlated between the north, east and altitude directions as well as between receivers. Similarly, let $\boldsymbol{x} \triangleq \mathrm{vec}(\mathbf{X})$, $\boldsymbol{p} \triangleq \mathrm{vec}(\mathbf{P})$ and $\boldsymbol{s} \triangleq \mathrm{vec}(\mathbf{S})$. Then, (2) can be rewritten as

$$\begin{aligned} H_0 &: \boldsymbol{x} \sim \mathcal{N}(\boldsymbol{p}, \boldsymbol{\Omega}), \\ H_1 &: \boldsymbol{x} \sim \mathcal{N}(\boldsymbol{s}, \boldsymbol{\Omega}). \end{aligned} \tag{4}$$

3. GENIE DETECTOR: KNOWN p AND s

Based on the system model described in Section 2, we will derive the Neyman-Pearson optimal detector. It is well known that the optimal detector, when the probability distribution under both hypotheses are perfectly known, is a (log-)likelihood-ratio test. The log-likelihood ratio of (4) is

$$\log \frac{f(\boldsymbol{x}|H_1)}{f(\boldsymbol{x}|H_0)} = \boldsymbol{x}^T \boldsymbol{\Omega}^{-1} (\boldsymbol{s} - \boldsymbol{p}) + \frac{1}{2} \boldsymbol{p}^T \boldsymbol{\Omega}^{-1} \boldsymbol{p} - \frac{1}{2} \boldsymbol{s}^T \boldsymbol{\Omega}^{-1} \boldsymbol{s} + c, \tag{5}$$

where c is a constant independent of the received data. When the true positions $\boldsymbol{p}$ and $\mathbf{s}$ are known, all terms that are independent of the received data $\boldsymbol{x}$ can be included in the decision threshold. Hence, the optimal detector is

$$\boldsymbol{x}^T \boldsymbol{\Omega}^{-1} (\boldsymbol{s} - \boldsymbol{p}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{6}$$

where η is a predetermined decision threshold. This is a genie detector in the sense that it is not realizable, since the true positions cannot be perfectly known. However, the genie detector serves as a performance limit for all other detectors.

In (6), the covariance matrix was assumed to be known and arbitrary. Consider the special case when the position errors are uncorrelated, with equal variance, both between position components and receivers, so that $\boldsymbol{\Omega} = \mathbf{I}$. Then, the optimal test statistic reduces to

$$\boldsymbol{x}^T (\boldsymbol{s} - \boldsymbol{p}) = \sum_{k=1}^{K} \mathbf{x}_k^T (\mathbf{s} - \mathbf{p}_k). \tag{7}$$

So far, we have assumed that all parameters are known under both hypotheses. In practice, that will not be the case. In particular, the main issue is that the true positions are unknown. Detection with unknown positions is dealt with in the following section.

4. GLRT WITH UNKNOWN POSITIONS

A well known, and often very well performing, method when there are unknown parameters is the generalized likelihood-ratio test (GLRT). That is, the unknown parameters are estimated using maximum-likelihood estimation, and the estimated parameters are used in the likelihood-ratio in lieu of the true parameters. That is, we wish to compute the maximum-likelihood (ML) estimates of the unknown positions $\mathbf{p}_k$ and $\mathbf{s}$. Since there is only a single observation available, the ML estimate of the receiver positions $\mathbf{p}_k$ is the observation itself, i.e.

$$\widehat{\mathbf{p}_k} = \mathbf{x}_k \tag{8}$$

The true spoofed position is the mean value of the observed position for each receiver. Hence, the ML estimate of the spoofed position is

$$\widehat{\mathbf{s}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}_k \tag{9}$$

By inserting the estimated positions $\widehat{\mathbf{p}_k}$ and $\widehat{\mathbf{s}}$ in the log-likelihood ratio (5), we obtain the GLRT

$$\boldsymbol{x}^T \boldsymbol{\Omega}^{-1} (\widehat{\boldsymbol{s}} - \widehat{\boldsymbol{p}}) + \frac{1}{2} \widehat{\boldsymbol{p}}^T \boldsymbol{\Omega}^{-1} \widehat{\boldsymbol{p}} - \frac{1}{2} \widehat{\boldsymbol{s}}^T \boldsymbol{\Omega}^{-1} \widehat{\boldsymbol{s}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{10}$$

where

$$\widehat{\boldsymbol{p}} = \mathrm{vec}\left(\begin{bmatrix} \mathbf{x}_1^T \\ \vdots \\ \mathbf{x}_K^T \end{bmatrix} \right) \text{ and } \widehat{\boldsymbol{s}} = \mathrm{vec}\left(\begin{bmatrix} \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}_k^T \\ \vdots \\ \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}_k^T \end{bmatrix} \right). \tag{11}$$

Moreover, the GLRT when the position errors are uncorrelated ($\boldsymbol{\Omega} = \mathbf{I}$) is

$$\begin{aligned} &\boldsymbol{x}^T (\widehat{\boldsymbol{s}} - \widehat{\boldsymbol{p}}) + \frac{1}{2} \widehat{\boldsymbol{p}}^T \widehat{\boldsymbol{p}} - \frac{1}{2} \widehat{\boldsymbol{s}}^T \widehat{\boldsymbol{s}} = \\ &\sum_{k=1}^{K} \left(\frac{1}{K} \sum_{l=1}^{K} \mathbf{x}_k^T \mathbf{x}_l - \frac{1}{2} \|\mathbf{x}_k\|^2 - \frac{1}{2} \left\| \frac{1}{K} \sum_{l=1}^{K} \mathbf{x}_l \right\|^2 \right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \end{aligned} \tag{12}$$

Note that this test can also be used if the covariance matrix is unknown, even though the position errors could actually be correlated.

5. DETECTION WITH UNKNOWN POSITIONS AND COVARIANCE

In most practical scenarios, the true positions $\mathbf{P}$ and $\mathbf{S}$, as well as the covariance matrix $\boldsymbol{\Omega}$, are unknown. To be able to detect the spoofer despite that these parameters are unknown, we will exploit that the spoofed position is equal for all receivers whereas the authentic positions are distinct for different receivers.

Estimating the covariance matrix in a similar manner as the unknown positions in Section 4 is impossible in this case unless a significant structure or a priori information is imposed. The reason is that it would require at least as many observations as the size of the covariance matrix ($3K$) for the sample covariance matrix to have full rank. Instead of trying to estimate the covariance matrix, we propose two detectors based on properties of the mean of the observation that differ between the two hypotheses. The proposed detectors will be explained in the following sections.

5.1. Mean Squared Distance (MSD)

The first approach is to use the spread of the distances between the receivers. If the receivers are being spoofed, the average distance should be close to zero, whereas if they are not being spoofed it should be strictly positive. We propose to use the following test

$$\sum_{k=1}^{K}\sum_{l=1}^{k}\|\mathbf{x}_k-\mathbf{x}_l\|^2 \underset{H_1}{\overset{H_0}{\gtrless}} \eta. \tag{13}$$

Since we assume that the relative positions are unknown, the expected value of the average distance is of course also unknown. If all distances were known, then the receivers' relative positions would be known too, and the scenario would be equivalent to the one in [15, 16]. Of course, knowledge of the distances could be exploited in our proposed test to set the decision threshold more appropriately to achieve the desired detection and false-alarm probabilities. In the current proposal, the decision threshold can be set based on a desired detection probability, which depends on the deviation from zero caused by the noise in the spoofed case. The false-alarm probability will then depend on the scenario in the non-spoofed case, for example the receivers' actual relative positions.

5.2. Singular Value Spread

Another approach is to exploit properties of the observation matrix, or rather the mean of the observation matrix. In the current model, $\mathbb{E}\{\mathbf{X}|H_0\} = \mathbf{P}$ and $\mathbb{E}\{\mathbf{X}|H_1\} = \mathbf{S}$. Since $\mathbf{S}$ is comprised of the single position $\mathbf{s}$, $\mathbb{E}\{\mathbf{X}|H_1\}$ has a single positive singular value whereas $\mathbb{E}\{\mathbf{X}|H_0\}$ has three positive singular values (provided that $K \geq 3$ and the receiver antennas are spread out in the three dimensions). The proposed method is based on these properties of the singular values of $\mathbf{X}$, or equivalently of the eigenvalues of $\mathbf{X}\mathbf{X}^H$.

There exist many other examples where properties of the eigenvalues of a sample covariance matrix have been used for detection. For example, the well known sphericity test [19] discriminates between equal and distinct eigenvalues by using the ratio of the arithmetic to the geometric mean. The sphericity test was originally derived to distinguish between correlated and white Gaussian distributions. An extension to the sphericity test was proposed in [20], where the eigenvalues were not restricted to be equal or distinct but there could be an arbitrary number of distinct values with known multiplicities. These works, however, exploit properties of the covariance matrix of the observed data, whereas we are interested in the mean value of the observation.

We propose to use a similar test statistic, working on the singular values of the observation matrix. Let σ_i, $i = 1, \ldots, 3$, be the singular values of the observation matrix $\mathbf{X}$ sorted in descending order. In our case, the mean of the observation matrix has a single non-zero singular value under H_1, but three positive singular values under H_0. Therefore, the proposed test is

$$\frac{\sigma_1}{\sigma_2+\sigma_3} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{14}$$

Note that this test can be performed without any knowledge of the covariance. However, if the covariance is known or can be estimated, one could prewhiten the received data and then perform an identical test based on the whitened data.

6. DETECTION WITH KNOWN LOCAL POSITIONS

Let $\widetilde{\mathbf{p}}_k$ denote the receiver positions parametrized in a local reference frame such that the origin is determined by the constraint $\sum_{k=1}^{K}\widetilde{\mathbf{p}}_k = \mathbf{0}$. That is, the global receiver positions are $\mathbf{p}_k = \mathbf{b} + \mathbf{R}\widetilde{\mathbf{p}}_k$, where $\mathbf{b}$ is the unknown origin of the local reference frame, and $\mathbf{R}$ is an unknown rotation matrix.

As a comparison, we will include the detector of [16], i.e.

$$-\left|\sum_{k=1}^{K}\sum_{l=1}^{2}\widetilde{\mathbf{p}}_{k,l}\mathbf{x}_{k,l}\right| - \delta\sum_{k=1}^{K}\widetilde{\mathbf{p}}_{k,3}\mathbf{x}_{k,3} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{15}$$

where δ is the ratio of the horizontal error variance to the vertical error variance. The detector (15) was proposed in [16] by extending the two dimensional GLRT of [15] to three dimensions by adding an extra term that includes the vertical component. The error variances were assumed in [16] to be equal in the north and east directions. To allow for unequal horizontal error variances, the detector is slightly generalized so that $\delta \triangleq (\mathbf{\Omega}_{1,1}+\mathbf{\Omega}_{2,2})/(2\mathbf{\Omega}_{3,3})$. When the horizontal error variances are equal that is equivalent to what was originally proposed in [16].

The detector (15) that was extended from two (horizontal) to three dimensions in [16] is not adapted to deal with the unknown three dimensional rotation, but only takes the horizontal rotation into account. A straightforward modification of the test statistic, to be able to deal with a three dimensional rotation, can be made by taking the absolute value of all terms rather than only of the first terms corresponding to the horizontal components. That is, the modified test statistic is

$$-\left|\sum_{k=1}^{K}\sum_{l=1}^{2}\widetilde{\mathbf{p}}_{k,l}\mathbf{x}_{k,l} + \delta\sum_{k=1}^{K}\widetilde{\mathbf{p}}_{k,3}\mathbf{x}_{k,3}\right| = -\left|\sum_{k=1}^{K}\widetilde{\mathbf{p}}_k^T\mathbf{\Lambda}\mathbf{x}_k\right|, \tag{16}$$

where $\mathbf{\Lambda}$ is a weight matrix. Note that in this special case, the weight matrix is used to compensate for the (known) error variances so that $\mathbf{\Lambda} = \mathrm{diag}(1, 1, \delta)$. If the variances are known, a better weight matrix would be $\mathbf{\Lambda} = \mathrm{diag}(1/\sigma_N^2, 1/\sigma_E^2, 1/\sigma_A^2)$, where σ_N^2, σ_E^2 and σ_A^2 denote the error variances in the north, east and altitude directions respectively (assumed to be equal for all receivers). Note that the detectors (15) and (16) cannot account for correlation between receivers. Even more generally, the weight matrix $\mathbf{\Lambda} = \mathbf{\Omega}^{-1}$ could be used to compensate for the full covariance.

7. NUMERICAL RESULTS

In the following, we will show some numerical evaluations of the detection performance, based on Monte-Carlo simulations. The eight true receiver positions were placed at the corners of a cube with side length $d = 5$ meters. The true spoofed position was placed in the center of the cube. Two versions of the detector (15) are included, one where the rotation of the receiver positions is perfectly aligned with the true rotation (labeled *no rot.*), and one where the rotation

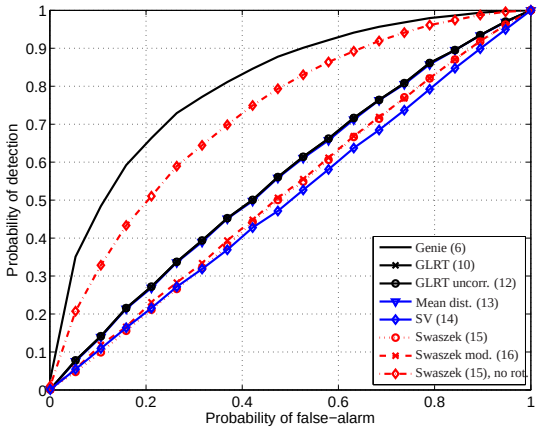

Fig. 1. ROC curve with uncorrelated errors and eight receivers positioned in the corners of a 5 meters cube.

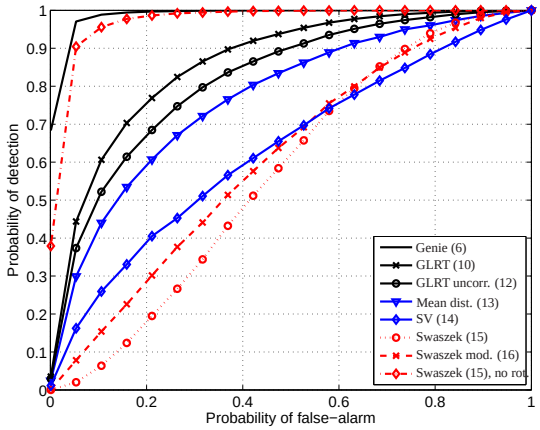

Fig. 2. ROC curve with correlated errors ($\rho = 0.3$) and eight receivers positioned in the corners of a 5 meters cube.

is unknown, which is modeled by a random rotation with uniformly distributed angles in three dimensions. The former case is, of course, equivalent to perfectly known rotation in three dimensions.

Figure 1 shows the receiver operating characteristics (ROC) for the detectors, with uncorrelated errors ($\mathbf{\Omega} = \sigma^2\mathbf{I}$, where $\sigma = 10$). As expected, the GLRT with and without knowledge of the covariance have equal performance since the errors are uncorrelated. In addition, the detector (15) (no rot.) that requires knowledge of the relative positions as well as their rotation, outperforms all of the detectors that do not have that knowledge, but still perform far from the optimal detector. When the rotation is unknown however, the detectors (15) and (16) perform very poorly. That is, knowledge of the local receiver positions can be quite useful if the platform rotation is known too, for example, by the use of gyros and accelerometers. The proposed GLRT detectors (10) and (12), and the mean distance detector (13) performs quite well, despite not having any knowledge of the true receiver positions. The proposed singular value detector (14) shows quite poor performance, and is also computationally more burdensome than the other detectors. Note that the performance in absolute numbers depends much on the scenario (number of receivers, receiver positions etc.), but the performance of different methods relative to each other is similar. For example, the distance between the receivers is smaller in these simulations as compared to [16], which deteriorates the performance for all detectors.

Figure 2 shows the ROC with a fixed correlation factor of 0.3 between all position components. The covariance matrix $\mathbf{\Omega}$ is also normalized such that $\|\mathbf{\Omega}\|_2 = \sigma^2$, to make the position errors comparable with those of Figure 1. In this case, when the errors are correlated, there is a clear difference between the two GLRT detectors (10) and (12). Moreover, we note that both of these detectors outperform the detector (15) of [16] when the rotation is unknown, although no knowledge of the receiver positions is required for (10) and (12). Note also that due to the unknown three dimensional rotation, the detector (15) performs even worse than flipping a coin at low probabilities of false alarm ($\lesssim 0.25$).

Figure 3 shows the ROC with a random covariance matrix $\mathbf{\Omega}$ for each realization. The covariance matrix is created as $\mathbf{\Omega} = \mathbf{H}\mathbf{H}^H$, where the coefficients of the $3 \times 5K$ matrix $\mathbf{H}$ are drawn from a white Gaussian distribution, and then normalized such that $\|\mathbf{\Omega}\|_2 = \sigma^2$. That is, the covariance matrix is Wishart distributed but normalized to make a fair comparison with the previous cases. In this case,

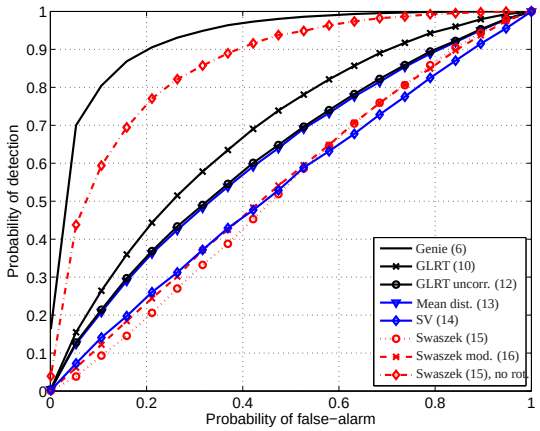

Fig. 3. ROC curve with a Wishart covariance matrix and eight receivers positioned in the corners of a 5 meters cube.

the average performance of the GLRT (12), without knowledge of the covariance, is very close to that of the mean distance detector (13). Again, the proposed singular value detector shows poor performance compared to the other detectors.

8. CONCLUDING REMARKS

We have proposed several new spoofing detectors that exploit the position solution from multiple COTS GPS receivers in different ways, without requiring any knowledge of the receiver positions. We have also proposed an extension to a previously proposed detector, that exploits knowledge of the local receiver positions but also takes into account an unknown three dimensional rotation. We have also shown by Monte-Carlo simulations that the proposed detectors perform well in many cases.

It should be noted that it is straightforward to include other metrics than the position solution from each receiver, such as the individual satellite pseudo ranges. That would only increase the dimension of the data model (1), and the derivation would follow in exactly the same way.

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