RATIONAL CONSUMER BEHAVIOR MODELS IN SMART PRICING

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ABSTRACT

A game-theoretic framework based on smart pricing in power grids that incorporates heterogeneous user preferences and renewable power uncertainty is considered. The system operator adopts an adaptive pricing policy that depends on total consumption and renewable generation. The pricing policy sets up a non-cooperative game of incomplete information among users with heterogeneous preferences. Selfish, altruistic and welfare maximizing user behavior models are proposed. Information exchange models in which users only have private information, communicate or receive broadcasted information are considered. For each pair of behavior and information exchange models, rational consumption strategy is characterized. Numerical analyses reveal that communication is beneficial for the expected aggregate payoff while it does not affect the expected net revenue of the system operator. Moreover, the additional information to the users helps reduce the variance of total consumption among runs increasing the accuracy of demand predictions.

Index Terms— Smart grid, demand response, game theory.

1. INTRODUCTION

Smart pricing policies emerge as prominent methods to alleviate the complications in power balancing caused by uncertainties both on the consumer and on the supply side in power systems. The uncertainty on the consumer side is caused by changes in user consumption preferences while the uncertainty on the supply side is due to renewable resources [1–4]. Smart meters that control the power consumption of customers, and enable information exchange between meters and the system operator (SO) provide the infrastructure to implement smart pricing policies [5].

Real-time pricing (RTP) is a smart pricing policy where price depends on instantaneous consumption of the population [2, 6, 7]. In RTP, the SO shares part of the risk and reward with its customers by setting price based on the total consumption which depends on user population behavior. Therefore, it is customary to propose game theoretic models of consumption behavior where users strategically reason about behavior of others to determine their consumption [2, 3, 6–12]. The specifics of the consumer behavior model and the information provided impact the welfare of the system and is critical in assessing the benefits of a pricing scheme [12]. Given a RTP scheme our goal in this paper is to characterize price anticipatory behavior models, in which users strategically reason about their impact on price, under different information exchange schemes and assess their impact on welfare and SO's net revenue.

The SO exercises a RTP mechanism in which customers agree to a linear price function that depends on the total consumption and a parameter to alleviate the renewable generation uncertainty (Section 2.1). The user utilities depend on heterogeneous consumption preferences and price (Section 2.2) [13, 14]. We propose three models of consumer behavior based on whether a user regards his selfish utility, the population's aggregate utility or the welfare (Section 2.3). As time progresses, the users' consumption decisions reveal information about their preferences which can be utilized in updating estimates of total consumption, and hence price. For this, we provide three information exchange models, namely, private, action sharing and broadcast (Section 2.4). In the private model, users only receive the SO's initial public signal. In action sharing there exists a communication network which enables users to exchange their consumption with immediate neighbors. In broadcast, the SO broadcasts the total consumption after each time step. We formulate each user behavior and information exchange model pair as a repeated game of incomplete information and characterize equilibrium behavior (Section 4). In [14] we provided a characterization of selfish players with private and communication information exchange models. In this paper, we extend the characterization to altruistic and welfare maximizing behavior and broadcast information model. Moreover, we comparatively analyze the effects of each pair of behavior and information exchange model on total consumption, aggregate user utility and SO's net revenue (Section 5).

Our findings can be summarized as follows. Providing more information to the users does not hurt the SO's expected net revenue and increases the expected aggregate utility. In addition, it reduces the uncertainty in total demand. Action sharing model eventually achieves the expected utility under full information when the network is connected. The positive effects of sharing reduce with increasing preference correlation. Preference correlation increase has a decreasing effect on the expected aggregate utility for all behavior models. Finally, the inefficiency due to selfish behavior diminishes as the number of customers grows.

2. DEMAND RESPONSE MODEL

There are N customers each equipped with a power consumption scheduler. Power consumption of user $i \in \{1, \dots, N\}$ at time $h \in \mathcal{H} := \{1, \dots, H\}$ is denoted by l_{ih}. The sum of the power consumed by N customers at time h yields the time slot total consumption $L_h := \sum_{i \in \mathcal{N}} l_{ih}$.

2.1. Real Time Pricing

The SO's cost of supplying L_h amounts of power is $C_h(L_h)$ units. When the generation cost per unit is constant, $C_h(L_h)$ is a linear function of L_h. More often, increasing the load L_h results in increasing unit costs as more expensive energy sources are brought online. This results in superlinear cost functions $C_h(L_h)$ with a customary model being the quadratic form

$$C_h(L_h) = \frac{1}{2}\kappa_h L_h^2, \tag{1}$$

for given constants $\kappa_h > 0$ that depend on the time slot h. The cost in (1) has been experimentally validated for thermal generators [15] and is otherwise widely accepted as a reasonable approximation [2, 6, 10].

The SO implements an adaptive pricing strategy whereby users are charged a slot-dependent price p_h that varies linearly with the total power consumption L_h. The SO owns renewable source plants and incorporates renewable source generation into the pricing strategy by introducing a random variable $\omega_h \in \mathbb{R}$ that depends on the renewable power produced at time h. The per-unit power price at time h is set as

$$p_h(L_h; \omega_h) = \gamma_h(L_h + \omega_h), \tag{2}$$

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where $\gamma_h > 0$ is a policy parameter to be determined by the SO based on its objectives. In [13], we present how the operator can pick the policy parameter to minimize PAR or achieve a desired rate of return. The random variable ω_h depends on the renewable source generation in h. That is, SO applies a price discount $\omega_h < 0$ when renewable sources yield above their nominal benchmark capacity, otherwise it implies a price increase. The specific dependence of ω_h with the realized renewable generation and the policy parameter γ_h, are part of the supply contract between the SO and its customers. We assume that the SO uses a model on the renewable power generation to estimate the value of ω_h at the beginning of time h. The mean estimate $\bar{\omega}_h := E_{\omega_h}[\omega_h]$ of the corresponding probability distribution P_{ω_h} is made available to all users prior to time h.

The SO's price function maps the total demand to the market price. Observe that the price $p_h(L_h;\omega_h)$ at time h becomes known *after* the end of the time slot. This is because prices depend on the total demand L_h and the value of ω_h which are unknown a priori.

2.2. Power consumer

User i's consumption at time h, l_{ih}, depends on his consumption preference for the time $g_{ih} > 0$. We model the preference g_{ih} as a random variable that may vary across time. User i receives a quadratically diminishing marginal utility that increases linearly with his preference g_{ih} and decreases quadratically with α_h, that is, $g_{ih}l_{ih} - \alpha_h l_{ih}^2$. The utility of i at time slot $h \in \mathcal{H}$ is then captured by the difference between the marginal utility of i with the monetary cost of consumption $l_{ih}p_h(L_h;\omega_h)$,

$$u_{ih}(l_{ih}, L_h; g_{ih}, \omega_h) = -l_{ih}p_h(L_h;\omega_h) + g_{ih}l_{ih} - \alpha_h l_{ih}^2. \quad (3)$$

Note that if the SO's policy parameter is set to $\gamma_h = 0$, the utility of user i is maximized by $l_{ih} = g_{ih}/2\alpha_h$ – see [1, 2] for similar formulations.

User i's utility depends on the total consumption at h L_h, that is, it depends on the consumption of other users denoted by $l_{-ih} := \{l_{jh} : j \in \mathcal{N} \setminus i\}$. l_{-ih} depends partly on respective preferences, i.e., g_{-ih}, which are, in general, *unknown* to user i. We assume, however, that there is a probability distribution $P_{\mathbf{g}_h}(\mathbf{g}_h)$ on the vector of preferences $\mathbf{g}_h := [g_{1h}, \dots, g_{Nh}]^T$ from which the preferences are drawn. We further assume that $P_{\mathbf{g}_h}$ is normal with mean $\bar{g}_h\mathbf{1}$ where $\bar{g}_h > 0$ and $\mathbf{1}$ is an $N \times 1$ vector of ones, and covariance matrix $\mathbf{\Sigma}_h$, $P_{\mathbf{g}_h}(\mathbf{g}_h) = \mathcal{N}(\mathbf{g}_h;\, \bar{g}_h\mathbf{1}, \mathbf{\Sigma}_h)$. We use the operator $E_{\mathbf{g}_h}$ to signify expectation with respect to the distribution $P_{\mathbf{g}_h}$ and σ_{ij}^h to denote the (i,j)th entry of the covariance matrix $\mathbf{\Sigma}_h$. Note that users have equal average preferences. In general, $\sigma_{ij}^h > 0$ to account for correlated preferences due to, e.g., common weather. We assume that if there is a change in the user preferences from time h to $h+1$, the distributions of $\mathbf{g}_h$ and $\mathbf{g}_{h+1}$ are independent.

At the beginning of time h the information available to users and the operator is the expected effect of renewable sources on price $\bar{\omega}_h$, the user preference distribution $P_{\mathbf{g}_h}$ and the parameters α_h and γ_h. We assume that $P_{\mathbf{g}_h}$ is correctly predicted by the SO based on past data and is announced to the users. The SO also announces the parameter γ_h and its expectation of ω_h. In addition, each user knows his own preference g_{ih}.

2.3. Consumer behavior models

The aggregate utility at time h is the sum of consumer utilities,

$$U_h(l_{ih}, l_{-ih}) := \sum_{i=1}^{N} u_{ih}(l_{ih}, L_h; g_{ih}, \omega_h), \quad (4)$$

and the net revenue of the SO defined as its revenue minus the cost (1)

$$NR_h(L_h;\omega_h) := p_h(L_h;\omega_h)L_h - C_h(L_h). \quad (5)$$

The welfare of the overall system at time h is the sum of the aggregate utility with the net revenue,

$$W_h(l_{ih}, l_{-ih}) := U_h + NR_h. \quad (6)$$

User i is *selfish* when he wants to maximize individual utility in (3). He is *altruistic* when he cares about the well-being of other users, that is, aims to maximize U_h in (4) given his information on preferences of others. Finally, user i might also consider the well-being of the entire system and aim to maximize *welfare* W_h in (6) given his information. We use the superscript $\Gamma \in$ {S, U, W} in $u_{ih}^{\Gamma}(l_{ih}, l_{-ih})$ to indicate that user i maximizes his selfish payoff S, aggregate utility U or welfare W.

2.4. Information exchange models

Consumption behavior of other users at time h l_{jh} can provide valuable information about the consumption preferences $\mathbf{g}_h$ in that time slot. This information is of use to user i in estimating consumption for time $h+1$ if the preferences of the users do not change in that time slot, that is, $\mathbf{g}_h = \mathbf{g}_{h+1}$. Otherwise, the information is not helpful in estimating behavior of others for time $h+1$ because the change in the preference distribution is independent. We present a list of possible information exchange models under the assumption that the preferences remain the same for a given amount of time starting from time h and lasting until there is a change in the preferences, that is, $\mathbf{g}_h = \mathbf{g}_0 := [g_{10}, \dots, g_{N0}]$ with prior distribution $P_{\mathbf{g}_0}$ for the time zone $\mathcal{T} = \{h \in \mathcal{H} : \mathbf{g}_h = \mathbf{g}_0\}$. If there is a change in the preference distribution we restart the information exchange process. The prediction of renewable source term P_{ω_h} is allowed to vary for $h \in \mathcal{T}$. We use I_{ih}^{Ω} to denote the set of information available to consumer i at time slot $h \in \mathcal{T}$ for the information exchange model Ω.

Private (P). The information specific to consumers is the merest possible when it consists of the private preference g_{i0}, that is, $I_{ih}^{P} = \{g_{i0}\}$.

Action Sharing (AS). Power control schedulers are interconnected via a communication network represented by a graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$ with its nodes representing the users $\mathcal{N} = \{1, \dots, N\}$ and edges belonging to the set $\mathcal{E}$ indicating communication between nodes. After time slot h, user i's neighbors, $\mathcal{N}_i := \{j \in \mathcal{N} : (j,i) \in \mathcal{E}\}$, send their consumption $l_{\mathcal{N}_i t} := [l_{i_1 t}, \dots, l_{i_{d(i)} t}]$ to user i where the vector of i's $d(i) := \#\mathcal{N}_i$ neighbors is denoted by $[i_1, \dots, i_{d(i)}]$. Hence, the information of user i at time slot $h \in \mathcal{T}$ contains self-preference g_{i0} and the consumption of his neighbors up to time $h-1$, that is, $I_{ih}^{AS} = \{g_{i0}, \{l_{\mathcal{N}_i t}\}_{t=1,\dots,h-1}\}$. We assume that the power consumption schedulers keep the received information private and that they know the network structure $\mathcal{G}$.

SO Broadcast (B). The SO collects all the user behavior at each time h and broadcasts the total consumption L_h, that is, $I_{ih}^{B} = \{g_{i0}, L_{1:h-1}\}$.

Behavior model, i.e., selfish (S), altruistic (U), or welfare (W) maximizer, and the information exchange model, i.e., private (P), action sharing (AS) or SO broadcast (B) determine the consumption decisions of user i. Next we define the user rational behavior for each behavior model using the solution concept Bayesian Nash equilibrium (BNE).

3. BAYESIAN NASH EQUILIBRIA

User i's load consumption at time $h \in \mathcal{T}$ is determined by his *belief* q_{ih}^{Ω} and *strategy* s_{ih}. The belief of i is a conditional probability distribution on $\mathbf{g}_0$ given I_{ih}^{Ω}, $q_{ih}^{\Omega}(\cdot) := P_{\mathbf{g}_0}(\cdot|I_{ih}^{\Omega})$. We use $E_{ih}^{\Omega}[\cdot] := E_{\mathbf{g}_0}[\cdot|I_{ih}^{\Omega}]$ to indicate conditional expectation with respect to belief of q_{ih}. In order to second-guess the consumption of other customers, user i forms beliefs on preferences given the common prior $P_{\mathbf{g}_0}$ and his information I_{ih}^{Ω}. His strategy maps any possible local observation that he may have to his consumption, that is, $s_{ih} : I_{ih}^{\Omega} \mapsto \mathbb{R}$ for any I_{ih}^{Ω}. In particular, user i's best response strategy is to maximize expected utility with respect to his belief q_{ih}^{Ω} given the strategies of other customers $\mathbf{s}_{-ih} := \{s_{jh}\}_{j \neq i}$,

$$BR^{\Gamma}(I_{ih}^{\Omega}; \mathbf{s}_{-ih}) = \arg\max_{l_{ih}} E_{ih}^{\Omega}\left[u_{ih}^{\Gamma}(l_{ih}, \mathbf{s}_{-ih}; g_{i0}, \omega_h)\right]. \quad (7)$$

Before we define the BNE solution concept, we state the following lemma that characterizes the general form of the best response function for all the consumer models Γ = {S, U, W}.

Lemma 1 *The best response strategy for the consumer behavior models $\Gamma \in \{S, U, W\}$ has the following general form*

$$BR^{\Gamma}(I_{ih}^{\Omega}; \mathbf{s}_{-ih}) = \frac{g_{i0} - \mu_h^{\Gamma}\bar{\omega}_h - \lambda_h^{\Gamma}\sum_{j\neq i}[E_{ih}^{\Omega}[s_{jh}])}{2(\tau_h^{\Gamma} + \alpha_h)} \tag{8}$$

where $\lambda_h^S = \mu_h^S = \tau_h^S = \gamma_h$, $\lambda_h^U = 2\gamma_h$, $\mu_h^U = \tau_h^U = \gamma_h$, and $\lambda_h^W = 2\kappa_h$, $\mu_h^W = 0$, $\tau_h^W = \kappa_h$.

The proof follows by taking the derivative of the corresponding utility with respect i's consumption l_{ih}, equating to zero and solving the equality for l_{ih}. A BNE strategy profile is such that each user maximizes his expected utility u_{ih}^{Γ} with respect to his own belief given that other user play with respect to BNE strategy.

Definition 1 *A Bayesian Nash equilibrium (BNE) strategy $\mathbf{s}^{\Gamma} := \{s_{ih}^{\Gamma}\}_{i\in\mathcal{N},h\in\mathcal{T}}$ for the user behavior model $\Gamma \in \{S, U, W\}$ is such that for all $i \in \mathcal{N}$, $h \in \mathcal{T}$, and $\{I_{ih}^{\Omega}\}_{i\in\mathcal{N},h\in\mathcal{T}}$,*

$$E_{ih}^{\Omega}\big[u_{ih}^{\Gamma}(s_{ih}^{\Gamma}, \mathbf{s}_{-ih}^{\Gamma}; g_{i0}, \omega_h)\big] \geq E_{ih}^{\Omega}\big[u_{ih}^{\Gamma}(s_{ih}, \mathbf{s}_{-ih}^{\Gamma}; g_{i0}, \omega_h)\big]. \tag{9}$$

A BNE strategy (9) is computed using beliefs formed according to Bayes' rule. Note that in BNE strategy profile, no user at any given point in time has a profitable deviation to another strategy – see [16–18] for more details on the concept. In (9), users keep beliefs on others' consumption, which is a function of their beliefs and strategies, to respond optimally. Equivalently, a BNE strategy is where users best respond as per (7) to best response strategies of others, that is, $s_{ih}^{\Gamma}(I_{ih}^{\Omega}) = BR(I_{ih}^{\Omega}; \mathbf{s}_{-ih}^{\Gamma})$ for all $i \in \mathcal{N}$, $h \in \mathcal{T}$ and I_{ih}^{Ω}. Using this notion of BNE and Lemma 1, next we characterize the unique linear BNE strategy for any Ω and Γ.

4. CONSUMERS' BAYESIAN GAME

It suffices for user i to keep an estimate of the preference profile $\mathbf{g}_0$ in order to keep an estimate of beliefs and strategies of other users [16]. We define the preference profile augmented with mean $\bar{g}_0$, $\tilde{\mathbf{g}} := [\mathbf{g}_0^T, \bar{g}_0]^T$. The mean and error covariance matrix of i's belief at time h is denoted by $E_{ih}^{\Omega}[\tilde{\mathbf{g}}]$ and $\mathbf{M}_{\tilde{\mathbf{g}}\tilde{\mathbf{g}}}^{i}(h) := E[(\tilde{\mathbf{g}} - E[\tilde{\mathbf{g}}|I_{ih}^{\Omega}])(\tilde{\mathbf{g}} - E[\tilde{\mathbf{g}}|I_{ih}^{\Omega}])^T]$, respectively. Next result shows that, for an information exchange model $\Omega \in$\{P, AS, B\}, there exists a unique BNE strategy that is calculated by a linear weighting of $E_{ih}^{\Omega}[\tilde{\mathbf{g}}]$ and the weights are obtained by solving a set of linear equations based on the behavior model $\Gamma \in$ \{S, U, W\} – see [19] for the proof.

Proposition 1 *Consider the Bayesian game defined by the payoff u_{ih}^{Γ} for $\Gamma \in \{S, U, W\}$. Let the information of customer i at time $h \in \mathcal{T}$ I_{ih}^{Ω} be defined by one of the information exchange models $\Omega \in \{P, AS, B\}$. Given the normal prior on the self-preference profile $\mathbf{g}_0$, the user i's mean estimate of the preference profile at time $h \in \mathcal{T}$ can be written as a linear combination of $\tilde{\mathbf{g}}$, that is, $E_{ih}^{\Omega}[\tilde{\mathbf{g}}] = \mathbf{T}_{i,h}^{\Omega}\tilde{\mathbf{g}}$ where $\mathbf{T}_{i,h}^{\Omega} \in \mathbb{R}^{N+1\times N+1}$ for all $h \in \mathcal{T}$, and the unique equilibrium strategy for i is linear in his estimate of the augmented self-preference profile,*

$$s_{ih}^{\Gamma}(I_{ih}^{\Omega}) = \mathbf{v}_{ih}^T E_{ih}^{\Omega}[\tilde{\mathbf{g}}] + r_{ih} \tag{10}$$

where $\mathbf{v}_{ih} \in \mathbb{R}^{N+1}$ and $r_{ih} \in \mathbb{R}$ are the strategy coefficients. The strategy coefficients are calculated by solving the following set of equations for the consumer behavior models $\Gamma \in \{S, U, W\}$

$$\mathbf{v}_{ih}^T\mathbf{T}_{i,h}^{\Omega T} + \rho_h^{\Gamma}\lambda_h^{\Gamma}\sum_{j\in\mathcal{N}\setminus i}\mathbf{v}_{jh}\mathbf{T}_{i,h}^{\Omega T}\mathbf{T}_{j,h}^{\Omega T} = \rho_h^{\Gamma}\mathbf{e}_i \quad \forall i \in \mathcal{N}, \tag{11}$$

$$r_{ih} + \rho_h^{\Gamma}\lambda_h^{\Gamma}\sum_{j\in\mathcal{N}\setminus i} r_{jh}^{\Gamma} = -\rho_h^{\Gamma}\mu_h^{\Gamma}\bar{\omega}_h \quad \forall i \in \mathcal{N} \tag{12}$$

where $\lambda_h^{\Gamma}, \mu_h^{\Gamma}, \tau_h^{\Gamma}$ are as defined in Lemma 1 for $\Gamma \in \{S, U, W\}$, $\rho_h^{\Gamma} = (2(\tau_h^{\Gamma} + \alpha_h))^{-1}$ and $\mathbf{e}_i \in \mathbb{R}^{N+1}$ is the unit vector.

Proposition 1 presents the computation of BNE consumption strategies at each time which is integrated with belief propagation. The scheduler repeatedly determines its consumption strategy given behavior model Γ and propagates its beliefs on $\tilde{\mathbf{g}}$ given available information based on the information exchange model Ω to use them in the next time. The Bayesian belief propagation follows the LMMSE sequential estimates at each time for the AS and B information models. The proof uses LMMSE sequential estimates to show that the beliefs remain Gaussian and the mean estimates are linear combinations of $\tilde{\mathbf{g}}$ at all times for all models. For each behavior $\Gamma \in$ \{S, U, W\} the user solves a different set of equations in (11)-(12). This user can do locally by keeping track of how others compute their beliefs [14]. For *Private* model, users do not receive any new information within the horizon hence their mean estimate of $\tilde{\mathbf{g}}$ is the same, that is, $\mathbf{T}_{i,h}^{P} = \mathbf{T}_{i,1}^{P}$ for $h \in \mathcal{T}$, which implies the set of equations (11)-(12) need to be solved only once at the beginning to determine the strategy for the time horizon. For *Action Sharing*, upon observing actions of his neighbors $l_{\mathcal{N}_i h}$, user i has new relevant information about $\tilde{\mathbf{g}}$ which it can use to better predict the total consumption in future steps. In *SO Broadcast* model, each user receives the total load at time h L_h that is useful in estimating price in the following time. We detail the local computations of scheduler i in an algorithm for the selfish and action sharing model in [14]. See [19] for specific changes for each model pair.

4.1. Price-taking consumer behavior

Users are price takers $\Gamma = \mathrm{K}$ when they do not anticipate their effect on price, that is, the selfish payoff in (3) depends on self consumption l_{ih} and price p_h. Given the price at time p_h, consumers maximize their payoff by $l_{ih}^K = (-p_h + g_{ih})/2\alpha_h$. Users are charged hourly prices p_h determined by the SO maximizing hourly expected net revenue, that is, $p_h = \max_p E[pL_h^K - C_h(L_h^K)]$ where $L_h^K = \sum_{j=1}^N l_{jh}^K$. Expected net revenue maximization results in $p_h = (2\alpha_h + \kappa_h)\bar{g}_h/(4\alpha_h + 2N\kappa_h)$. Note that information models do not affect price-taking behavior.

5. NUMERICAL ANALYSIS

We analyze the performance under the behavior models $\Gamma \in$ \{S, U, W, K\} and the information exchange models $\Omega \in$\{P, AS, B\}. In each pair of price anticipating behavior $\Gamma \in$ \{S, U, W\} and information exchange model consumers behave rationally following BNE strategy in Proposition 1. Price takers follow the model in Section 4.1. We consider average consumption $\bar{L} := \sum_h L_h/H$, aggregate utility $U = \sum_h U_h/H$, net revenue $NR = \sum_h NR_h/H$ and welfare $W = \sum_h W_h/H$ as the performance metrics of the model.

The setup contains $H = 24$ hours. We let the SO's cost parameter be $\kappa_h = 1$ for $h \in \mathcal{H}$. The policy parameter is $\gamma_h = 1.2$ for all times – see [13] for an extensive analysis on the effects of γ_h. There are $N = 10$ users. We consider a geometric network on a 3 mile by 5 mile radius with a connection threshold of 2 miles. We analyze the effect of the population size N and the network structure in Section 5.1. Each user has a decay equal to $\alpha_h = 1$ for $h \in \mathcal{H}$. In order for the information sharing models to be relevant we assume that the preferences $\mathbf{g}_h$ are realized once and at the beginning of period as per the discussion in Section 2.4, that is, $\mathbf{g}_h = \mathbf{g}$. The preference $\mathbf{g}$ is normal with mean $\bar{g} = 30$ with identical variance $\sigma_{ii} = 4$ and homogeneous correlation σ_{ij}. We analyze the effect of correlation σ_{ij} on the performance metrics by varying $\sigma_{ij} \in \{0, 1, 2, 3\}$. We further set $\omega_h = \omega$ for all $h \in \mathcal{H}$. Unless otherwise stated, we let ω have normal distribution with mean $\bar{\omega} = 0$ and variance $\sigma_{\omega} = 2$.

We consider 100 instantiations of the random variables $\mathbf{g}$ and ω for each $\sigma_{ij} \in \{0, 1, 2, 3\}$. We compute the expected values of average consumption, aggregate utility and net revenue $(E\bar{L}, EU, ENR)$ by taking an average of all runs for a given correlation coefficient σ_{ij}.

Our findings can be summarized as follows – see Table 1 in [19] for complete results. With increasing correlation σ_{ij}, EU and ENR decrease for each Γ and Ω pair, e.g., when $\sigma_{ij} = 0$ to $\sigma_{ij} = 3$, EU decreases from 187 to 182.5 for $\Gamma = \mathrm{U}$. The ENR of the SO is the largest when $\sigma_{ij} = 0$ and $\Gamma = \mathrm{K}$. However, the SO's ENR for $\Gamma =$

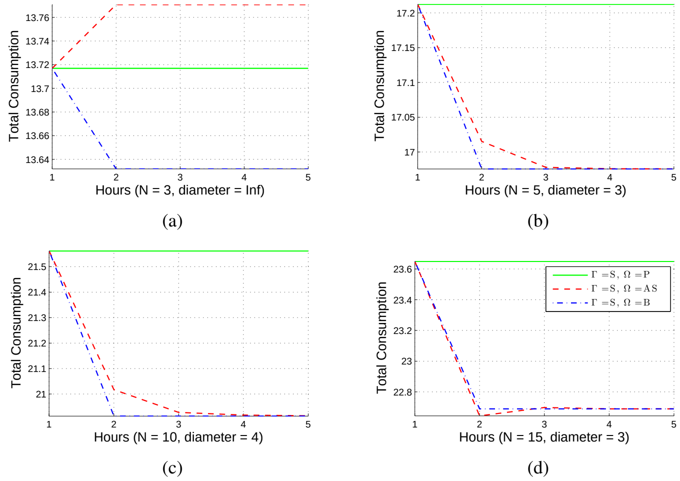

Fig. 1. L_h over time for Γ =S and Ω $\in$\{P, AS, B\} for $N = \{3, 5, 10, 15\}$. When the network is connected, AS model converges to B model in the number of steps equal to the diameter of the network.

K drops from $ENR = \$122$ when $\sigma_{ij} = 0$ to $ENR = \$6.3$ when $\sigma_{ij} = 3$. Moreover, for $\Gamma =$ K, we observe that the variance of ENR increases when σ_{ij} changes from $\sigma_{ij} = 0$ to $\sigma_{ij} = 3$. Among $\Gamma \in$\{S, U, W\} SO attains the highest ENR when $\Gamma =$ S, $ENR = \$67$ for $\Omega =$ P and $\sigma_{ij} = 0$. Furthermore, when $\Gamma \in$ \{S, U, W\}, the effect of correlation coefficient on SO's ENR is small, e.g., $ENR = \$66$ when $\sigma_{ij} = 3$ for $\Gamma =$ S and $\Omega =$ P. ENR drops significantly when $\Gamma =$ U, e.g., the ENR drops to \$20 when $\Gamma =$ U. The model pair $\Gamma =$ W and $\Omega =$ B, achieves the highest EW for all σ_{ij}, e.g., $EW = 216$ when $\sigma_{ij} = 0$. Among anticipatory behaviors $\Gamma \in$ \{S, U, W\}, the lowest $EW = 168$ is when $\Gamma =$ S and $\Omega =$ P for $\sigma_{ij} = 0$. The loss due to selfishness is more remarkable than the loss due to keeping information private, that is, $EW = 172$ when $\Gamma =$ S and $\Omega =$ B, or $EW = 211$ when $\Gamma =$ W and $\Omega =$ P. The positive effect of communication on EW is expected since information exchange helps users estimate price better. However, this does not imply that the utility of each user is improved [20, 21].

Providing more information to the consumers is always beneficial to the expected aggregate utility EU for $\Gamma \in$ \{S, U, W\} and the information affects neither ENR nor $E\bar{L}$. Consequently, Ω $\in$\{AS, B\} improves EW. We observe that the improvement in B model is larger than AS model. This is because in AS users learn about $\tilde{\mathbf{g}}$ through their neighbors, that is, it takes longer in AS for the users to eventually learn the sufficient statistic for a connected network than in the B model. The impact of AS and B models vanishes as the correlation approaches $\sigma_{ij}/\sigma_{ii} = 1$. We further consider the variance of $\bar{L}$ as a measure of the uncertainty of SO on demand. We observe that the variance for P model is always larger than AS or B models. In sum, the SO can allow users to share their information and expect the user utility to increase and variance of average consumption to drop without any decrease in the ENR.

5.1. Effect of population size N and renewable uncertainty $\bar{\omega}$

Figs. 1(a)-(d) exhibit the total consumption with respect to hours for the population size $N = \{3, 5, 10, 15\}$, respectively. Each line corresponds to a different information exchange model Ω for $\Gamma =$ S – see the legend in Fig. 1(d). The network is determined by randomly placing N individuals on a 3 mile×5 mile area and connecting them with the threshold connectivity of 2 miles. The network diameter is displayed in the horizontal axis along with the population size for each plot. We observe that when the network is connected (Figs. 1(b)-(d)), the total consumption in AS model converges to the total consumption in B model. Note that full information is achieved when the SO broadcasts L_h and σ_{ij} is homogeneous, that is, L_h is a sufficient statistic of price [20]. The convergence time is in the order of the network diameter. When the network is not connected (Fig. 1(a)), the convergence does not occur.

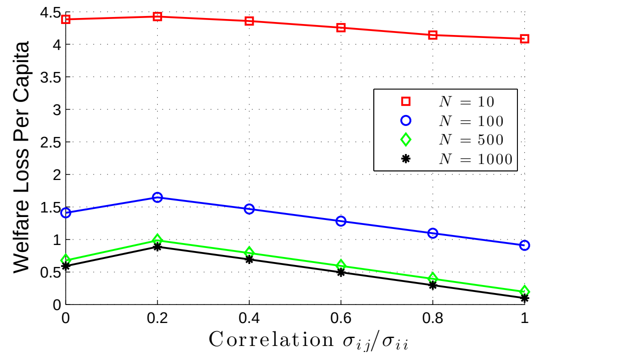

Fig. 2. Expected welfare loss per capita EWL/N for N with respect to correlation coefficient σ_{ij}/σ_{ii}. EWL is the difference in EW when Γ =W, Ω =B and EW when Γ =S, Ω =P. EWL/N decreases with N.

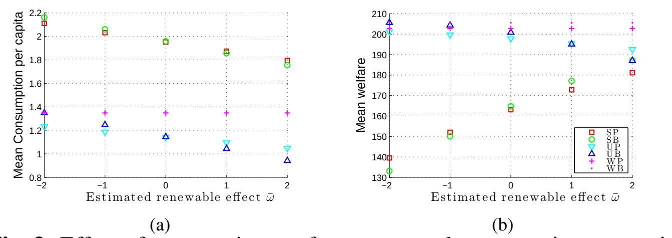

Fig. 3. Effect of mean estimate of ω, $\bar{\omega}$, on total consumption per capita $E\bar{L}/N$ (a) and welfare EW (b). $\bar{\omega} \in \{-2, -1, 0, 1, 2\}$ and $\sigma_{ij}/\sigma_{ii} = 0.6$. For each Γ $\in$\{S,U,W\} we consider $\Omega \in \{P, B\}$. Increasing $\bar{\omega}$ affects the EW positively when $\Gamma =$ S, and negatively when $\Gamma =$ U.

We further examine the effect of N on the EW loss per capita in Fig. 2. EW loss, EWL, is the difference between the EW for $\Gamma =$ W and $\Omega =$ B and EW for $\Gamma =$ S, and $\Omega =$ P. Expected welfare loss per capita normalizes EWL by the N, that is, EWL/N. The EWL incorporates inefficiencies due to selfish behavior and information. From Fig. 2, we observe that the inefficiency disappears as N increases. Furthermore, the correlation coefficient σ_{ij}/σ_{ii} can increase welfare loss for $\sigma_{ij}/\sigma_{ii} < 0.2$, otherwise when $\sigma_{ij}/\sigma_{ii} > 0.2$ increases, it has a decreasing effect on EWL. From discussion above, increase in correlation coefficient has a decreasing effect on the EW. That is, increasing σ_{ij} is more detrimental when $\Gamma =$ W and $\Omega =$ B than when $\Gamma =$ S and $\Omega =$ P. This is due to the fact that as $\sigma_{ij}/\sigma_{ii} \to 1$, the informational inefficiency disappears.

Figs. 3(a)-(b) plot the total consumption per capita $E\bar{L}/N$ and mean welfare EW respectively when $\bar{\omega} \in \{-2, -1, 0, 1, 2\}$ with $\sigma_{ij}/\sigma_{ii} = 0.6$. Based on the best response of W behavior in (8), W user is not affected by the changes in $\bar{\omega}$. Since the increase in $\bar{\omega}$ implies an increase in price, the $E\bar{L}/N$ drops for S and U behaviors (Fig. 3(a)). Because the S users have higher consumption than W users, the decrease in consumption benefits EW of S users. Similarly, the U users have lower consumption than W users, hence further decrease in consumption due to increase in $\bar{\omega}$ detriments EW. Conversely, decreasing $\bar{\omega}$, can improve EW for U users above EW for W users – see Fig. 3(b) when $\bar{\omega} = -2$.

6. CONCLUSION

We considered rational behavior models under information exchange models for a power market with heterogeneous user preferences and a SO. The SO exercised a RTP policy which set up a game of non-cooperative game of incomplete information for the users. We showed that when the users exchange consumption levels or the SO broadcasts aggregate demand information, the expected aggregate utility increases and demand variance decreases without affecting SO's net revenue.

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