PARAMETRIC FRUGAL SENSING OF MOVING AVERAGE POWER SPECTRA

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ABSTRACT

Wideband spectrum sensing is one of the core components of cognitive radio. A novel *frugal sensing* scheme was recently proposed by Mehanna *et al*, aiming to crowdsource spectrum sensing operations to a network of sensors transmitting randomly filtered power measurement bits to a fusion center (FC). The ambient power spectrum is then estimated at the FC using a non-parametric approach. Here, it is assumed that the primary signal admits a Moving Average (MA) parametrization, and the frugal sensing problem is revisited from a parametric spectral estimation point of view. We show that the problem of estimating admissible MA parameters (and thus the MA power spectrum) from single bit quantized data can be formulated as a non-convex Quadratically Constrained Quadratic Program (QCQP). This is NP–Hard in general, but semidefinite-relaxation (SDR) can be employed to obtain approximate solutions. Simulations reveal the superior performance of the SDR technique over the globally optimal solution obtained from the non-parametric formulation, when the MA assumption is valid.

1. INTRODUCTION

Wideband spectrum sensing is a crucial prerequisite for cognitive radio, since it forms the basis for adaptive spectrum sharing. While the focus of most prior work has been on reconstruction of the signal's *Fourier spectrum*, only the *power spectrum* (PS) [1] is relevant for certain sensing applications e.g., cognitive radio and radio astronomy. The power spectrum can be estimated by taking the Fourier transform of a finite set of autocorrelation lags. Utilizing the fact that power measurements are linear in the autocorrelation, it was shown in [1] that by exploiting a low order correlation model, a finite length autocorrelation sequence can be estimated at sub-Nyquist sampling rates by gathering enough measurements to build an overdetermined system of equations. If spectral information is available *a priori* in the form of carrier frequencies and spectral masks, then sampling rate requirements can be relaxed even further [2].

In this paper, a network sensing scenario is considered, comprising scattered low-end sensors with limited communication capabilities, each of which reports a single randomly filtered power measurement bit to a fusion center (FC). By exploiting the spatial diversity of the sensors, such distributed sensing schemes are able to overcome issues related to fading, shadowing and the hidden terminal problem. Furthermore, such schemes allow the task of spectrum sensing to be crowdsourced to handheld wireless devices. Power spectrum estimation in this context was first considered in [3], where it was shown that adequate spectrum sensing at the FC can be performed even with few bits, by utilizing a Linear Programming (LP) formulation which exploits the autocorrelation parametrization and other spectral non-negativity properties. This is in contrast to the PS estimation methods [1, 2] which assume availability of analog amplitude samples (i.e., finely quantized bitstreams) at the FC. This is very reasonable when the measurements are co-located, but far less appealing in distributed sensing scenarios, because it is more prone to sensing and communication errors, demanding in terms of communication resources, and it also has an adverse impact on sensor battery lifetime.

One lesson from classical spectral estimation based on analog measurements [4] is that if we have prior information that the signal of interest admits a representation in terms of a parametric model of a certain order, then we should use it to improve estimation performance. Adopting a parametric model provides a more parsimonious representation of the PS as compared to a non-parametric model, since the former typically uses fewer parameters to describe the spectrum. This is the main motivation for our present work, where we assume that the signal admits a Moving Average (MA) representation. From an application perspective, this model is well motivated for representing digital communication signals which are pulse shaped using finite impulse response (FIR) filters, and transmitted over wireless channels which are also commonly modeled as FIR filters.

Considering the same network sensing scenario as in [3], we seek to estimate the MA parameters from single bit quantized data. The PS estimate is obtained by simply taking the magnitude square of the Discrete-time Fourier Transform (DTFT) of the parameters. Exploiting the MA parametrization, it is shown that the parameter estimation problem from 1 bit data can be formulated as a non-convex QCQP, which is NP–Hard in general [5]. Hence, we resort to the technique of semidefinite relaxation (SDR) [6] in order to obtain polynomial time approximate solutions to the problem. SDR is often used in conjunction with *randomization* procedures to obtain high quality sub-optimal solutions for a large class of NP–Hard problems which are of engineering interest (see [6] and references therein). The SDR technique is compared against the non-parametric LP formulation in [3], followed by an additional step which imposes MA structure on the autocorrelation estimate provided by the former method. Simulations indicate the superior performance of the SDR method when the MA parametrization is valid.

2. SYSTEM MODEL

The idea of *Frugal Sensing* was first introduced in [3], where it was demonstrated that wideband power spectrum estimation from few bits is possible by employing a network of scattered low-end sensors with low rate communication capabilities. Consider a network sensing scenario, where M scattered sensors take measurements of the ambient signal power and report to a FC. Sensors process (shifted, attenuated, and possibly multipath-filtered versions) of the wide-sense stationary (WSS) signal $x(t)$, whose Nyquist rate samples form the discrete time sequence $x(n)$. As shown in [3], the Nyquist sampling rate requirement can be relaxed by using an equivalent analog processing and integration chain. Automatic gain control is employed

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to eliminate the effects of path loss, shadowing, and frequency-flat fading, while the effects of frequency-selective fading (which varies from sensor to sensor) can be mitigated, provided sensors average out their measurements over many fading states (see [3] for a detailed explanation). Timing offsets and phase shifts are allowed, since the power spectrum is invariant with respect to these factors. The Moving Average representation of $x(n)$ is given by

$$x(n) = \sum_{k=0}^{q} h(k)w(n-k) \tag{1}$$

where $\mathbf{h} = [h(0), h(1), ..., h(q)]^T$ are the model parameters, $w(n) \sim \mathcal{CN}(0,1)$, and q is the model order. Each sensor $m \in \{1, 2, \cdots, M\}$ filters $x(n)$ using a wideband FIR filter with complex binary pseudo-noise (PN) impulse response $g_m(n)$ of length K:

$$g_m(n) = \begin{cases} (1/\sqrt{2K})(\pm 1 \pm j) & : 0 \le n \le K-1 \\ 0 & : otherwise \end{cases} \tag{2}$$

The filter output is given by $z_m(n) = \sum_{k=0}^{K-1} g_m^*(k)x(n-k)$, where $\mathbf{g}_m = [g_m(0), \ldots, g_m(K-1)]^T$ are the filter tap weights and $\mathbf{x} = [x(n), x(n-1), \ldots, x(n-K+1)]^T$ are the tap inputs. The filter impulse response $\mathbf{g}_m$ can be generated by a PN shift register, whose initial seed is unique to each sensor (e.g., its serial number) and is known to the FC. The use of random PN filters promotes diversity, simplifies the convolution operation (no multiplications are required) and eliminates the need for coordination among sensors. We denote the average power of the WSS signal $z_m(n)$ by $\rho_m = \mathbb{E}[|z_m(n)|^2] = \mathbb{E}[|\mathbf{g}_m^H\mathbf{x}|^2] = \mathbf{g}_m^H\mathbf{R}_x\mathbf{g}_m$, where $\mathbf{R}_x = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$ is the $K \times K$ Toeplitz-Hermitian autocorrelation matrix of $\mathbf{x}$. Each sensor estimates ρ_m by averaging over N samples to obtain

$$\hat{\rho}_m = \frac{1}{N}\sum_{n=0}^{N-1} |z_m(n)|^2 \tag{3}$$

Under appropriate ergodic mixing conditions [7, p. 171], we have that $\lim_{N\to\infty} \hat{\rho}_m = \rho_m$. Finally, each sensor compares its estimate ρ_m to a single, fixed threshold t. If $\hat{\rho}_m \ge t$, then a $'1'$ is transmitted to the FC, otherwise a $'0'$ is transmitted. We introduce the set notation $\mathcal{M}_a := \{m : \hat{\rho}_m \ge t\}$ and $\mathcal{M}_b := \{m : \hat{\rho}_m < t\}$, with $M_a = |\mathcal{M}_a|$ and $M_b = |\mathcal{M}_b|$ such that $M_a + M_b = M$. Thus, on receipt of a $'1'$ (or a $'0'$) from a sensor, the FC infers that $\mathbf{g}_m^H\mathbf{R}_x\mathbf{g}_m \ge t$ (or $\mathbf{g}_m^H\mathbf{R}_x\mathbf{g}_m < t$), assuming sufficient averaging to ensure that sample averages converge to ensemble averages. Since it is only required to ensure that the sign of each inequality is not reversed, sample averaging requirements are relaxed relative to high rate quantization. The FC is required to estimate the ambient PS of $x(n)$ from these inequalities, which are linear in the autocorrelation sequence $r(k)$. In the following section, it is shown that by utilizing the MA parametrization of $x(n)$, these inequalities can be written explicitly in terms of the MA parameters $\mathbf{h}$.

3. PROBLEM FORMULATION

Since $x(n)$ is an MA(q) process, we can express $\mathbf{R}_x$ as

$$\mathbf{R}_x = r(0)\mathbf{\Theta}_0^K + \sum_{k=1}^{\min(K-1,q)} (r(k)\mathbf{\Theta}_k^K + r^*(k)\mathbf{\Theta}_{-k}^K) \tag{4}$$

where $\mathbf{\Theta}_k^K$ is the $K \times K$ elementary Toeplitz matrix with ones on the k^{th} diagonal and zeros elsewhere, and $\mathbf{r}_x = [r^*(K-1), \ldots, r^*(1), r(0), r(1), \ldots, r(K-1)]^T$ is the autocorrelation sequence. The upper limit on the summation is due to the fact that $r(k) = 0, \forall\ |k| > q$, and hence, depending on whether K is set to be larger than q or not, we get the corresponding number of summation terms. Using (4), each $\rho_m = \mathbf{g}_m^H\mathbf{R}_x\mathbf{g}_m$ can be expressed as

$$\mathbf{g}_m^H\mathbf{R}_x\mathbf{g}_m = \mathbf{g}_m^H\Bigg(r(0)\mathbf{\Theta}_0^K + \sum_{k=1}^{\min(K-1,q)} (r(k)\mathbf{\Theta}_k^K + r^*(k)\mathbf{\Theta}_{-k}^K)\Bigg)\mathbf{g}_m \tag{5}$$

$$= \underbrace{\mathbf{g}_m^H\mathbf{\Theta}_0^K\mathbf{g}_m}_{c_{m,0}} r(0) + \sum_{k=1}^{min(K-1,q)} \left(\underbrace{\mathbf{g}_m^H\mathbf{\Theta}_k^K\mathbf{g}_m}_{c_{m,k}} r(k) + \underbrace{\mathbf{g}_m^H\mathbf{\Theta}_{-k}^K\mathbf{g}_m}_{c_{m,-k}} r^*(k)\right) \tag{6}$$

$$= c_{m,0}r(0) + \sum_{k=1}^{\min(K-1,q)} (c_{m,k}r(k) + c_{m,-k}r^*(k)) \tag{7}$$

where in (7), $c_{m,k} = \mathbf{g}_m^H\mathbf{\Theta}_k^K\mathbf{g}_m$ represents the k^{th} lag of the deterministic autocorrelation sequence of the m^{th} broadband filter with impulse response $\mathbf{g}_m$. The autocorrelation sequence of an MA(q) process can be characterized in terms of $\mathbf{h}$ as follows

$$r(k) = \begin{cases} \sum_{i=0}^{q-|k|} h^*(i)h(i+|k|) & : |k| \le q \\ 0 & : |k| > q \end{cases} \tag{8}$$

$$= \begin{cases} \mathbf{h}^H\mathbf{\Theta}_k^{(q+1)}\mathbf{h} & : |k| \le q \\ 0 & : |k| > q \end{cases} \tag{9}$$

Substituting the expression for $r(k)$ given by (9) in (7), we obtain

$$\mathbf{g}_m^H\mathbf{R}_x\mathbf{g}_m = \mathbf{h}^H\Bigg(c_{m,0}\mathbf{\Theta}_0^{(q+1)} + \sum_{k=1}^{min(K-1,q)} (c_{m,k}\mathbf{\Theta}_k^{(q+1)} + c_{m,-k}\mathbf{\Theta}_{-k}^{(q+1)})\Bigg)\mathbf{h} \tag{10}$$

$$= \mathbf{h}^H\mathbf{C}_m\mathbf{h} \tag{11}$$

where by construction each $\mathbf{C}_m$ matrix is also Toeplitz, Hermitian and positive semi-definite [13]. Hence, the linear inequalities in $\mathbf{r}_x$ are equivalent to the quadratic inequalities in the MA parameters $\mathbf{h}$. For determining the PS, we first seek to estimate an $\mathbf{h}$ that satisfies the given set of quadratic inequalities. The total energy of the signal $\mathbb{E}[|x(n)|^2] = r(0) = \|\mathbf{h}\|_2^2$ is chosen as a cost function to minimize, consistent with the premise of cognitive radio, where it is assumed that most of the spectrum is idle at most times. After determining the parameter estimate $\hat{\mathbf{h}}$, the PS estimate is computed as $\hat{S}(e^{j\omega}) = |\hat{H}(e^{j\omega})|^2$, where $\hat{H}(e^{j\omega})$ is the DTFT of $\hat{\mathbf{h}}$. Note that the phase of $\mathbf{h}$ cannot be estimated even from analog output power measurements, let alone from quantized ones; but this ambiguity is immaterial for our purposes, because it does not affect the power spectrum, which is what we are ultimately after. The problem of estimating an admissible $\mathbf{h}$ can therefore be formulated as

$$\underset{\mathbf{h}\in\mathbb{C}^{p+1}}{\text{minimize}} \quad \|\mathbf{h}\|_2^2 \tag{12a}$$

$$\text{subject to} \quad \mathbf{h}^H\mathbf{C}_m\mathbf{h} \ge t,\ m \in \mathcal{M}_a \tag{12b}$$

$$\mathbf{h}^H\mathbf{C}_m\mathbf{h} < t,\ m \in \mathcal{M}_b \tag{12c}$$

which is a QCQP problem. However, since the constraint set (12b) represents the intersection of the exteriors of multiple co-centered ellipsoids, the problem is non-convex and NP–Hard in general. In

the following section, we relax the problem and propose a polynomial time approximation algorithm to obtain approximate solutions.

4. SEMIDEFINITE RELAXATION

The problem (12) can be equivalently recast as follows

$$\begin{aligned} \underset{\mathbf{H}\in\mathbb{C}^{(q+1)\times(q+1)}}{\text{minimize}} \quad & \text{Trace}(\mathbf{H}) && \text{(13a)}\\ \text{subject to} \quad & \text{Trace}(\mathbf{C}_m\mathbf{H}) \geq t,\ m\in\mathcal{M}_a && \text{(13b)}\\ & \text{Trace}(\mathbf{C}_m\mathbf{H}) < t,\ m\in\mathcal{M}_b && \text{(13c)}\\ & \mathbf{H}\succeq\mathbf{0}, && \text{(13d)}\\ & \text{rank}(\mathbf{H}) = 1 && \text{(13e)} \end{aligned}$$

where we have defined $\mathbf{H} = \mathbf{h}\mathbf{h}^H$ and used the cyclic property of the trace operator to write $\mathbf{h}^H\mathbf{C}_m\mathbf{h} = \text{Trace}(\mathbf{C}_m\mathbf{H})$. The non-convexity in the reformulated problem has been isolated in the form of the rank-1 constraint on $\mathbf{H}$. Dropping the rank constraint, we obtain the following relaxed problem

$$\begin{aligned} \underset{\mathbf{H}\in\mathbb{C}^{(q+1)\times(q+1)}}{\text{minimize}} \quad & \text{Trace}(\mathbf{H}) && \text{(14a)}\\ \text{subject to} \quad & \text{Trace}(\mathbf{C}_m\mathbf{H}) \geq t,\ m\in\mathcal{M}_a && \text{(14b)}\\ & \text{Trace}(\mathbf{C}_m\mathbf{H}) < t,\ m\in\mathcal{M}_b && \text{(14c)}\\ & \mathbf{H}\succeq\mathbf{0} && \text{(14d)} \end{aligned}$$

which is the Lagrangian bi-dual [8] of (12) and has the form of a Semidefinite Programming Problem (SDP). Hence, (14) can be solved efficiently to global optimality using modern interior point methods, at a worst case computational complexity of $O((q+1)^{6.5})$ [9]. Note that solving the rank relaxed problem does not solve the non-convex QCQP (12) in general. However, several post-processing techniques have been developed which use rank relaxation to generate approximate solutions for the original NP–Hard problem from the optimal solution of the relaxed problem. This may be done via *randomization* techniques [6], which have a considerably lower complexity cost compared to solving the relaxed SDP.

Randomization Algorithm: Denote the optimal solution of (14) by $\mathbf{H}_{\text{opt}}$. If it turns out that $\text{rank}(\mathbf{H}_{\text{opt}}) = 1$, then its principal component is the globally optimal solution for (12). However, in general, $\mathbf{H}_{\text{opt}}$ is not rank 1, in which case we use the following *randomization* approach to obtain approximate solutions for (12) from $\mathbf{H}_{\text{opt}}$.

• If $\mathbf{H}_{\text{opt}}$ is approximately rank 1, then we keep its principal component $\mathbf{h}_{\text{pc}} = \sqrt{\lambda_m}\mathbf{q}_m$ as a possible candidate solution, where λ_m and $\mathbf{q}_m$ denote the principal eigen-value and eigen vector of $\mathbf{H}_{\text{opt}}$ respectively. We attempt to convert $\mathbf{h}_{\text{pc}}$ into a feasible solution for (12) by scaling it to satisfy both sets of constraints of (12). We first determine the minimum scaling α for which $\mathbf{h}_{\text{pc}}$ satisfies all the constraints in the set (12b), which can be determined as

$$\alpha = \sqrt{\frac{t}{\min\limits_{m\in\mathcal{M}_a}(\mathbf{h}_{\text{pc}})^H\mathbf{C}_m(\mathbf{h}_{\text{pc}})}} \qquad (15)$$

Thus, we obtain a new candidate vector $\tilde{\mathbf{h}}_{\text{A}} = \alpha\mathbf{h}_{\text{pc}}$. If the candidate vector also satisfies all the constraints in the set (12c), then it is kept as a candidate solution. However, if it violates one or more constraints in (12c), it is discarded.

• In general, $\mathbf{H}_{\text{opt}}$ will not be approximately rank 1. In this case, we employ the technique of *Gaussian randomization* [6] in order to obtain a feasible solution for (12). A total of L candidate Gaussian random vectors $\{\mathbf{h}_l^c\}_{l=1}^L \sim \mathcal{N}(\mathbf{0}, \mathbf{H}_{\text{opt}})$ are drawn, from which the 'best' is chosen by the following approach. Each vector $\mathbf{h}_l^c$ is scaled by a factor β_l until it satisfies the set (12b) and then it is checked if the set (12c) is also satisfied for the same scaling. The scaling factor β_l is determined as

$$\beta_l = \sqrt{\frac{t}{\min\limits_{m\in\mathcal{M}_a}(\mathbf{h}_l^c)^H\mathbf{C}_m(\mathbf{h}_l^c)}} \qquad (16)$$

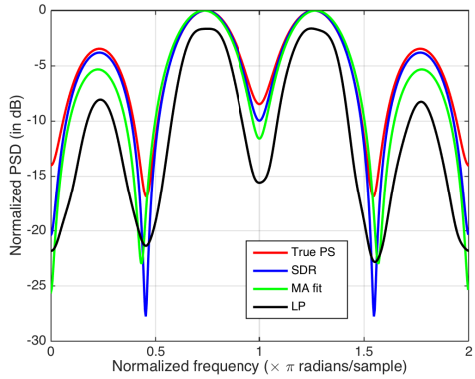
If $\beta_l\mathbf{h}_l^c$ violates any of the constraints in the set (12c), it is discarded. Otherwise, it is kept as a possible solution. Amongst such candidate Gaussian vectors which can be scaled to yield feasible solutions, the one with the minimum cost is chosen as the final approximate solution $\tilde{\mathbf{h}}_{\text{B}}$.

• In most cases, the previously outlined approaches fail in obtaining a feasible solution. If this is the case, we propose to drop the convex constraint set (12c) and scale the candidate vectors to satisfy the non-convex constraints (12b) only. For example, when $\mathbf{h}_{\text{pc}}$ is scaled by α, the vector $\tilde{\mathbf{h}}_{\text{C}} = \alpha\mathbf{h}_{\text{pc}}$ satisfies the set (12b) , but we do not check for violations in (12c). We employ a similar technique for Gaussian randomization, where each random vector is scaled by β_l to yield a solution that is feasible for the set (12b) only, following which the minimum cost candidate is chosen as the solution $\tilde{\mathbf{h}}_{\text{D}}$. Our basic intuition for dropping the set (12c) stems from the fact that the set (12b) is more informative, since it corresponds to the activity detection set. Furthermore, the choice of cost function (12a) places an upper bound on each quadratic term in the set (12b) by virtue of the Rayleigh-Ritz criterion which upper bounds the Rayleigh quotient by its principal eigenvalue. Thus, it is expected that this method will also produce good quality approximate solutions. Overall, after solving an instance of (14) to obtain $\mathbf{H}_{\text{opt}}$, the algorithm proceeds in the following manner. First, it is checked if $\text{rank}(\mathbf{H}_{\text{opt}}) = 1$, in which case, $\mathbf{h}_{\text{pc}}$ is the globally optimal solution of the problem (12). Otherwise, we check if it is possible to obtain an approximate feasible solution $\tilde{\mathbf{h}}_{\text{A}}$ or $\tilde{\mathbf{h}}_{\text{B}}$. If both $\tilde{\mathbf{h}}_{\text{A}}$ and $\tilde{\mathbf{h}}_{\text{B}}$ exist, then the one having the smaller cost is chosen as the solution. In case a feasible solution cannot be obtained, the set (12c) is dropped and a pair of solutions $\tilde{\mathbf{h}}_{\text{C}}$ and $\tilde{\mathbf{h}}_{\text{D}}$ are obtained. Again, the one having smaller cost is chosen as the solution.

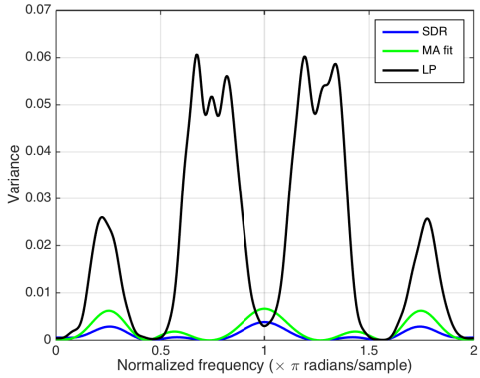
5. NUMERICAL RESULTS

In order to benchmark our SDP relaxation method, we compare its performance against a two-step estimation approach, where in the first step a non-parametric approach is adopted - namely, the autocorrelation function is estimated using the MA-agnostic LP formulation in [3]. The MA parametrization is utilized in the second step to fit an MA autocorrelation sequence by solving a Semidefinite Quadratic Linear Programming (SQLP) problem, as described in [4, p. 131]. The spectral estimate is obtained by taking the DTFT of the MA autocorrelation estimate. The various optimization problems were modeled using YALMIP [10], with the relaxed SDP and SQLP problems being solved using the generic cone program solver SeDuMi [11], while MOSEK [12] was used for solving the LP problem. The Normalized Mean Squared Error (NMSE) criterion was used as a performance criterion, where $\text{NMSE} = \mathbb{E}\left[\frac{\|S_x-\hat{S}_x\|_2^2}{\|S_x\|_2^2}\right]$, S_x being the true spectrum and $\hat{S}_x$ being the estimated spectrum, with both spectra normalized by their peak values. The expectation is taken over the random signal and the random impulse response of the wideband FIR filters.

In order to illustrate the effectiveness of our proposed approach, we consider a sensing scenario with $M = 100$ sensors, setting the broadband filter length $K = 24$ and the threshold such that $M_a = 25$. In figure 1a, each PS estimate was normalized by its peak value, and the results were averaged over 500 Monte-Carlo trials, for a signal generated by a real MA(4) model. The model or-



(a) Mean Normalized Spectra



(b) Variance of Normalized Spectra

Fig. 1: Illustrative example for a real MA(5) model

der was assumed to be known in this case. The quality of the PS estimates obtained from the SDR and the MA fitting techniques, using only 100 bits, is very satisfactory. The NMSE values obtained from the LP, MA fitting and SDR estimates were $0.2966, 0.0366$ and 0.0073 respectively, thus showcasing the superior performance of the parametric approach, in spite of the fact that in 99.8% of the trials, SDR failed to obtain a feasible solution for the non-convex QCQP, while both the LP and MA fitting formulations can be solved to global optimality in polynomial time. This is because SDR utilizes the MA parametrization from the outset, while the two-step MA fitting method makes use of the MA representation only in the second step. In spite of failing to obtain a feasible solution in most cases, the SDR method still produces a high quality sub-optimal solution (in terms of spectral NMSE) by the constraint dropping technique mentioned in Section 4. In figure 1b, the spectral variance is plotted as a function of the normalized frequency. The LP estimate exhibits very high variance about its peak, which explains why the peak of the mean normalized LP estimate in figure 1a is below 0 dB. The MA fitting estimate exhibits lower variance, but the SDR exhibits the least variance across all frequencies.

We present another simulation in Figure 2 where 50 MA models of order 9 were randomly generated. Setting $M = 80$ sensors, $K = 30$, the threshold t was varied in order to vary M_a, the number of sensors reporting above threshold. Knowledge of the true model order is assumed. For each model realization, the spectral NMSE for each M_a was averaged over 100 Monte-Carlo trials, with the final result obtained by averaging across all model realizations.

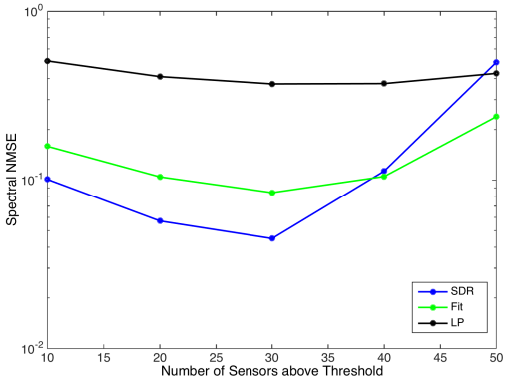


Fig. 2: NMSE vs M_a for MA(9) models

The superior performance of the SDR technique is again noted, even though in the overwhelming majority of cases it is unable to yield a feasible solution, and we have to resort to dropping the convex constraints. The non-parametric method is significantly worse-off in comparison. Although the two-step MA model fitting method brings about an improvement in the quality of the PS estimate, it still cannot match the SDR estimate, except when more than half the sensors report above threshold. Extensive simulations across a wide range of model orders revealed that the SDR technique always outperforms the other approaches, and the optimal choice of threshold for which the SDR estimate attains its lowest NMSE value corresponds to roughly $25 - 35\%$ sensors reporting above the threshold.

6. CONCLUSIONS

A network sensing scenario was considered, where a network of scattered, low-end sensors with limited communications capabilities transmit randomly filtered power measurement bits to a FC. The development of parametric power spectral estimation techniques in this context was pursued. Under the assumption that the primary signal admits a Moving Average time-series representation, the problem of estimating admissible MA parameters, and thus the MA spectrum, can be formulated as a non-convex QCQP, which is NP–Hard in general. In order to obtain polynomial-time approximate solutions, we use rank relaxation followed by a randomization procedure. Due to the two-sided constraints of the non-convex QCQP, this rarely yields a feasible solution. However, a high quality (albeit infeasible) sub-optimal estimate can be obtained by dropping the convex constraint set. The method was tested against the non-parametric LP formulation in [3], as well as a two-step LP plus MA model fitting approach. Simulations revealed the superior performance of the SDR technique, in terms of spectral NMSE and spectral variance, over the other two methods, thus showcasing the advantage of using the MA parametrization from the outset, even though it can only be solved approximately. For further developments, regarding the choice of the parameters M_a, K, t and their effects on the estimation performance, and scenarios where only an upper bound on the true model order is known, we refer the reader to [13].

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