WAVELET-BASED COMPRESSED SPECTRUM SENSING FOR COGNITIVE RADIO WIRELESS NETWORKS

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ABSTRACT

Spectrum sensing is an essential functionality of cognitive radio wireless networks (CRWNs) that enables detecting unused frequency sub-bands for dynamic spectrum access. This paper proposes a compressed spectrum sensing framework by (i) constructing a sparsity basis in wavelet domain that helps compressed sensing at sub-Nyquist rates and (ii) applying a wavelet-based singularity detector on the reconstructed signal to identify available frequency sub-bands with low complexity. In particular, for the compressed sensing, an optimized Haar wavelet basis is employed to sparsely represent piecewise constant (PWC) signals which closely approximates the frequency spectrum of a sensed signal. Our simulation results show that our proposed framework outperforms existing compressed spectrum sensing methods by providing higher accuracy at lower sampling rates.

Index Terms— Compressed sensing, wavelet transform, spectrum sensing, cognitive radio, dynamic spectrum access.

1. INTRODUCTION

Cognitive radio wireless network (CRWN) is a novel communication paradigm that provides dynamic spectrum access to users in an opportunistic manner. As opposed to traditional licensed (i.e., fixed) spectrum access networks, CRWN increases spectrum allocation by allowing its users to access licensed bands without interfering with the *primary* (licensed) users. Thereby, *secondary* (unlicensed) users can exploit available spectrum bands to increase both their capacity and the overall bandwidth utilization without interfering with primary users. For this purpose, *spectrum sensing* is an important requirement that enables users to adapt their spectrum use by detecting unused frequency sub-bands (i.e., spectrum holes). However, accurate and efficient spectrum sensing is one the most challenging problems in CRWNs [1]. In general, spectrum sensing has high complexity, since CRWNs typically operate at wideband frequencies requiring very large number of measurements at Nyquist sampling rates. In order to reduce the complexity of spectrum sensing without degrading its accuracy, this paper proposes a wavelet-based compressed spectrum sensing framework where (i) an optimized *Haar-Fourier basis* (i.e., the Fourier transform followed by a Haar wavelet transform) is constructed for effective sparse recovery at sub-Nyquist sampling rates, and (ii) a wavelet-based singularity detector is then employed to identify spectrum holes within the operating frequency band. The main advantage of our framework is that the proposed basis construction in Haar-Fourier domain provides a sparse signal representation helping effective signal recovery. This is mainly because the frequency spectrum of a sensed signal can be closely approximated as a *piecewise constant* (PWC) signal which is sparse in Haar wavelet domain.

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In the literature, multiple spectrum sensing approaches have been proposed (refer to [1] for a detailed overview on CRWNs). Yucek and Arslan [2] survey different types of spectrum sensing approaches where our problem of interest is commonly referred as *wideband spectrum sensing*. In [3] and [4], authors develop optimal thresholding strategies focusing on accurate spectrum sensing, but they do not consider sensing at sub-Nyquist rates with low complexity. More similar to our work, Tian and Giannakis [5] propose a compressed sensing framework where the sparsity basis is chosen as the Fourier basis, and more recently, Cohen and Eldar [6] derive the minimal sampling rate for perfect spectrum reconstruction and investigate different reconstruction techniques for both digital and analog systems. However, in both [5] and [6], authors assume that the received signal is sparse in frequency (Fourier) domain which does not generally hold, especially when the wideband channel is busy. In contrast, our proposed basis construction in Haar-Fourier domain (which is the main contribution of this paper) leads to a better sparse representation regardless of how the channel is utilized.

The rest of the paper is organized as follows. Section 2 formulates the problems in our compressed spectrum sensing framework. In Section 3, we discuss the proposed solution. Simulation results are presented in Section 4, and Section 5 draws some conclusions.

2. PROBLEM FORMULATION

In what follows, we first formulate the general spectrum sensing problem, then we extend this formulation by defining the compressed spectrum sensing problem. Finally, we formulate the optimal orthogonal basis design problem that is used to find a sparse representation of a set of signals for effective compressed spectrum sensing. In our formulations, the main assumptions are:

- The signals of interest can be closely approximated with piecewise constant (PWC) signals in frequency domain (see Fig.1).
- The channel noise is modeled as additive white Gaussian noise (AWGN), and the noise floor can be estimated using the measurements from unused sub-bands.
- The operating frequency range (i.e., minimum and maximum frequency) is known by all CRWN devices.

2.1. General Spectrum Sensing Problem

We start by formulating the general spectrum sensing problem originally presented in [7]. Let us suppose that a CRWN's operating frequency range is $B = [f_0, f_N]$ and has N sub-bands $B_1, ..., B_N$. The goal is identifying used and unused spectrum sub-bands. In particular, for a received signal $r(t)$ of the following general form,

$$r(t) = \sum_{n=1}^{N} \alpha_n p_n(t) + w(t) \tag{1}$$

we define its power spectral density (PSD) as

$$S_r(f) = \lim_{T\to\infty} E\left[\frac{1}{T}\left|\int_0^T r(t)e^{-i2\pi ft}dt\right|^2\right] \tag{2}$$

that is

$$S_r(f) = \sum_{n=1}^{N} \alpha_n^2 S_n(f) + S_w(f) \qquad f \in [f_0, f_N] \tag{3}$$

where $p_n(t)$ is the signal whose frequency spectrum is

$$S_n(f) = \begin{cases} 1 & \text{if } f \in B_n \\ 0 & \text{if } f \notin B_n \end{cases} \tag{4}$$

that has support only in sub-band $B_n = [f_{n-1}, f_n]$, α_n is the signal magnitude in sub-band B_n, and $w(t)$ is an additive white Gaussian noise with power spectral density $S_w(f)$. Thus, the spectrum sensing problem can be formally defined as finding the following parameters based on the received signal $r(t)$,

- $\{f_n\}_{n=1}^{N-1}$: Frequency boundaries identifying N frequency sub-bands $\{B_n\}_{n=1}^{N}$.
- $\{\alpha_n^2\}_{n=1}^{N}$: Square magnitude of signal at each sub-band $\{B_n\}_{n=1}^{N}$ where $\alpha_n^2 \approx 0$ implies B_n is unused.

Fig.1 illustrates the power spectral density of a received signal and the parameters that we need to characterize for the spectrum sensing purpose.

2.2. Compressed Sensing Problem

In order to effectively solve the spectrum sensing problem formulated in Section 2.1, it is essential to accurately acquire the received signal $r(t)$ and its frequency spectrum $S_r(f)$. However, depending on the operating frequency range (B), required sampling rates can be very high. To reduce complexity of the spectrum sensing process, compressed sensing (CS) frameworks have been proposed to obtain reasonable frequency spectrum estimates at sub-Nyquist sampling rates [6].

In compressed sensing, the goal is to design a $K \times M$ dictionary matrix $\mathbf{D} = \mathbf{\Phi\Psi}$ with the restrictive isometry property (RIP) [8, 9] such that (i) a given signal $\mathbf{r} \in \mathbb{R}^M$ is sparse in an orthogonal basis (or frame) $\mathbf{\Psi}$, and (ii) a matrix $\mathbf{\Phi} \in \mathbb{R}^{K\times M}$ ($K < M$) reduces the number of measurements (i.e., sampling rate). After determining the dictionary $\mathbf{D}$, CS approaches perform a sparse recovery of the signal $\mathbf{r}$ from its CS measurements $\mathbf{y} = \mathbf{\Phi r} = \mathbf{\Phi\Psi x}$ where $\mathbf{x}$ is the sparse representation of $\mathbf{r}$ in transform domain (i.e., $\mathbf{x}^* = \mathbf{\Psi}^t\mathbf{r}$). Formally, this can be written as following optimization problem,

$$\hat{\mathbf{x}}_0 = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_0 \quad \text{subject to } \mathbf{y} = \mathbf{Dx}. \tag{5}$$

Note that, this problem is known to be NP-hard due to the l_0 pseudo-norm in the objective function. However, if there exists a sufficiently sparse solution, the problem can be relaxed to an l_1 minimization problem [8], where the relaxed solution $\hat{\mathbf{x}}_1$ closely approximates $\hat{\mathbf{x}}_0$.

2.3. Orthogonal Sparsity Basis Design

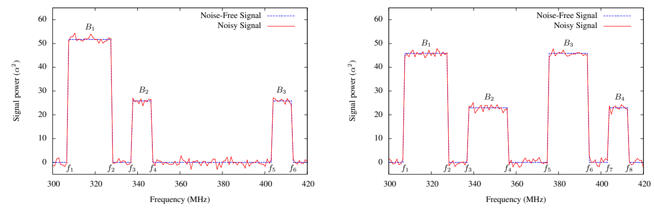
In compressed sensing, it is crucial to design a basis (or a frame) providing a sparse representation for a class of signals. This is because better compression and reconstruction can be achieved when sparsity of signals is maximized through some basis. In this work, we focus on orthogonal sparsity basis design which can be posed as an optimization of the following problem,

$$\hat{\mathbf{\Psi}} = \arg\min_{\mathbf{\Psi}} \|\mathbf{\Psi}^t\mathbf{Y}\|_0 \quad \text{subject to } \mathbf{\Psi}^t\mathbf{\Psi} = \mathbf{I} \tag{6}$$

where $\|\mathbf{A}\|_0$ is the number of non-zero elements in matrix $\mathbf{A}$, $\mathbf{Y} \in \mathbb{R}^{M\times S}$ is the matrix of S training signals with length M, and $\mathbf{\Psi}$ is the $M \times M$ basis matrix. This problem can be also relaxed to an l_1 minimization by changing its objective function to $\|\mathbf{\Psi}^t\mathbf{Y}\|_1$ which is equal to sum of absolute values of elements in matrix $\mathbf{\Psi}^t\mathbf{Y}$. Note that the signals of interest in this problem are piecewise constant. Since the piecewise constant signals are known to be sparse in Haar wavelet domain by construction [10, 11], we further constrain our orthogonal basis design to be in Haar wavelet domain. In particular, we reduce the problem stated in (6) to designing a tree-structured Haar wavelet filterbank, where a basis can be fully defined by a tree $\mathcal{T}$ (i.e., wavelet packet) [10]. Thus, the reduced problem is

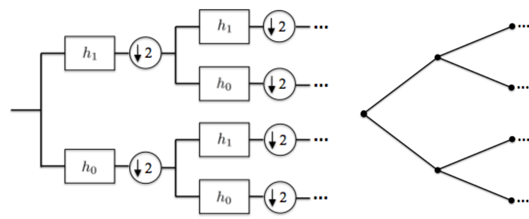
$$\min_{\mathcal{T}} \|\mathbf{H}_{\mathcal{T}}^t\mathbf{Y}\|_0 \quad \text{subject to } \mathbf{H}_{\mathcal{T}}^t\mathbf{H}_{\mathcal{T}} = \mathbf{I} \tag{7}$$

where $\mathbf{H}_{\mathcal{T}}$ denotes the basis matrix determined based on a Haar wavelet packet tree $\mathcal{T}$. Fig.2 illustrates the wavelet packet abstraction of multi-stage filterbanks in terms of trees. The problem in (7) is a special case of the standard wavelet packet construction problem [12] where in our case the objective function promotes sparsity.

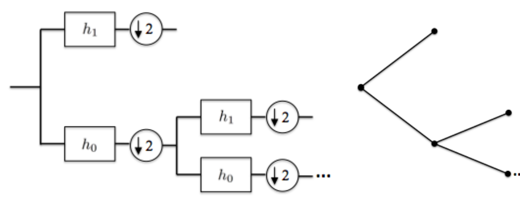


(a) Sparse channel (b) Busy channel

Fig. 1: Signals of interest can be closely approximated with piecewise constant functions.



(a) Symmetric filterbank



(b) Dyadic filterbank

Fig. 2: Two-channel filterbank decompositions and their wavelet packet (tree) abstractions. The vertices of the trees correspond to wavelet coefficients.

3. PROPOSED SOLUTION

We now provide details of our proposed solution, including (i) a method to find a sparsity basis ($\mathbf{\Psi}$) in Haar wavelet domain, (ii) the solution to the compressed spectrum sensing, and (iii) the wavelet-based singularity detection method to identify frequency sub-bands from reconstructed signal.

3.1. Haar Wavelet Packet Design

In order to find the best wavelet packet tree solving the optimization problem stated in (7), we propose a two-step solution: We first perform a wavelet packet analysis on PWC training signals in $\mathbf{Y}$ and construct a vertex-weighted tree where weights reflect sparsity. Then, a tree pruning algorithm is employed to find the best sub-tree.

Let us assume that each signal in $\mathbf{Y}$ has length $M = 2^{J+1}$ where J is a non-negative integer. For wavelet packet analysis, training signals are first transformed using J-level symmetric Haar wavelet decomposition (see Fig.2(a)). Then, the sparsity (i.e., l_0 pseudo-norm) of the resulting wavelet coefficients is calculated at each level of decomposition. Note that wavelet coefficients at each branch of the wavelet filterbank correspond to a vertex of a tree (i.e., black nodes in Fig.2). Thus, we define a vertex-weighted tree associated to J-level symmetric wavelet decomposition where the vertex-weights are the sparsity of the corresponding wavelet coefficients. After defining the tree, we employ a sub-tree pruning algorithm to find the minimum cost sub-tree ($\mathcal{T}^*$) leading to best basis $\hat{\mathbf{H}} = \mathbf{H}_{\mathcal{T}^*}$. In particular, we use the tree-pruning algorithm originally developed by Coifman and Wickerhauser for entropy based wavelet packet selection [12]. The same algorithm is also used for RD-optimized wavelet packet selection in [13].

Note that the proposed method requires a noise-free training signal set ($\mathbf{Y}$) which may not be available in practice. In case of noisy training signals, we can simply modify the objective function in (7) by replacing sparsity measure with l_1 norm and/or including regularization terms. If a training signal set is unavailable, secondary (unlicensed) users can learn about the frequency use patterns by observing the channel for a certain amount of time before constructing a Haar-Fourier basis. In addition, frequency bands allocated for primary users can be known a priori so that secondary users can decide on a basis by simulating on randomly generated PWC signals that match pre-allocated frequency intervals. Basis selection can be also made based on expected length of transitions in PWC signals of interest. For example, higher frequency utilization leads to PWC signals with larger transition length. Typically, as the average transition length increases, a dyadic Haar wavelet decomposition with larger number of decompositions results in a favorable sparsity basis.

3.2. Compressed Spectrum Sensing

After constructing the sparsity basis $\mathbf{\Psi}$, our next goal is defining the measurement matrix $\mathbf{\Phi}$ that realizes spectrum sensing at sub-Nyquist rates. For this purpose, let us suppose that the received signal $r(t)$ is sampled at Nyquist rate which leads to M samples vectorized in $\mathbf{r}_t \in \mathbb{R}^M$. In frequency domain, $\mathbf{r}_f = \mathbf{F}_M\mathbf{r}_t$ is approximately PWC where $\mathbf{F}_M$ is $M \times M$ discrete Fourier transform (DFT) matrix. In the CS literature, assuming that a signal $\mathbf{r}$ is k-sparse with respect to basis $\mathbf{\Psi}$, it has been shown that if the measurement matrix $\mathbf{\Phi} \in \mathbb{R}^{K\times M}$ is i.i.d. Gaussian, then sparse recovery algorithms can recover signals from $K = O(k\log(M))$ measurements [14]. If $\mathbf{\Phi}$ is a random row matrix of the DFT matrix, a similar number of measurements is sufficient in practice [15]. Based on this result, we select our measurement matrix as $\mathbf{\Phi} = \mathbf{R}\mathbf{F}_M^{-1}$ where $\mathbf{R}$ is a $K \times M$ random row matrix obtained by randomly removing rows of identity matrix $\mathbf{I}_M$ with dimension $M \times M$, and $\mathbf{F}_M^{-1}$ is the inverse DFT matrix.

Hence, for the CS problem stated in (5) our dictionary is $\mathbf{D} = (\mathbf{R}\mathbf{F}_M^{-1})\hat{\mathbf{H}}$ and measurement vector is $\mathbf{y} = \mathbf{R}\mathbf{F}_M^{-1}\mathbf{r}_f$. The ultimate goal is reconstructing the original (sparse) Haar wavelet coefficients, $\mathbf{x}^* = \hat{\mathbf{H}}^t\mathbf{r}_f$, from CS measurements in $\mathbf{y}$ using dictionary $\mathbf{D}$. With respect to time domain, (i) coefficients $\mathbf{x}^* = \hat{\mathbf{H}}^t\mathbf{F}_M\mathbf{r}_t$ are in Haar-Fourier domain, and (ii) measurements in $\mathbf{y} = \mathbf{R}\mathbf{F}_M^{-1}\mathbf{F}_M\mathbf{r}_t = \mathbf{R}\mathbf{r}_t$ are actually random samples in time-domain which is useful in practice. For reconstruction we use basis pursuit algorithm in Section 4, yet a less complex algorithm such as orthogonal matching pursuit [16] can be employed if complexity is a concern.

3.3. Identifying Spectrum Bands: Spectrum Hole Detection

After successfully recovering the frequency spectrum $S_r(f)$, we can now identify the used/unused spectrum sub-bands in CRWN. Our proposed solution has two steps: (i) We detect transition locations (discontinuities) in $S_r(f)$ corresponding to maximum/minimum frequency of each sub-band. (ii) We estimate signal power within each sub-band, and then decide whether it is used/unused sub-band based on relative signal power levels. Although it is possible to identify spectrum holes by simple thresholding, the proposed solution is more robust to noise and reconstruction errors.

To detect signal transitions in the frequency spectrum, we adopt a wavelet-based approach exploiting the results presented in [7, 17]. In particular, we use the *multiscale wavelet products* approach originally developed in [7] which provides robust detection under noisy signals. By finding the local maxima of multiscale wavelet products, we estimate the frequency boundary locations within (f_0, f_N) as

$$\{\hat{f}_n\}_{n=1}^{N_e} = \arg\underset{f}{\text{maxima}}\,\{|U_d\{S_r(f)\}|\} \quad f \in (f_0, f_N) \quad (8)$$

where N_e is the number of detected frequency boundaries and $U_d\{\cdot\}$ denotes the d-scale wavelet product formulated as

$$U_d\{S_r(f)\} = \prod_{j=1}^{d} W'_{s=2^j}\{S_r(f)\}. \quad (9)$$

where $W'_s\{\cdot\}$ operator is the first derivative of Gaussian wavelet with scale s. Note that $\hat{f}_0 = f_0$ and $\hat{f}_{N_e+1} = f_N$ based on the third assumption in Section 2. After detecting $\{\hat{f}_n\}_{n=1}^{N_e}$, we estimate average signal power at each sub-band in $\{\hat{B}_n\}_{n=1}^{N_e+1}$, that is

$$\mathcal{E}_n = \frac{1}{\hat{f}_n - \hat{f}_{n-1}}\int_{\hat{f}_{n-1}}^{\hat{f}_n} S_r(f)df \quad \forall n = \{1, 2, ..., N_e + 1\} \quad (10)$$

which can provide good estimates of the original $S_r(f)$ since it is PWC. Thus, for each frequency band $\hat{B}_n$, the square magnitude of the signal, α_n^2, can be estimated from $\mathcal{E}_n$

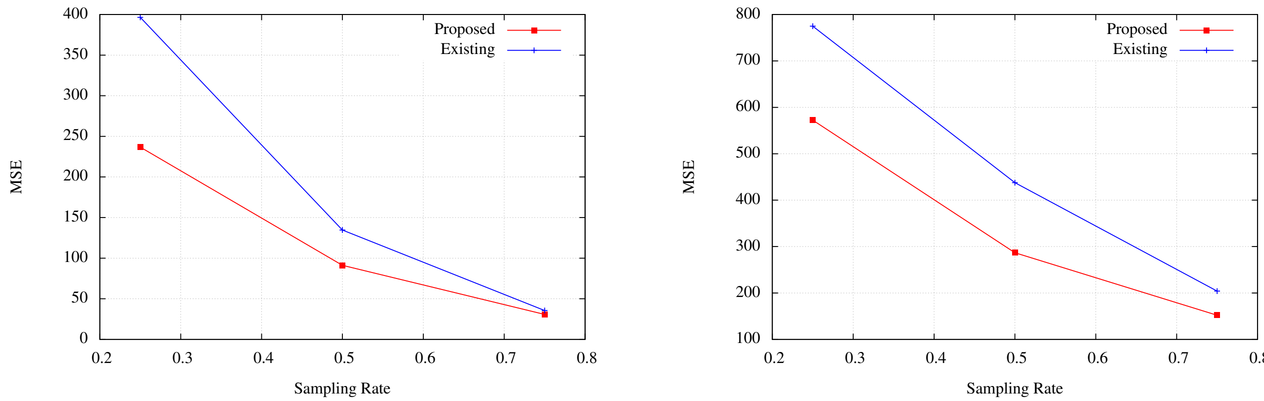
$$\hat{\alpha}_n^2 = \mathcal{E}_n - \min_i \mathcal{E}_i, \quad \forall n = \{1, 2, ..., N_e + 1\} \quad (11)$$

where the subtracted term suppresses the noise floor introduced by the channel based on the second assumption in Section 2.

4. RESULTS

In this section, we demonstrate the spectrum sensing performance of our approach in terms of accuracy/efficiency trade-off and compare it against the method proposed in [5]. To simulate each approach, we first perform compressed spectrum sensing at different sampling rates where the basis pursuit algorithm is used for sparse signal recovery. Based on the reconstructed signals, we identify spectrum holes by using the method presented in Section 3.3. Then, the spectrum sensing performances are evaluated in terms of two different metrics: (i) Mean square error (MSE) of reconstructed signal with respect to original (noise-free) signal, (ii) the spectrum hole detection performance in terms of receiver operating characteristics (ROCs).

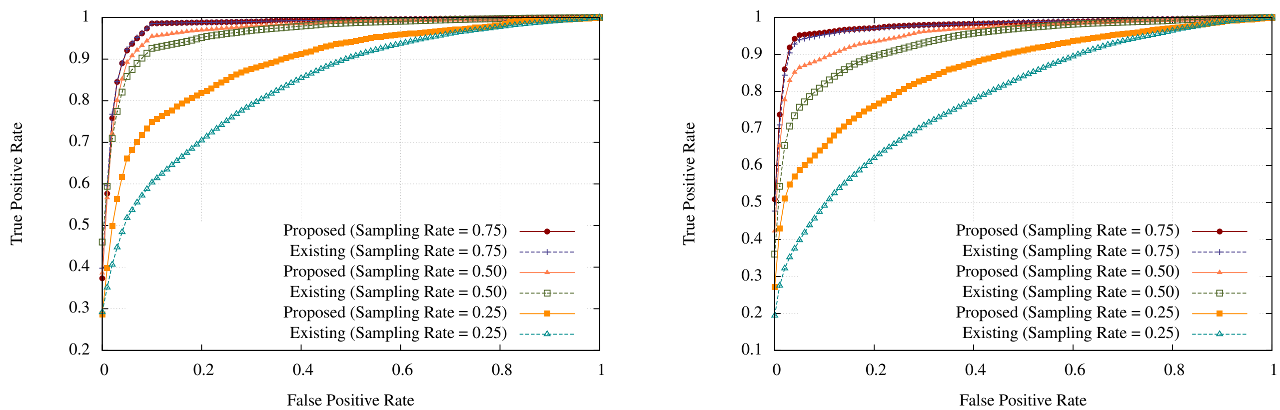
To evaluate the performance of the proposed approach, our experiments are set up as follows. Throughout the simulations, the



(a) 30% utilized (sparse) channel

(b) 70% utilized (busy) channel

Fig. 3: Sparse recovery performances of our proposed approach and existing method [5] in terms of mean square error (MSE).



(a) 30% utilized (sparse) channel

(b) 70% utilized (busy) channel

Fig. 4: Spectrum hole detection performances of our proposed approach and existing method [5] in terms of ROCs at different sampling rates.

wideband of interest is in the range of $B = [f_0 = 300, f_N = 420]$ MHz, and the frequency points determining the sub-bands, i.e., $\{f_n\}_{n=1}^{N-1}$, are first randomly selected and then fixed. In practice, this is analogous to a government allocating different spectrum bands to primary (licensed) users. In order to find the sparsity basis ($\mathbf{\Psi}$), we generate a noise-free training set $\mathbf{Y}$ (see in (6)) by picking i.i.d Bernoulli random vectors $\mathbf{b} \in \{0,1\}^N$ with success rate p for each sub-band $\{B_n\}_{n=1}^{N}$. If $b_n = 1$, this implies sub-band B_n is utilized, otherwise it is empty. Then, we randomly assign high ($\alpha^2 = 20$) and medium ($\alpha^2 = 10$) signal power values for the utilized sub-bands. The test dataset is independently generated by following the same procedure, yet an AWGN is added on top. In our experiments, we use a dataset with 100 training and 20 test samples. Note that, the success rate, p, corresponds to overall channel utilization, and we do our experiments for $p = 0.3$ and $p = 0.7$ which approximately leads to 30% (sparse) and 70% (busy) channel utilization, respectively. For each p value, the training procedure (i.e., solving the problem stated in (7)) leads to two different basis. Specifically, 3-level and 4-level dyadic Haar wavelet packet transforms are constructed for $p = 0.3$ and $p = 0.7$, respectively. Since the measurement matrix ($\mathbf{\Phi}$) is random, we repeat our simulations 30 times at different sampling rates. To detect spectrum holes, we employ multiscale wavelet products with $d = 5$ scales (see in Section 3.3).

The average spectrum sensing performance is evaluated in terms of MSE and ROCs, which are shown in Figs. 3 and 4, respectively. The results show that the proposed approach outperforms the existing method in both sparse recovery and spectrum hole detection at any sampling rate. Also, the proposed method performs better for both sparse and busy spectrum use cases. The performance difference between two methods decreases if the channel utilization is lower or the sampling rate is higher.

5. CONCLUSIONS

Based on our observations from the experimental results, the following conclusions are drawn:

- An optimized basis in Haar-Fourier domain can significantly improve spectrum sensing in terms of accuracy/efficiency trade-off.
- The proposed approach significantly outperforms the existing work at low sampling rates (i.e., low complexity), for both high and low channel utilization scenarios.
- The performance of existing work is similar to our method only if the channel use is sparse and sampling rate is high (which means higher complexity).
- Practically, Haar-Fourier basis training can be done for different channel utilization rates where we can estimate utilization by some means and use the corresponding basis.

6. REFERENCES

[1] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Comput. Netw.*, vol. 50, no. 13, pp. 2127–2159, Sep. 2006.

[2] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications." *IEEE Communications Surveys and Tutorials*, vol. 11, no. 1, pp. 116–130, 2009.

[3] Z. Quan, S. Cui, A. H. Sayed, and H. Poor, "Wideband spectrum sensing in cognitive radio networks," in *IEEE International Conference on Communications, 2008. ICC '08.*, May 2008, pp. 901–906.

[4] Z. Quan, S. Cui, and A. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 28–40, Feb 2008.

[5] Z. Tian and G. Giannakis, "Compressed sensing for wideband cognitive radios," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing, 2007. ICASSP 2007.*, vol. 4, April 2007, pp. 1357–1360.

[6] D. Cohen and Y. Eldar, "Sub-nyquist sampling for power spectrum sensing in cognitive radios: A unified approach," *IEEE Transactions on Signal Processing*, vol. 62, no. 15, pp. 3897–3910, Aug 2014.

[7] Z. Tian and G. Giannakis, "A wavelet approach to wideband spectrum sensing for cognitive radios," in *Proc. 1st Intl. Conf. on Cognitive Radio Oriented Wireless Networks and Coms. (CROWNCOM), Mykonos*, 2006.

[8] D. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, April 2006.

[9] M. F. Duarte and R. G. Baraniuk, "Spectral compressive sensing," *Applied and Computational Harmonic Analysis*, vol. 35, no. 1, pp. 111 – 129, 2013.

[10] M. Vetterli and J. Kovačevic, *Wavelets and Subband Coding*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1995.

[11] M. Unser and P. Tafti, "Stochastic models for sparse and piecewise-smooth signals," *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 989–1006, March 2011.

[12] R. Coifman and M. Wickerhauser, "Entropy-based algorithms for best basis selection," *IEEE Transactions on Information Theory*, vol. 38, no. 2, pp. 713–718, March 1992.

[13] K. Ramchandran and M. Vetterli, "Best wavelet packet bases in a rate-distortion sense," *IEEE Transactions on Image Processing*, vol. 2, no. 2, pp. 160–175, Apr 1993.

[14] E. Candes and M. Wakin, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 21–30, March 2008.

[15] J. Tropp, M. Wakin, M. Duarte, D. Baron, and R. Baraniuk, "Random filters for compressive sampling and reconstruction," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing, 2006. ICASSP 2006.*, vol. 3, May 2006, pp. 872–875.

[16] Y. Pati, R. Rezaiifar, and P. S. Krishnaprasad, "Orthogonal matching pursuit: recursive function approximation with applications to wavelet decomposition," in *Proc. the Twenty-Seventh Asilomar Conference on Signals, Systems and Computers, 1993.*, vol. 1, Nov 1993, pp. 40–44.

[17] S. Mallat and W. L. Hwang, "Singularity detection and processing with wavelets," *IEEE Transactions on Information Theory*, vol. 38, pp. 617–643, 1992.