ACCURATE KERNEL-BASED SPECTRUM SENSING FOR GAUSSIAN AND NON-GAUSSIAN NOISE MODELS

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ABSTRACT

This paper introduces a spectrum sensing scenario based on kernel theory which compares favorably against the conventional Energy Detector (ED) in a cognitive radio system. The so-called Kerenlized Energy Detector (KED) can provide superior accuracy in the case of non-Gaussian noise. The incorporation of the nonlinear kernel function in the KED test statistics allows for the development of a nonlinear algorithm capable of considering both higher order and Fractional Lower Order Moments (FLOMs) in the sensing task. Simulation results show that the proposed semi-blind kernelized spectrum sensing algorithm is much robust against impulsive noises and displays a considerably better detection performance than the conventional ED in practical impulsive man-made noises which are generally modeled as the Laplacian and the α-stable distributions. Moreover, for the Gaussian signal and noise model, the performance of the KED scheme is almost identical to that of the conventional ED.

Index Terms— Cognitive radio, spectrum sensing, non-Gaussian impulsive noises, α-stable noise model.

1. INTRODUCTION

Cognitive Radio (CR) has been recognized as a promising solution dealing with the scarcity of the radio spectrum [1]. In the CR technology, unauthorized users, known as Secondary Users (SUs), exploit unutilized bands, known as spectrum holes, when and where no Primary User (PU) exists in their vicinity using these spectrum holes. Toward this goal, SUs must sense the spectrum accurately to detect the existing idle bands for opportunistic usage, and vacate the occupied bands when a PU starts its transmission. This prevents from interfering and degrading the PU's performance. These fundamental requirements give rise to the challenging issue of spectrum sensing which is a crucial task in Cognitive Radio Networks (CRNs). Many spectrum sensing schemes are presented in the literature [2, 3], among which Energy Detector (ED) and Higher order statistics (HOS) based spectrum sensing methods are more considered in this paper. HOS-based algorithms, such as the multiple cumulants-based spectrum sensing method in [4], have a great potential to deal with non-Gaussian signals. However, the computational cost and the implementation burden of these methods are overwhelming. The conventional ED has been recognized as one of the most practical spectrum sensing schemes due to its simplicity and the ease of implementation. Moreover, energy detector is a semi-blind procedure which is independent of PU's signal properties, but it requires the noise statistics to compute the sensing threshold. However, the performance of HOS-based methods, as well as the second order moment based schemes, degrades drastically for the α-stable noise model where $\alpha < 2$. Despite the extensive literature on the spectrum sensing domain, most of them have concentrated on the Gaussian noise assumption which undergo a drastic performance degradation when the Gaussianity assumption of the noise fails. Although, the Gaussian assumption is justified for the thermal noise, a vast range of experiments conducted in various indoor and outdoor environments invalidates the Gaussian assumption for impairments caused by wireless channels. More precisely, communication signals in indoor environments are affected by the man-made impulsive noises caused by electrical equipments such as microwave ovens and devices with electromechanical switches. For outdoor environments, main sources of impulsive noises are power lines, buildings in radar clutter and lightning in the atmosphere [5]. Recently, a handful of spectrum sensing methods has been presented in the non-Gaussian impulsive-type noise environments which are generally developed based on the fractional lower order statistics [6, 7].

Motivated by the simplicity of the conventional ED and the capability of higher order and Fractional Lower Order Moments-based (FLOMs) algorithms for non-Gaussian signals [8], in this paper, we propose the Kernelized Energy Detector (KED) which implicitly encompasses higher order and fractional lower order statistics through incorporating nonlinear kernel functions in the sensing procedure. As will be explained in Section 3, the kernel theory is considered as an appropriate choice in the spectrum sensing task capable of realizing a good balance among the three major requirements, sensing accuracy, computational complexity and the detection problem in the presence of impulsive noises. In addition to a moderate complexity, the proposed KED test is much robust against impulsive man-made noises leading to a considerably better detection performance than that of the conventional ED in practical impulsive noise environments. For the case of α-stable noise scenario where $\alpha < 2$, numerical evaluations indicate a superior sensing capability of the KED scheme compared to that of the conventional ED and the FLOMs-based methods, while the performance of the conventional ED degrades drastically due to the infiniteness of the second and higher order moments. In addition, for the case of the Laplacian noise, the proposed KED scheme demonstrates 2.5 dB better performance than that of the conventional ED for a fixed detection probability.

The rest of the paper is organized as follows. In Section 2, the system model and the spectrum sensing problem are introduced. Some fundamental concepts of the kernel theory and the proposed KED method are presented in Section 3. Section 4 provides some simulation results, and finally in Section 5, an overview of the results and conclusions are presented.

2. SYSTEM MODEL AND PROBLEM DESCRIPTION

This paper considers a non-cooperative spectrum sensing scenario where each SU performs the PU's detection task individually. We assume that SUs are equipped with P antennas. The available data for spectrum sensing is denoted as an $N \times P$ matrix $\mathbf{X} = [\mathbf{x}_1 \ \mathbf{x}_2 \ \ldots \ \mathbf{x}_P]$, where N is the cardinality of the sample set and each column $\mathbf{x}_k = [x_{k,1}, \ldots, x_{k,N}]^T$, $k = 1, ..., P$, denotes the complex valued received signal vector in k^{th} antenna of the SU. The received data matrix $\mathbf{X}$ can be thought of as an over-sampled signal from one antenna with factor P. The spectrum sensing problem for each SU is represented by the following binary hypotheses test:

$$\begin{cases} H_0: & \mathbf{x}_k = \mathbf{w}_k, \\ H_1: & \mathbf{x}_k = \mathbf{s}_k + \mathbf{w}_k, \end{cases} \qquad k = 1, 2, \ldots, P, \tag{1}$$

where H_0 and H_1 indicate the hypothesis relating to the absence and presence of the PU, $\mathbf{s}_k = [s_{k,1}, \ldots, s_{k,N}]^T$ represents the PU's signal samples, and $\mathbf{w}_k = [w_{k,1}, \ldots, w_{k,N}]^T$ is the additive noise. Throughout this paper, the additive noise is modeled as the Gaussian and non-Gaussian (Laplacian and α-stable impulsive noises) distributions. The received signal samples are assumed to be independent and identically distributed (i.i.d.). Moreover, we assume that the signal and the noise are independent. For the performance evaluation of our proposed scheme, we use the false alarm probability, $P_{fa} = P(H_1|H_0)$, and the detection probability, $P_d = P(H_1|H_1)$.

The pure noise samples requirement is inevitable in semi-blind spectrum sensing schemes such as the energy detector, where accurately estimated noise statistics are essential to measure the sensing threshold precisely [9]. Moreover, in the sensing schemes based on goodness-of-fit tests, pure noise samples are necessary in the sensing task to measure the similarity of the received signal distribution to the noise distribution. For this reason, we adopt a two stage spectrum sensing scenario to shed light on the practical issues relating to our proposed semi-blind kernelized spectrum sensing algorithm where in analogy with the conventional ED, the noise statistics are needed to determine the sensing threshold. A two stage spectrum sensing scenario, consisting of two Sensing Periods (SP), is adopted as a practical procedure supported by the IEEE 802.22 [10] standard to extract pure noise samples in the fine sensing stage and use these samples in the subsequent fast sensing steps. During the fine SP, accurate detection algorithms are employed in considerably longer time spans, while the fast sensing should be done much faster to meet the online requirements of cognitive radio.

3. KERNELIZED ENERQY DETECTOR

In this section, we first briefly explain the fundamental concepts of kernel theory and then propose a new spectrum sensing scheme, namely Kernelized Energy Detector (KED), for the CRN model introduced in Section 2.

Kernel Theory: Kernel theory has been enjoying considerable popularity during the last two decades due to its ability in solving complex real world problems through efficient nonlinear learning algorithms. Kernels are similarity measures which can be thought as a dot product in a feature space $\mathcal{H}$. In kernel methods, data points $\mathbf{x}$ belonging to the input space $\mathcal{X}$ are mapped into a feature space $\mathcal{H}$ with a greater dimension using a nonlinear feature map Φ as $\Phi : \mathcal{X} \longrightarrow \mathcal{H}, \ \ \mathbf{x} \longrightarrow \Phi(\mathbf{x})$ [11]. For these algorithms, different feature maps yield completely different feature spaces, where each one displays a different performance. Therefore, choosing a proper feature map is a crucial task depending on the problem under scrutiny. Generally, a kernel function is the dot product of data points in the feature space. Kernel trick is referred to the process of substituting the inner product with an equivalent kernel function which proposes an efficient measure through which kernel methods are used without explicitly knowing the feature mapping and the kernel function provides sufficient information in this context [12]. By employing the kernel trick, any linear algorithm in the literature, which is based on inner product, can be easily kernelized via replacing an inner product with the kernel function $K(\mathbf{x}_i, \mathbf{x}_j) = \langle \Phi(\mathbf{x}_i), \Phi(\mathbf{x}_j) \rangle_{\mathcal{H}}, \ \forall \, \mathbf{x}_i, \mathbf{x}_j \in \mathcal{X}$, where $K(.,.)$ is a positive semi-definite function [11].

KED Scheme: Energy detector is a simple spectrum sensing algorithm which can be easily implemented in CR systems. It only measures the power of the received signal and ignores higher order and Fractional Lower order moments (FLOMs) of data. On the other hand, it is shown in [4] that Higher Order Statistics (HOS) have a promising performance in non-Gaussian signal processing where multiple cumulants-based sensing schemes are employed. However, HOS-based methods require considerably large number of samples to reach a satisfactory estimation which in turn leads to an overwhelming computational complexity. Moreover, these HOS-based methods do not have the capability of handling impulsive noises properly, specially for the α-stable noise model where $\alpha < 2$. FLOMs are distinguished as a powerful mean for impulsive signals processing [8]. Motivated by the simplicity of the energy detector and capability of HOS and FLOMs based algorithms to deal with non-Gaussian signals, the Kernelized Energy Detector (KED) is proposed which has a moderate complexity and can be easily implemented in analogy with the conventional ED. The KED incorporates higher order and fractional lower order moments of the received signal samples in the spectrum sensing process through employing the kernel methods. In one point of view, the KED method is considered as an improved version of the conventional ED, especially in the practical case of non-Gaussian noise. From another perspective which stems from the fact that kernels are similarity measures, the KED can be interpreted as an algorithm that employs the similarity of the received signal samples for the spectrum sensing task. Thus, as will be seen in this section, the proposed KED method is derived by applying the kernel theory to the conventional ED. The test statistic of the conventional energy detector [13, 14], defined as $T_{ED} = \sum_{i=1}^{N} \sum_{j=1}^{P} |x_{i,j}|^2$, has a form of the dot product, i.e., $T_{ED} = \sum_{i=1}^{N} \langle \mathbf{x}_i, \mathbf{x}_i \rangle$, where $\mathbf{x}_i$ represents each row of the received data matrix $\mathbf{X}$ defined in Section 2, which paves the way for the promotion of the conventional energy detector using the kernel trick. Thus, we can replace the dot product in the test statistic of the energy detector with an arbitrary kernel function as follows:

$$T_{KED} = \sum_{i=1}^{N} K(\mathbf{x}_i, \mathbf{x}_i), \tag{2}$$

where $K(\mathbf{x}_i, \mathbf{x}_i)$ is a nonlinear kernel function which depends on the received signal vector $\mathbf{x}_i$. As an example, if the kernel function $K(\mathbf{x}_i, \mathbf{x}_i)$ is a polynomial kernel of the second degree, our test statistic in (2) will become as $T_{KED} = \sum_{i=1}^{N} \left(\|\mathbf{x}_i\|^2 + 1 \right)^2 = \sum_{i=1}^{N} \left(\|\mathbf{x}_i\|^4 + 2 \|\mathbf{x}_i\|^2 + 1 \right)$. It is seen that higher order moments of the received signal as well as the power of the signal in the conventional ED are employed in the sensing task using (2), while the KED in (2) does not take into account the joint moments of the received signal samples, i.e., the interaction of the samples. As will be proved in Lemma 1, $\mathbb{E}\{\langle \Phi(\mathbf{x}_i), \Phi(\mathbf{x}_j) \rangle\} = \mathbb{E}\{K(\mathbf{x}_i, \mathbf{x}_j)\}$ incorporates the effect of the higher order and fractional lower order statistics and considers the interaction of the received signal samples in the sensing task. Based on the above property of $\mathbb{E}\{K(\mathbf{x}_i, \mathbf{x}_j)\}$, a more improved KED test statistic than (2) is proposed as

$$T_{KED,U} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} K(\mathbf{x}_i, \mathbf{x}_j), \tag{3}$$

where $K(\mathbf{x}_i, \mathbf{x}_j)$ is a nonlinear kernel function of two received signal vector $\mathbf{x}_i$ and $\mathbf{x}_j$ corresponding to each row of matrix $\mathbf{X}$. Moreover, the test statistic in (3) is an unbiased estimation for $\mathbb{E}\{K(\mathbf{x}_i, \mathbf{x}_j)\}$. According to the U-statistics theory, the KED test statistic in (3) can be more simplified as $T_{KED,U} = \binom{N}{2}^{-1} \sum\sum_{1 \leq i < j \leq N} K(\mathbf{x}_i, \mathbf{x}_j)$, if the nonlinear kernel function $K(\mathbf{x}_i, \mathbf{x}_j)$ is symmetric [15]. According to the Hoeffding's inequality [16, p. 25], we know that the KED test statistic in (3) is highly concentrated around its mean value which is a vital requirement of any sensing scheme in order to distinguish two hypotheses H_0 and H_1. In other words, for a bounded kernel function $a \leq K(x_i, x_j) \leq b$, where $P = 1$, deviation of the KED test statistic from its mean value is sufficiently small, i.e.,

$$P\left(|T_{KED,U} - \mathbb{E}\{T_{KED,U}\}| \geq \varepsilon\right) \leq \mathrm{e}^{\frac{-N\varepsilon^2}{(b-a)^2}}, \tag{4}$$

where ε is a positive real value. In the case of Gaussian kernel function $K(x_i, x_j) = e^{-\frac{\left(x_i - x_j\right)^2}{2c^2}}$, where c is the bandwidth of the kernel and x_i and x_j are the received signal samples at the SU, we have $0 \leq K(x_i, x_j) \leq 1$ which is independent of the statistical distribution of the received samples whether they belong to the bounded or unbounded distributions[1]. Thus, it is concluded that in the case of Gaussian kernel function, inequality (4) is always satisfied regardless of the signal and the noise distributions in the sensing task. This in turn represents that the KED test statistic in (3) is robust for different noise scenarios leading to a suitable sensing scheme for impulsive man-made noises. As will be shown later, the main advantage of the proposed KED in (3) is a better detection performance for the case of non-Gaussian noise; moreover, it is much more robust against impulsive man-made noises than the classical ED. Optimality of the proposed KED test statistic is investigated in [17] where it is proved that in the case of utilizing a proper kernel function satisfying the finiteness of the KED test statistic (as mentioned in (4)), an optimal spectrum sensing scheme could be obtained for various Gaussian and non-Gaussian noise scenarios. Finally, although some general properties of the kernel functions which can be considered in the KED scheme are presented in this paper and in [17], it is beyond the scope of this paper to design different kernels to fulfill these properties.

Lemma 1 *Let assume $K(x_i, x_j)$ is a nonlinear kernel function. Then,* 1) *$\mathbb{E}\{K(x_i, x_j)\}$ can be represented by higher order moments and cumulants of two random variables x_i and x_j, where the expectation is computed with respect to x_i and x_j.* 2) *The kernelized spectrum sensing defined as a nonlinear moment, $\mathbb{E}\{K(x_i, x_j)\}$, employs the FLOMs in the sensing process.*

$$\mathbb{E}\{K(x_i, x_j)\} = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{K(0,0)^{(k_1,k_2)}}{k_1! k_2!} m_{X_i X_j}(k_1, k_2). \tag{5}$$

$$\mathbb{E}_{X_i X_j}\{K(x_i, x_j)\} = \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\ell!} \mathbb{E}_{X_i X_j}\{K(x_i, x_j)^{(n,\ell)}\} \times c_{X_i X_j}(\ell+1, n), \tag{6}$$

where the superscripts in the parenthesis indicates order of derivation and $m_{X_i X_j}(k_1, k_2)$ and $c_{X_i X_j}(\ell+1, n)$ are the joint moment and cumulat of random variables X_i and X_j, respectively.

$$\mathbb{E}\{K(x_i, x_j)\} = \sum_{k=1}^{M} \left[\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} F_{\alpha,\beta}(m,n) \mathbb{E}\{x_i^{m\alpha} x_j^{n\beta}\} \right], \tag{7}$$

where D^{α} is the fractional derivative operator for $0 < \alpha \leq 1$ [18] and $F_{\alpha,\beta}(m,n) = \frac{1}{\Gamma(\alpha m+1)\Gamma(\beta n+1)} (D^{\alpha})^m \Phi_k(x_i) (D^{\beta})^n \Phi_k(x_j)$.

Proof: This Lemma can be proved using the Taylor series expansion and the definitions of moment generating and cumulant generating functions which is eliminated due to the space constraints.

Corollary 1: As a result of Lemma 1, the KED test statistic (3) which is compatible with the U-statistics based estimation of $\mathbb{E}\{K(x_i, x_j)\}$, defined as $\beta = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} K(x_i, x_j)$, includes the higher order joint statistics and FLOMs of the received signal samples.

Practical methodology associated with the KED algorithm in the two stage spectrum sensing context is presented in [17] which is not provided here due to the space considerations.

4. NUMERICAL RESULTS

In this section, some numerical results are presented to evaluate the performance of the proposed KED algorithm in practical impulsive man-made noises to confirm our analysis in Section 3. The simulation setup follows a one-primary and K-secondary users scenario where each SU performs the spectrum sensing task individually. BPSK and QPSK modulated signals are considered for PU's signal, and the bandwidth of the Gaussian kernel function, c, is considered to be fixed for the KED algorithm. We generate pure noise samples corresponding to the predetermined noise distributions: Gaussian, Laplacian or α-stable, based on which sensing threshold is measured for a pre-specified false alarm probability. The measured threshold is then employed to sense the PU's signal. We assume that the cardinality of the received signal samples set in the sensing task is 200 and 400. Additionally, the number of iterations in each simulation is $10,000$. The performance of the KED scheme is compared with that of the conventional ED which has been extensively investigated in the literature, e.g. [9, 13]. Moreover, a fair comparison can be achieved in this case since both the conventional ED and the proposed KED scheme are semi-blind methods requiring noise statistics in detection process. Detection probability versus SNR and the Receiver Operating Characteristic (ROC) curves are employed as the metrics to demonstrate the performance of the KED and the conventional ED schemes in various noise scenarios.

Fig. 1 illustrates the ROC curves for the proposed KED scheme compared with the conventional ED for the QPSK signal, SNR=-8dB, and both in Laplacian and Gaussian noise scenarios. As depicted in Fig. 1, the KED scheme provides almost the same detection performance with the conventional ED in the Gaussian Noise (GN) model as expected, however, for the Laplacian Noise (LN), the KED algorithm achieves a considerably better performance than the ED. This better performance is mainly due to the fact that the KED scheme employs higher order and fractional lower

[1] The upper bound is due to the fact that power of the exponential in the Gaussian kernel is always negative and $K(x_i, x_j)$ is maximized when the power of the exponential is zero , i.e., $\forall x_i, x_j$ then $(x_i - x_j)^2 \geq 0$.

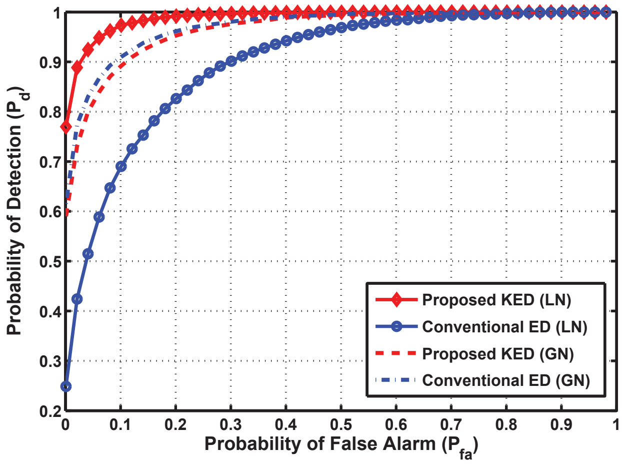

Fig. 1. ROC curves for the PU with QPSK modulated signal for both Laplacian and Gaussian noise scenarios, where the KED and the ED algorithms consider $N = 200$ and SNR $= -8$dB.

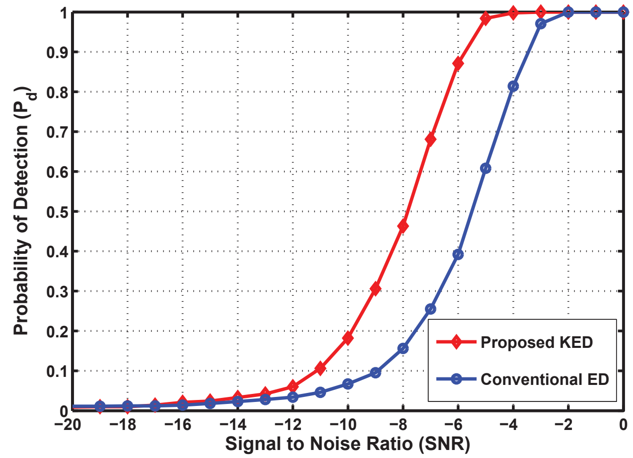

Fig. 2. P_d versus SNR for a PU with QPSK and the Laplacian noise for the KED and the ED algorithms where N = 200 and $P_{fa} = 0.01$.

order moments for the LN, while in the GN, higher order moments provide redundant information where no performance improvement is achieved. Fig. 2 shows the detection probability of the KED algorithm compared to the conventional ED for different SNR values among $[-20, 0]$dB for the Laplacian noise and QPSK signal where the false alarm probability is considered to be $P_{fa} = 0.01$. As seen in Fig. 2, the KED scheme displays about 2.5 dB better performance than the conventional ED. To investigate the performance of the proposed KED test in various impulsive noise models, the α-stable distribution is employed in our simulations. As previously mentioned, the second or higher order moments of the α-stable distribution is infinite for $\alpha < 2$. It should be noted that due to the infiniteness of the noise variance for the α-stable model, the SNR cannot be measured as SNR$=\frac{\mathbb{E}\{\|s\|^2\}}{\mathbb{E}\{\|w\|^2\}}$. Instead, the SNR for the above scenario should be computed using the Generalized Signal-to-Noise Ratio defined as GSNR$=\frac{\mathbb{E}\{\|s\|^2\}}{M_\gamma}$ where γ is the dispersion parameter of the α-stable distribution and M_γ denotes the averaged value of it. Fig. 3 demonstrates the ROC curves for the KED and the ED schemes when the noise follows an α-stable distribution for different values of $\alpha = 0.5, 1, 1.5, 2$, shift parameter $\nu = 0$, $N = 400$ and GSNR$= -2$dB. It is worth mentioning that α-stable distribution for $\alpha = 1$ and $\nu = 0$ coincides with the Cauchy distribution, while $\alpha = 2$ determines the Gaussian distribution. As represented in Fig. 3, the proposed KED scheme has significantly better detection performance for $\alpha < 2$ than the conventional ED scheme and it is much more robust against various impulsive noises, while the convectional ED fails to detect the PU's signal in the existence of these α-stable impulsive noises. The better performance of the KED scheme for $\alpha < 2$ is due to the fact that the Gaussian kernel function $K(x_i, x_j) = e^{-\frac{(x_i - x_j)^2}{2c^2}}$ is bounded among zero and one regardless of the noise density. According to (4), this leads to a well concentrated test in various noise scenarios which is crucial for a spectrum sensing method.

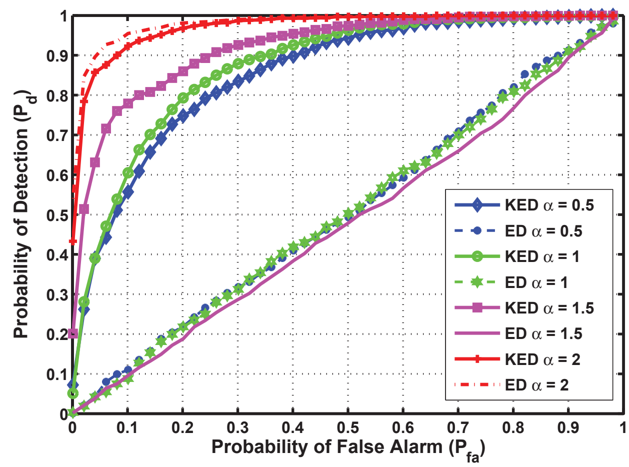

Fig. 3. ROC curves for a PU with BPSK modulated signal and the α-stable noise distribution with various values of α for the KED and the ED algorithms, where $N = 400$, GSNR $= -2$dB.

5. CONCLUSION

In this paper, we proposed a robust spectrum sensing scheme, namely the kernelized energy detector, for the practical impulsive man-made noises. The proposed KED algorithm significantly outperforms the conventional ED for the case of α-stable noise model when $\alpha < 2$, while the performance of the conventional ED degrades considerably due to the infinite variance of the α-stable distribution. The better detection performance of the KED test can be attributed to the kernel theory which enables us to develop nonlinear algorithms capable of employing the higher order and fractional lower order statistics in the sensing procedure. In addition, incorporation of bounded kernel functions in the KED method leads to concentrated test statistics around its mean value, according to Hoeffding's inequality, which results in a high detection accuracy in Gaussian and non-Gaussian impulsive noise scenarios.

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