DOUBLE DIFFERENTIAL TRANSMISSION FOR TWO-WAY RELAY SYSTEMS WITH UNKNOWN CARRIER FREQUENCY OFFSETS

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ABSTRACT

In this paper, an amplify-and-forward two-way relay system with unknown carrier frequency offsets (CFOs) is considered. A double differential transmission scheme is proposed to achieve successful two-way relaying transmission without any CFOs information. The average symbol error rate (SER) performance of the proposed scheme is analyzed and a closed-form upper bound of the average SER is derived. Simulation results are provided to validate the proposed scheme.

Index Terms— Two-way relay, double differential modulation, carrier frequency offset.

1. INTRODUCTION

Two-way relay systems, where two source terminals simultaneously send information to each other with the help of a relay terminal, have drawn much interest due to their potential of improving spectral efficiency compared to conventional four-phase one-way relaying systems [1, 2].

When the channel information is unavailable, several differential transmission strategies for two-way relay systems based on both decode-and-forward (DF) and AF protocols are proposed in [3]. An analog network coding scheme with differential modulation is proposed in [4] for AF two-way relay systems without any knowledge of channel information. Like traditional differential schemes, these proposed differential schemes requires the channel to remain static over at least two time slots [3] [4]. However, the presence of carrier frequency offset (CFO) due to the relative motion of the transmitter and receiver, makes the block fading channels time varying and breaks the basic assumption of differential schemes.

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For time varying channels with CFO, double differential modulation is a preferred choice, and has been used in cooperative communications. For DF cooperative communication systems with single relay, double differential modulation is considered in [5]. Distributed double differential modulation for multiple relay terminals using DF protocol is proposed in [6]. Double differential modulation for AF based cooperative communication with random CFOs is studied in [7]. Different from conventional cooperative communications, in AF two-way relay systems, due to the sharing of spectral resources, the signal sent by one source is delivered not only to the other source, but also back to the original source as self-interference through the relay. Such self-interference signal prevents the direct application of double differential modulation.

In this paper, a new double differential transmission scheme is proposed for AF two-way relay systems without the knowledge of CFOs. The symbol error rate (SER) performance of the proposed scheme is analyzed and an analytical SER upper bound is derived. The simulation results coincide with the theoretical analysis and show that the proposed scheme outperforms existing differential two-way relaying or double differential one-way relaying schemes.

2. SYSTEM MODEL

Consider a two-way relay system with three terminals: two source terminals T_1 and T_2, one relay terminal T_3. The terminal $T_i, i = 1, 2, 3$, is moving at a velocity of v_i, which can change with time. Assume that all terminals are half duplex and equipped with single antenna. T_1 and T_2 need to transmit information to each other through the help of T_3. In a conventional one-way relay system, it takes 4 time slots for T_1 and T_2 to finish the exchange of one symbol. While in a two-way relay system, it takes 2 time slots.

The channel of each link is assumed to be a Rayleigh block fading channel. All links are assumed to be perturbed by different CFOs caused by Doppler effect. Assume that the phases caused by CFOs are randomly distributed over $[-\pi, \pi\rangle$

and independent of each other [7]. These random offsets are assumed to remain fixed for at least three two-way transmissions.

In Phase I of a conventional two-way relay communication, T_1 and T_2 transmits their signals to T_3 simultaneously. In Phase II, T_3 amplifies the received signal with a factor β and transmits to the source terminals T_1 and T_2. When the channels' CFOs are unknown, neither T_1 nor T_2 can extract the other's information with the presence of self-interference signal. Therefore, double differential modulation can not be implemented directly in this case.

3. PROPOSED DOUBLE DIFFERENTIAL TRANSMISSION SCHEME

Since all the links are assumed to be block fading channels, the channel gains do not change during the channel coherence time. Meanwhile, the CFOs of the links change randomly. It is difficult for the source terminals to estimate the CFOs for both T_1-T_3 and T_2-T_3 links. However, it is easy for $T_i, i = 1, 2$ to get the channel gain of its own link. Before the two-way transmission, a signal sequence of length L is first transmitted from the relay terminal T_3 to the source terminals T_1 and T_2. Let $[x[1], \cdots, x[L]]$ be the transmitted signal sequence. Assume that $|x(l)|^2 = 1, \forall l \in [1, L]$. The received signal at $T_i, i = 1, 2$, is

$$r_{T_i}[l] = \sqrt{P_T} h_{i3} e^{j\omega_{3i}(l-1)} x[l] + n_{T_i}[l], l = 1, 2, \cdots, L \quad (1)$$

where P_T is the transmit power, $h_{ij} \sim CN(0, \sigma_{ij}^2)$ is the complex channel coefficient of $T_i \to T_j$ link, σ_{ij}^2 is the channel variance, $i \neq j, i, j \in \{1, 2, 3\}$, $n_{T_i}[l]$ is the additive complex white Gaussian noise with mean 0 and variance σ^2. Each source terminal can estimate its channel gain by $|\hat{h}_{i3}|^2 = \frac{\sum_{l=1}^{L} |r_{T_i}[l]|^2}{LP_T}, i = 1, 2$. After that, T_1 and T_2 begin to transmit their information signals.

Assume that there are K two-way transmissions during the channel coherence time. For the kth transmission ($k = 1, 2, \cdots, K$), the information symbol of T_1 and T_2 are $s_1[k]$ and $s_2[k]$ respectively. They come from a normalized M-PSK constellation $\mathcal{A}$, i.e., the average symbol energy of $\mathcal{A}$ is 1. After double differential modulation, the transmitted signal $u_i[k]$ of T_i can be obtained from $s_i[k]$ as follows,

$$\begin{aligned} &g_i[k] = g_i[k-1] s_i[k], \\ &u_i[k] = u_i[k-1] g_i[k], k = 2, 3, \cdots, K, i = 1, 2 \end{aligned} \quad (2)$$

with $u_i[0] = u_i[1] = g_i[1] = 1$. Since $|s_i[k]| = 1$, it follows from (2) that $|g_i[k]| = |u_i[k]| = 1$. In Phase I of the kth transmission, the received signal at T_3 can be written as

$$y[k] = \sum_{i=1}^{2} \sqrt{P_i} h_{i3} e^{j\omega_{i3}(2k-2)} u_i[k] + n_3[k]. \quad (3)$$

where P_i is the transmit power of $T_i, i = 1, 2$, $\omega_{ij} = 2\pi f_{ij}$, and f_{ij} is the unknown CFO of $T_i \to T_j$ link, $i \neq j, i, j \in \{1, 2, 3\}$, $n_3[k]$ is the additive complex white Gaussian noise with mean 0 and variance σ^2.

Consider the Doppler effect, the received frequency is $f = \frac{c+v_r}{c+v_s} f_0$, where f_0 is the carrier frequency, c is the velocity of waves in the medium, v_r is the velocity of the receiver relative to the medium, which is positive if the receiver is moving towards the source, and negative in the other direction. v_s is the velocity of the source relative to the medium, which is positive if the source is moving away from the receiver, and negative in the other direction. The speeds v_r and v_s are small compared to c. Therefore, we can find out that the CFOs caused by relative motion of the terminals have the following relationship: $f_{i3} = f_{3i}, i = 1, 2$. Then T_3 forwards βy^* to T_1 and T_2 in Phase II, where $(\cdot)^*$ represents the operation of conjugation. The received signal at T_1 becomes

$$\begin{aligned} r_1[k] = &\sqrt{P_1 P_3} \beta |h_{13}|^2 u_1^*[k] \\ &+ \sqrt{P_2 P_3} \beta h_{23}^* h_{13} e^{j\triangle\omega(2k-2)} u_2^*[k] + n_1'[k] \end{aligned} \quad (4)$$

where $\triangle\omega = \omega_{13} - \omega_{23}$ and $n_1'[k] = \sqrt{P_3} \beta h_{13} e^{j\omega_{13}(2k-2)} n_3^*[k] + n_1[k]$. Assume that the transmit powers and the channel statistic information are known at the terminals, the factor $\beta = \sqrt{\frac{1}{\sum_{i=1}^{2} P_i \sigma_{i3}^2 + \sigma^2}}$. The first term of (4) can be subtracted by T_1, which is called self-interference cancellation. The received signal at T_1 can be rewritten as

$$r_1[k] = \sqrt{P_2 P_3} \beta h_{23}^* h_{13} e^{j\triangle\omega(2k-2)} u_2^*[k] + \tilde{n}_1[k] \quad (5)$$

where $\tilde{n}_1[k] = n_1'[k] + n_e[k]$, and $n_e[k] = \sqrt{P_1 P_3} \beta \triangle_{|h_{13}|^2} u_1^*[k]$ with $\triangle_{|h_{13}|^2} = |h_{13}|^2 - |\hat{h}_{13}|^2$. In the following, only the processing at T_1 is illustrated since the processing at T_2 is similar. By using (2), we get the following equations

$$\begin{aligned} &r_1[k] = r_1[k-1] g_2^*[k] e^{2j\triangle\omega} + \eta_1[k] \\ &r_1^*[k] r_1[k-1] = s_2[k] r_1^*[k-1] r_1[k-2] + \tilde{\eta}_1[k] \end{aligned} \quad (6)$$

where $\eta_1[k] = -\tilde{n}_1[k-1] g_2^*[k] e^{2j\triangle\omega} + \tilde{n}_1[k]$ and $\tilde{\eta}_1[k] = \eta_1^*[k] r_1[k-2] g_2^*[k-1] e^{2j\triangle\omega} + \eta_1[k-1] r_1^*[k-1] g_2[k] e^{-2j\triangle\omega} + \eta_1^*[k] \eta_1[k-1]$. Therefore, without the knowledge of CFOs, the information symbol from T_2 can be decoded at T_1 as

$$\hat{s}_2[k] = \arg\min_{s_2 \in \mathcal{A}} |r_1^*[k] r_1[k-1] - s_2 r_1^*[k-1] r_1[k-2]|^2 \quad (7)$$

4. AVERAGE SER PERFORMANCE ANALYSIS

In this section, average SER performance is analyzed when perfect self-interference cancellation is performed. Because $n_i[k]$ is the statistically independent Gaussian random variables with mean 0 and variance σ^2 for different i and k, we can get the statistical characteristics of the equivalent noise $\tilde{\eta}_1[k]$ as follows.

$$\begin{aligned} &\mathrm{E}\tilde{\eta}_1[k] = 0 \\ &\mathrm{D}\tilde{\eta}_1[k] = 4\beta^2 P_2 P_3 |h_{23}|^2 |h_{13}|^2 \sigma_e^2 + 2\sigma_e^4 \end{aligned} \quad (8)$$

where $\sigma_e^2 = (\beta^2 P_3 |h_{13}|^2 + 1)\sigma^2$. From (6), the signal power of $s_2[k]$ is $(\beta^2 P_2 P_3 |h_{23}|^2 |h_{13}|^2 + \sigma_e^2)^2$. Therefore, the SNR of the double differential two-way transmission is SNR $= \frac{(\gamma+1)^2}{4\gamma+2}$, where $\gamma = \frac{\beta^2 P_2 P_3 |h_{23}|^2 |h_{13}|^2}{(\beta^2 P_3 |h_{13}|^2+1)\sigma^2}$. To make the analysis mathematically feasible, we take the following high SNR approximation SNR $\approx \frac{\gamma}{4} + \frac{1}{8}$. Let $\gamma_{13} = \frac{P_3|h_{13}|^2}{\sigma^2}$, and $\gamma_{23} = \frac{P_2|h_{23}|^2}{\sigma^2}$, γ can be written as $\gamma = \frac{\gamma_{23}\gamma_{13}}{\gamma_{13}+\bar{\gamma}_s+1}$, where $\bar{\gamma}_s = \frac{P_1\sigma_{13}^2 + P_2\sigma_{23}^2}{\sigma^2}$.

Since the channel coefficients are independent complex Gaussian random variables, the channel gains $|h_{i3}|^2, i = 1, 2$, are independent exponential random variables with parameters $\frac{1}{\sigma_{i3}^2}, i = 1, 2$, respectively. The probability density function (PDF) of γ can be derived as [8]

$$f_\gamma(\gamma) = 2\frac{\bar{\gamma}_s + 1}{\bar{\gamma}_{13}\bar{\gamma}_{23}} e^{-\frac{\gamma}{\bar{\gamma}_{23}}} K_0\left(b\sqrt{\gamma}\right) + \frac{2}{\bar{\gamma}_{13}\bar{\gamma}_{23}}\sqrt{\frac{\gamma(\bar{\gamma}_s+1)\bar{\gamma}_{13}}{\bar{\gamma}_{23}}} e^{-\frac{\gamma}{\bar{\gamma}_{23}}} K_1\left(b\sqrt{\gamma}\right) \tag{9}$$

where $\bar{\gamma}_{13} = \frac{P_3\sigma_{13}^2}{\sigma^2}$, $\bar{\gamma}_{23} = \frac{P_2\sigma_{23}^2}{\sigma^2}$, $b = 2\sqrt{\frac{\bar{\gamma}_s+1}{\bar{\gamma}_{13}\bar{\gamma}_{23}}}$, $K_0(\cdot)$ and $K_1(\cdot)$ denote the zeroth-order and the first order modified Bessel function of the second kind respectively. Then, the PDF of SNR can be given as $f_{SNR}(x) = 4f_\gamma\left(4(x - \frac{1}{8})\right)$.

The SER conditioned on SNR for the double differential two-way transmission is $P_e(\text{SNR}) = 2Q\left(\sqrt{2\text{SNR}}\sin\left(\frac{\pi}{M}\right)\right)$ [9]. It is known that Chernoff bound of Q-function is $Q(x) \leq \frac{1}{2}e^{-\frac{x^2}{2}}, x > 0$. By using the approximated SNR, the Chernoff bound of the average SER can be written as

$$\begin{aligned} P_e &\leq \int_0^\infty \frac{1}{2} e^{-x\sin^2\left(\frac{\pi}{M}\right)} f_{SNR}(x)dx \\ &= 2a_1^{-\frac{1}{2}} c_1 b^{-1} \frac{\bar{\gamma}_s + 1}{\bar{\gamma}_{13}\bar{\gamma}_{23}} W_{-\frac{1}{2},0}\left(\frac{b^2}{4a_1}\right) \\ &+ a_1^{-1} c_1 b^{-1} \frac{2}{\bar{\gamma}_{23}} \sqrt{\frac{(\bar{\gamma}_s+1)}{\bar{\gamma}_{13}\bar{\gamma}_{23}}} W_{-1,\frac{1}{2}}\left(\frac{b^2}{4a_1}\right) \end{aligned} \tag{10}$$

where $a_1 = \frac{1}{4}\sin^2\left(\frac{\pi}{M}\right) + \frac{1}{\bar{\gamma}_{23}}$ and $c_1 = e^{\frac{b^2}{8a_1} - \frac{1}{8}\sin^2\left(\frac{\pi}{M}\right)}$, $W_{\lambda,\mu}(\cdot)$ is the Whittaker function [10]. In [11], a tight upper bound of Q-function is given as $Q(x) \approx \frac{1}{4}e^{-\frac{2}{3}x^2} + \frac{1}{12}e^{-\frac{x^2}{2}}, x > 0$. By using the approximated SNR and the bound in [11], the average SER can be upper bounded by

$$\begin{aligned} P_e &\leq \int_0^\infty \left(\frac{1}{6} e^{-x\sin^2\left(\frac{\pi}{M}\right)} + \frac{1}{2} e^{-\frac{4}{3}x\sin^2\left(\frac{\pi}{M}\right)}\right) f_{SNR}(x)dx \\ &= \sum_{i=1}^{2}\left[k_i a_i^{-\frac{1}{2}} c_i b^{-1} \frac{\bar{\gamma}_s+1}{\bar{\gamma}_{13}\bar{\gamma}_{23}} W_{-\frac{1}{2},0}\left(\frac{b^2}{4a_i}\right)\right. \\ &\left. + k_i a_i^{-1} c_i b^{-1} \frac{1}{\bar{\gamma}_{23}} \sqrt{\frac{\bar{\gamma}_s+1}{\bar{\gamma}_{13}}} W_{-1,\frac{1}{2}}\left(\frac{b^2}{4a_i}\right)\right] \end{aligned} \tag{11}$$

where $k_1 = \frac{1}{3}, k_2 = 1, a_2 = \frac{1}{3}\sin^2\left(\frac{\pi}{M}\right) + \frac{1}{\bar{\gamma}_{23}}$, and $c_2 = e^{\frac{b^2}{8a_2} - \frac{1}{6}\sin^2\left(\frac{\pi}{M}\right)}$.

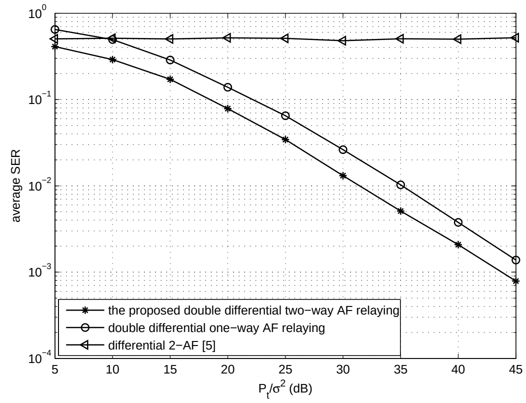

Fig. 1. Comparison of average SER performance of the proposed scheme with existing schemes

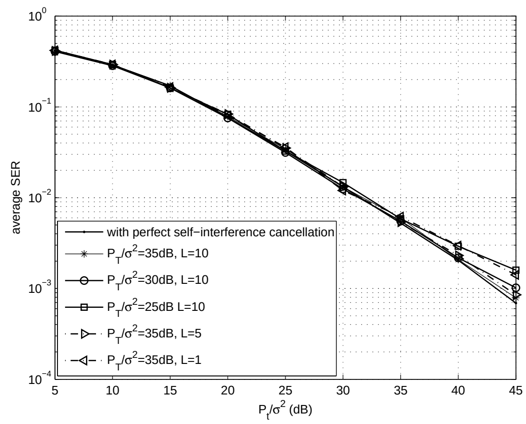

Fig. 2. Average SER performance of the proposed double differential two-way relay scheme with different overheads

5. SIMULATION RESULTS

In the simulations, the transmit power of each terminal is set as $P_i = P_t$, for $i = 1, 2, 3$. The channel variances are $\sigma_{ij}^2 = 1, i, j = 1, 2, 3, i \neq j$. The channels are perturbed by independent random CFOs, and the phases caused by the CFOs are uniformly distributed in the range of $[-\pi, \pi\rangle$.

The average SER performance of the proposed scheme is compared with existing schemes in Fig. 1. The transmit SNR of the sequence transmitted from T_3 is $P_T/\sigma^2 = 35dB$, and $L = 10$. A differential network coding scheme is proposed for two-way wireless communications in [3]. 2-AF with perfect CSI of [3] is considered here. BPSK is used for both schemes. It is shown that the differential scheme in [3] can not work due to the presence of unknown CFOs. The proposed scheme is also compared with the double differential

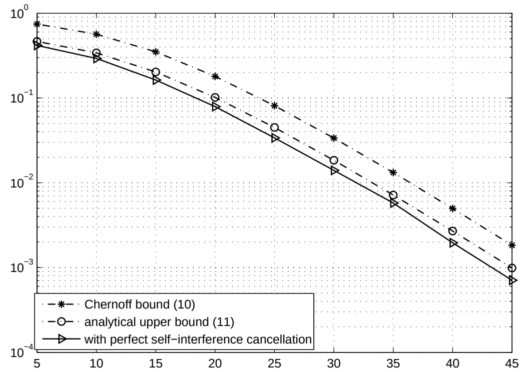

Fig. 3. Analytical SER bounds and simulated SER performance of the proposed double differential two-way relay scheme with random CFOs

one-way AF relaying (DD 1-AF) scheme. Since the DD 1-AF scheme consumes 4 time slots for the source terminals to exchange one symbol, QPSK is used for the DD 1-AF. It can be seen from Fig.1 that the proposed double differential two-way AF relaying scheme outperforms the DD 1-AF scheme with a performance gain of nearly $3dB$.

Fig. 2 shows the average SER performance of the proposed scheme with different overheads. The average SER performance with perfect self-interference cancellation is given as a benchmark. It can be seen that when $P_T/\sigma^2 = 35dB, L = 10$, the average SER performance is almost the same to the performance with perfect self-interference cancellation. When L reduces to 5, the average SER performance becomes slightly worse for high SNR. The similar degradation occurs when P_T/σ^2 reduces to $30dB$ and $L = 10$. When L or P_T/σ^2 reduces more, the average SER performance degrades more for high SNR.

Fig. 3 gives the analytical performances and the simulation result of the proposed scheme with unknown random CFOs. Chernoff bound and the upper bound in (11) are presented. The simulated SER performance with perfect self-interference cancellation is given for comparison. It can be seen that both bounds follow the shape of the simulated SER curve. Moreover, the upper bound in (11) is much tighter than Chernoff bound of the average SER.

6. CONCLUSION

In this work, we propose a double differential two-way relay scheme for amplify-and-forward two-way relay systems with unknown CFOs. A tighter upper bound of average SER is derived. Simulation results show that the proposed scheme performs well under the existence of random CFOs.

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