DIVERSITY COMBINING IN WIRELESS RELAY NETWORKS WITH PARTIAL CHANNEL STATE INFORMATION

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ABSTRACT

We develop a novel diversity combiner for joint reception in wireless relay networks with unknown relay-to-destination channel. The partially available channel information poses an obstacle for utilization of the relay diversity. We tackle this problem by formulating a unified second-order cone programming (SOCP) receiver which jointly detects and decodes transmitted signals from both source and relay nodes. Our results demonstrate that the proposed diversity combiners can achieve performance close to that of the maximum-ratio combiner based on full channel information.

Index Terms— Diversity combiner, relay network, second-order cone programming, partial channel information.

1. INTRODUCTION

Spatial diversity has proven effective in overcoming channel fading distortions by delivering reliable transmission in wireless communications. Spatial diversity leverages antenna array at the transmitter and/or the receiver side [1]. However, many mobile devices in cellular networks or wireless sensor networks may not be able to support sufficient antennas because of size, cost, and hardware limitations. In such environment, user cooperation or distributed antennas can cooperatively form virtual-antenna-array systems [2] for spatial diversity. In such cooperative communication networks, wireless users share and coordinate their resources to enhance transmission diversity and reception quality. This idea is particularly attractive in wireless environments due to the diverse channel qualities, limited energy, and bandwidth resources. Through cooperations, users experiencing deep fade can benefit from diversity channels of their partners to achieve the desired quality of service [3, 4].

Typically, a source node first broadcasts its messages to both relays and destination. Upon reception, relays will process the source signal before forwarding it to the destination. Relays typically follow certain forwarding protocols, such as demodulation/decode-and-forward [2], amplify-and-forward [5], coded-cooperation [6], and compress-and-forward [7]. In this work, we focus on the demodulation-and-forward (DF) protocol. Furthermore, we assume mutually orthogonal transmissions of source and relays in this work.

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Despite the much improved reliability by relay cooperation, wireless channel fading and noises can still lead to substantial performance loss and detection errors. In practice, forward error correction code is adopted in most wireless systems. In particular, low-density parity check (LDPC) codes have been gaining popularity owing to their near capacity performance [8] typically achieved via the popular sum-product algorithm (SPA). Instead of the non-linear SPA decoder based on belief propagation, Feldman *et al.* [9] presented a linear programming (LP) decoder. Nonetheless, one disadvantage associated with LP decoding is the exponential growth of parity check inequalities. Provided that the LDPC code is sufficiently long or the parity check matrix has relatively large row weights, the scale of the resulting LP can be prohibitive. Motivated by complexity reduction, authors in [10] presented an adaptive cutting plane technique.

Broadly, many research works within the framework of relay networks focused on node selection [11], power allocation [12], relay protocol design [13], distributed code construction [14], etc. We notice, however, few works are directed at the joint detection and decoding in relay networks [15, 16, 17, 18], especially when only partial channel state information (CSI) is available at the receiver. Partial CSI represents a scenario when cooperative relay nodes do not have the luxury of forwarding (some or all) training symbols to the destination for relay-to-destination channel estimation whereas the source node does transmit sufficient pilots to facilitate source-to-destination CSI estimation. Without full CSI, traditional diversity combining techniques, such as maximum-ratio combining (MRC), are no longer applicable. In this work, we propose a novel SOCP based diversity receiver for effective utilization of the relay diversity given partial CSI. In particular, our primary objective is the accurate detection of source signals through diversity combining and subspace separation. In order to achieve stronger robustness and better performance, we further integrate the LDPC code constraints into the joint detection and decoding problem formulation, and develop an adaptive procedure for complexity reduction.

2. PROBLEM DESCRIPTION

Consider the relay network illustrated in Fig. 1. The source (S) and destination (D) have a direct channel link h_{SD}, and there is a relay (R) node helping with the transmission. Denote the source-to-relay channel and the relay-to-destination channel h_{SR} and h_{RD}, respectively. Because of mobile device limitations mentioned earlier, we consider single antenna devices. In this manuscript, we assume all the channels to be Rayleigh fading, that is, $h_{SD} \sim \mathcal{CN}(0, \sigma_{SD}^2)$, $h_{SR} \sim \mathcal{CN}(0, \sigma_{SR}^2)$ and $h_{RD} \sim \mathcal{CN}(0, \sigma_{RD}^2)$. In reality, farther distance implies smaller channel gains because of path-loss. Thus, we let $\sigma_{SD}^2 < \sigma_{SR}^2$ and $\sigma_{SD}^2 < \sigma_{RD}^2$. A further assumption is the quasi-stationary channels that remain static under the transmission of multiple codewords.

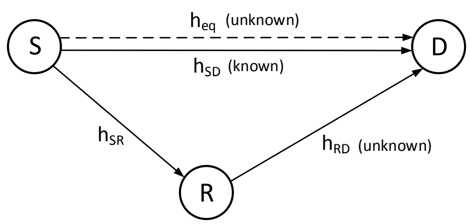

Fig. 1. A cooperative (single) relay network.

The source and relay can be paired in an opportunistic fashion. Depending on the specific protocol, the source can find a nearby relay node with good inter-channel quality so that few detection errors occur at the relay. Denote the instantaneous received signal-to-noise ratio (SNR) of S-R link and R-D link as $\gamma_{SR} = |h_{SR}|^2\overline{\gamma}$ and $\gamma_{RD} = |h_{RD}|^2\overline{\gamma}$, respectively, where $\overline{\gamma}$ is the average transmission SNR. Following the analysis presented in [18], the S-R-D link end-to-end bit error probability (BEP) is given by

$$\begin{aligned} P_{eq}^b(\gamma_{SR}, \gamma_{RD}) = {} & [1 - P_{SR}^b(\gamma_{SR})]P_{RD}^b(\gamma_{RD}) \\ & + [1 - P_{RD}^b(\gamma_{RD})]P_{SR}^b(\gamma_{SR}), \end{aligned} \tag{1}$$

where $P_{SR}^b(\gamma_{SR})$ and $P_{RD}^b(\gamma_{RD})$ are the BEPs of S-R link and R-D link, respectively. Assuming an equivalent channel $h_{eq} \sim \mathcal{CN}(0, \sigma_{eq}^2)$, the corresponding average SNR on the equivalent channel is

$$\overline{\gamma}_{eq} = \{Q^{-1}[P_{eq}^b(\gamma_{SR}, \gamma_{RD})]\}^2/\alpha|h_{eq}|^2, \tag{2}$$

where $Q(\cdot)$ is the Q-function and α is a constant depending on the underlying constellation, e.g., $\alpha = 1$ for 4-QAM.

For the sake of notational simplicity, we use h_1 to represent h_{SD}, and h_2 to represent h_{RD} or h_{eq} in the case of relay node with or without detection errors. Because of the time-divided orthogonality, the non-overlapping received signals from source and relay can be written as

$$\mathbf{r}[k] = \begin{bmatrix} r_1[k] \\ r_2[k] \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} x[k] + \begin{bmatrix} n_1[k] \\ n_2[k] \end{bmatrix}, \quad 1 \le k \le N \tag{3}$$

where $x[k]$ is the signal transmitted from the source, $r_1[k]$ and $r_2[k]$ are the received signals, and $n_1[k]$ and $n_2[k]$ are additive white Gaussian noise (AWGN) at the receiver.

3. BASIC DIVERSITY COMBINER FORMULATION

From the known channel h_1, we can estimate $x[k]$ through the direct link alone via maximum likelihood (ML)

$$\min_{x[k]} \sum_{k=1}^{N} \|h_1 x[k] - r_1[k]\|_2. \tag{4}$$

To utilize the information from relay node, we introduce a linear diversity combiner $\boldsymbol{\theta}$ such that the received symbols from source and relay together form the desired channel input $x[k]$. Ideally, we should have perfect recovery $x[k] = \boldsymbol{\theta}^H\mathbf{r}[k]$. However, in the presence of noise, it is more practical to use a small scalar $\tau[k] \ge 0$ to bound the error $\|x[k] - \boldsymbol{\theta}^H\mathbf{r}[k]\|_2$. Therefore, a minimization problem can be formulated as

$$\begin{aligned} \min_{\boldsymbol{\theta}, x[k]} \quad & \sum_{k=1}^{N} \|h_1 x[k] - r_1[k]\|_2 \\ \text{s.t.} \quad & \|x[k] - \boldsymbol{\theta}^H\mathbf{r}[k]\|_2 \le \tau[k], \quad 1 \le k \le N, \end{aligned} \tag{5}$$

where $[x[1] \ \ldots \ x[N]]$ and $\boldsymbol{\theta}$ are the variables.

For additional information, we exploit the orthogonality of signal and noise subspaces for noise suppression. As explained in [19], the singular value decomposition (SVD) is performed on a frame of received signals

$$[\mathbf{r}[1] \ \ldots \ \mathbf{r}[N]] = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^H = [\underbrace{\mathbf{U}_s}_{1} \underbrace{\mathbf{U}_n}_{1}]\boldsymbol{\Lambda}\mathbf{V}^H \tag{6}$$

where $\mathbf{U}_s$ spans the signal subspace and $\mathbf{U}_n$ spans the pure noise subspace. Since we do not expect $\boldsymbol{\theta}$ to amplify the received noise, we impose the classic subspace separation

$$\mathbf{U}_n^H\boldsymbol{\theta} = 0. \tag{7}$$

We caution that ill-designated $\tau[k]$'s may render the optimization infeasible. Therefore, we include $\tau[k]$'s as variables and lift them into the cost function. By introducing auxiliary variable $t[k]$ to account for $\|h_1 x[k] - r_1[k]\|_2$, we arrive at a disjoint SOCP (**D-SOCP**) formulation as follows

$$\begin{aligned} \min_{\boldsymbol{\theta}, x[k]} \quad & \sum_{k=1}^{N} (t[k] + \tau[k]) \\ \text{s.t.} \quad & \|h_1 x[k] - r_1[k]\|_2 \le t[k], \quad 1 \le k \le N, \\ & \|x[k] - \boldsymbol{\theta}^H\mathbf{r}[k]\|_2 \le \tau[k], \quad 1 \le k \le N, \\ & \mathbf{U}_n^H\boldsymbol{\theta} = 0, \\ & t[k], \tau[k] \ge 0, \quad 1 \le k \le N. \end{aligned} \tag{8}$$

4. INTEGRATION OF LDPC CODE CONSTRAINTS

To further improve the diversity receiver performance, we advocate the integration of channel code information at receiver in a unified optimization process. Instead of applying turbo iterations between the ML detector and the SPA decoder, we consider a set of linear constraints that are generated from the LDPC parity checks [9].

Consider an (N_c, K_c) LDPC code. Let $\mathcal{M}$ and $\mathcal{N}$ be the set of check nodes and variable nodes of the parity check matrix $\mathbf{H}$, respectively, i.e., $\mathcal{M} = \{1, \ldots, N_c - K_c\}$ and $\mathcal{N} = \{1, \ldots, N_c\}$. Denote the neighbor set of the m-th check node as $\mathcal{N}_m$. For a subset $\mathcal{F} \subseteq \mathcal{N}_m$ with odd cardinality $|\mathcal{F}|$, the explicit characterization of fundamental polytope is given in [9] by the parity check inequalities

$$\sum_{n \in \mathcal{F}} f[n] - \sum_{n \in (\mathcal{N}_m \setminus \mathcal{F})} f[n] \leq |\mathcal{F}| - 1, \quad \forall m \in \mathcal{M}, \tag{9}$$

plus the box constraints for each bit

$$0 \leq f[n] \leq 1, \quad \forall n \in \mathcal{N}. \tag{10}$$

To integrate the constraints (9) and (10) into the problem (8), we need to connect the symbol $x[k]$ and bit $f[n]$. For the 4-QAM with Gray mapping, the relationship between $x[k]$ and $f[n]$ can be described by

$$x[k] = (2f[2k-1] - 1) + j(1 - 2f[2k]). \tag{11}$$

5. UNIFIED SOCP AND CUTTING PLANE

5.1. Unified Second-Order Cone Program

We now are ready to summarize the optimization problem unified with LDPC code constraints. Since the log-likelihood ratio (LLR) of each bit is available through the S-D link, we can modify our objective function by adding the cost term $\sum_{n=1}^{N_c} \gamma[n] f[n]$, where $\gamma[n]$ denotes the LLR of bit $f[n]$. Moreover, different weights λ_t, λ_τ and λ_f are placed on $t[k]$'s, $\tau[k]$'s and $f[n]$'s to address the direct-link detection, diversity combining and LLR cost term, respectively. Therefore, we have the following unified SOCP (**U-SOCP**) formulation

$$\begin{aligned}
\min_{\boldsymbol{\theta}} \quad & \lambda_t \sum_{k=1}^{N} t[k] + \lambda_\tau \sum_{k=1}^{N} \tau[k] + \lambda_f \sum_{n=1}^{N_c} \gamma[n] f[n] \\
\text{s.t.} \quad & \|h_1 x[k] - r_1[k]\|_2 \leq t[k], \quad 1 \leq k \leq N, \\
& \|x[k] - \boldsymbol{\theta}^H \mathbf{r}[k]\|_2 \leq \tau[k], \quad 1 \leq k \leq N, \\
& \mathbf{U}_n^H \boldsymbol{\theta} = 0, \\
& x[k] = (2f[2k-1] - 1) + j(1 - 2f[2k]), \\
& \sum_{n \in \mathcal{F}} f[n] - \sum_{n \in (\mathcal{N}_m \setminus \mathcal{F})} f[n] \leq |\mathcal{F}| - 1, \\
& 0 \leq f[n] \leq 1, \quad \forall n \in \mathcal{N} \\
& t[k], \tau[k] \geq 0, \quad 1 \leq k \leq N.
\end{aligned} \tag{12}$$

In order to apply **U-SOCP** to a standard solver, we split the real and imaginary parts of the complex symbols. In fact, the variables are $\mathrm{Re}\{x[k]\}$, $\mathrm{Im}\{x[k]\}$, $\mathrm{Re}\{\boldsymbol{\theta}\}$, $\mathrm{Im}\{\boldsymbol{\theta}\}$, $f[n]$ by doubling the auxiliary variables $t^R[k]$, $t^I[k]$, $\tau^R[k]$, $\tau^I[k]$. In most cases, several codewords are transmitted in one quasi-stationary period. Considering the complexity, we only take one codeword in this optimization problem to obtain the optimum $\boldsymbol{\theta}$ before using it on all received signals within the coherence interval.

5.2. Complexity Reduction by Cutting Planes

We notice that the parity check inequalities (9) grow exponentially fast with the code weight. Particularly, for a parity check matrix $\mathbf{H}$ with weight d_m in the m-th row, we will have $\sum_{m \in \mathcal{M}} 2^{d_m - 1}$ constraints correspondingly. Hence, we are motivated to reduce the number of parity check constraints and thus reduce the overall complexity. As described in [10], an adaptive cutting plane method can effectively reduce the complexity at no performance loss. Hence, we introduce an adaptive SOCP (**A-SOCP**) receiver as follows

- S1 Initialize the **U-SOCP** without parity constraints (9).
- S2 Solve the current SOCP to obtain the bits $\{f[n]\}_{n \in \mathcal{N}}$ and round them to 0's and 1's.
- S3 If cuts (violated constraints) are found by substituting $\{f[n]\}_{n \in \mathcal{N}}$ into the parity inequalities, add them to the current SOCP and return to S2; otherwise, go to S4.
- S4 Use $\boldsymbol{\theta}$ to combine the signals in the whole frame before decoding the source bits.

6. NUMERICAL RESULTS

We present numerical results for the proposed diversity receiving algorithms **D-SOCP**, **U-SOCP** and **A-SOCP** for relay node with and without demodulation errors. It is worthwhile to note that recovered signals from the proposed diversity combiners are unquantized and therefore can be processed by sum-product algorithm (SPA) for further performance gains. The source transmits 4-QAM signals, and sends 30 pilot symbols for accurate channel estimation based on least squares. To set some benchmarks, MRC with full CSI and ML utilizing only direct-link CSI are compared against the proposed algorithms. Throughout this simulation section, we set channel gains to $\sigma_{SD}^2 = 0.7$ and $\sigma_{SR}^2 = \sigma_{RD}^2 = \sigma_{eq}^2 = 1$. In our figure legends, "Hard" means the bit error rate (BER) produced by hard decisions, whereas "Soft" represents the results processed by SPA.

First, we demonstrate the BER comparisons when relay detection is error-free in Fig. 2. The LDPC code we use is a (1024,512) regular code of column weight 2. The weighting coefficients in the SOCP cost functions are empirically set to $\lambda_t = 1$, $\lambda_\tau = 100$ and $\lambda_f = 10$. As shown, our proposed **D-SOCP** and **U-SOCP** outperform the direct-link ML by more than 8dB in SNR. Both are only slightly inferior to MRC with hard decisions and **U-SOCP** loses to MRC merely by 2dB after SPA. Comparison of results from our SOCPs after SPA, the **U-SOCP** shows substantial gain over **D-SOCP** in high SNR regime, while **A-SOCP** achieves exactly the same performance as **U-SOCP**, albeit with substantially lower complexity.

Now we proceed to show a more realistic scenario with relay demodulation errors. In Fig. 3, we illustrate the result of equivalent channel modeling. On the left, the BERs of S-R-D channel and equivalent channel are compared. The results confirm that the equivalent channel modeling is quite

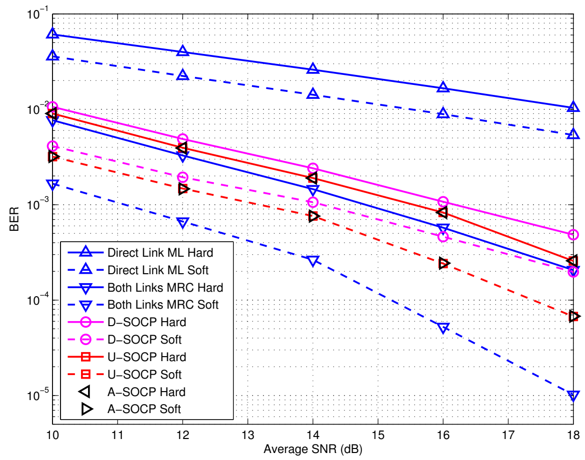

Fig. 2. BER comparisons of ML, MRC, D-SOCP, U-SOCP and A-SOCP in error-free relay detection.

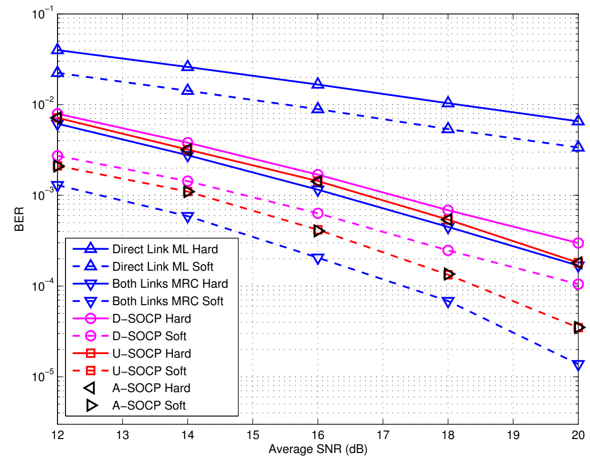

Fig. 4. BER comparisons of ML, MRC, D-SOCP, U-SOCP and A-SOCP with relay detection errors.

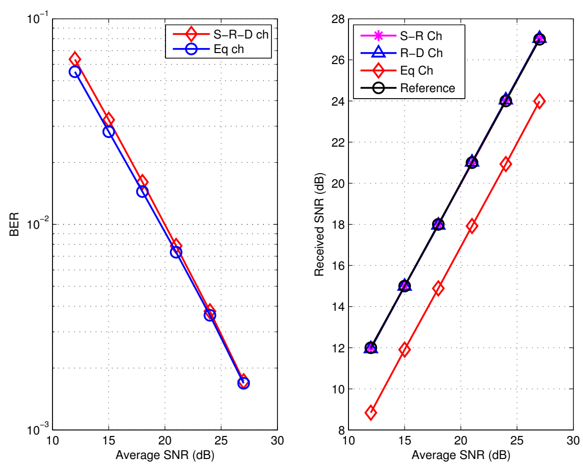

Fig. 3. Equivalent channel modeling: Left – BER comparisons; Right – SNR comparisons.

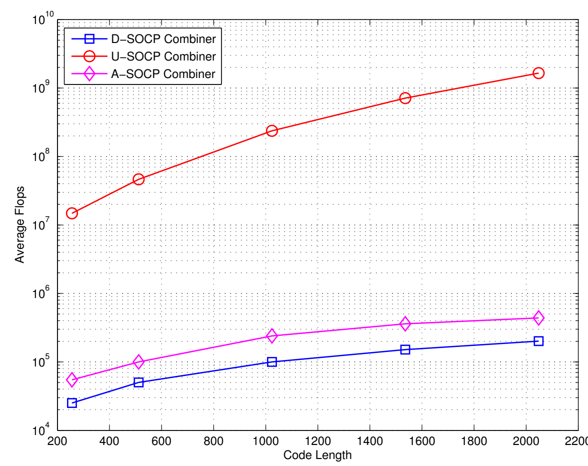

Fig. 5. Flops comparisons of D-SOCP, U-SOCP and A-SOCP.

accurate. Moreover, results from the right figure show that the received SNR of the equivalent channel exhibits a 3dB loss.

Next, the BER performance under relay demodulation errors is shown in Fig. 4. The simulation parameters are identical to those in the error-free relay case. Similarly as before, the direct-link ML performs worst while MRC performs the best. However, the performance gaps between the proposed algorithms and MRC are much smaller after hard decisions on the linear combiner outputs. Even after SPA processing, MRC is superior to **U-SOCP** only by 1dB.

Lastly, we examine the complexity aspect of the proposed diversity receivers. We use a plurality of LDPC codes here, instead of the single (1024,512) code used earlier. All the LDPC codes in this test are regular codes with column weight 3 and code rate 1/2. Hence, the code length is the only factor affecting the number of code constraints. By using cutting plane, only a small fraction of parity check inequalities are incorporated in the joint optimization problem, especially in the high SNR regime, for which the complexity is substantially reduced. This can be verified by the Flops read from MOSEK [20], as shown in Fig. 5. We can clearly see significant complexity reduction from **U-SOCP** to **A-SOCP**.

7. CONCLUSION

This work formulates a receiver optimization problem for diversity combining in cooperative relay networks when partial channel information from the relay nodes is unavailable. We presented a convex SOCP problem to account for channel uncertainty. We further proposed to integrate the LDPC code constraints forming a unified SOCP framework. We incorporated cutting planes to significantly reduce the complexity of the unified SOCP without performance degradation. Future works will address more efficient bandwidth utilization in the LDPC-coded relay network, and consider extensions to multi-hop and multi-branch relay networks.

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