MULTIUSER COOPERATIVE TRANSMISSION THROUGH SUPERPOSITION MODULATION BASED ON BRAID CODING

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***Abstract*—This paper investigates a cooperative transmission scheme for a multi-source single-destination system through signal-superposition-based braid coding. The source nodes take turns to transmit, and each time, a source "overlays" its new data together with (some or all of) what it overhears from its partner(s) using signal superposition, in a way similar to French-braiding the hair. We demonstrate how the resultant braid coding can be effectively employed in M-to-1 data collection networks to achieve progressive cooperation. We analyze two subclasses of braid coding, the nonregenerative and the regenerative cases, and, using the pairwise error probability (PEP) as a figure of merit, derive the optimal weight parameters theoretically for each class. For the regenerative case, a modified Viterbi maximum-likelihood (ML) estimator is proposed, whose complexity is linear to the message length. We compute the (Euclidean) free distance, and identify the memory size that strikes the best balance between performance and complexity. The proposed cooperative framework based on braid coding is general and subsumes several previous superposition modulation-based cooperative schemes as its special case. Simulations confirm the efficiency of the proposed schemes.**

***Index Terms*—cooperative communication, superposition modulation**

I. INTRODUCTION

Consider a multi-source single-destination M-to-1 cooperative system where two or more sources communicate, and at the same time help one another communicate, to the common destination. The majority of the practical schemes presented in the literature employ some form of time division, where the source allocates a portion of its transmission time to assist the other(s) [1] [2]. Such schemes are simple, but as Information Theory points out, time division alone is insufficient to achieve the capacity of a multiple access channel (MAC) [3].

To attain a higher spectral efficiency, researchers have also looked into more sophisticated techniques involving network coding, superposition modulation, and other forms of signal overlay. For example, [4] proposed a method where each user in the cooperative cohort transmits a superposed signal comprising of its own data and all the relaying data. A variation of this scheme considered cleaning up the relaying data (e.g. via decode-and-forward [5] [6] [7]), before constructing the new superposition signal. It is shown in [8] and [9] that leveraging useful coding tools such as iterative decoding can effectively enhance the cooperative performance. The signal superposition (aka superposition modulation) cooperative schemes based on amplify-and-forward and decode-and-forward are compared in [10]. The strategy in [11] pulled in the feedback channel from the destination to facilitate a higher gain. Cooperative schemes based on two-dimensional superposition modulation are also studied [12] [13]. There is also the proposal of extending superposition relaying to the code domain [14], and it is shown that effective coding gain can be achieved with only a judiciously-designed code-book and a sophisticated iterative receiver.

Though the implementation simplicity, the question remaining is how to best process and overlay signals at user side, and how to efficiently decode them at destination in M-to-1 system. The solution we develop here is progressive cooperation schemes using what we call the *braid coding*, a real-domain coding process similar to how a person French braids his/her hair: As each user takes its turn to transmit, it combines its own data (fresh data to be transmitted) with what it hears from the system in the preceding time slot (previously-transmitted data to be relayed), using appropriate processing and weights for each. Our scheme is general, and subsumes several previous superposition modulation-based schemes [4] [5] as its special cases. We classify braid coding into the regenerative and the nonregenerative types, characterize their properties, and show that the key lies in the appropriate choice of weights and constraint length of braid coding.

In *nonregenerative* braid coding, each user takes in the relay data without any decoding (or detection) effort, and blends it right into the fresh data. The advantage is the operational simplicity on the user end and the ability to achieve a full diversity gain [4], but the decoding complexity at the common destination can be high, and there is a good chance for error propagation (when the inter-user channels are less than desirable). We formulate the scheme as a real-domain recursive convolutional code $1/(a_0 + a_1 D)$, where D stands for the delay, and a_0 and a_1 are the weights factors for the fresh data and the relay data, respectively; In comparison, *regenerative* braid coding requires each user to decode and clean up the relay data, before reassembling some of them together with its fresh data. We show that the scheme can be formulated as a general real-domain non-recursive convolutional code $(b_0+b_1D+\cdots+b_mD^m)$ of memory m. When $m=1$, the regenerative braid code implements the same superposition modulation discussed in [5]. Using the pair-wise error probability (PEP) as a figure of merit, we derive the globally optimal values for the weights a_i and b_i that achieve the PEP optimality in every transmission and for arbitrary m. In previous related literature, the values are obtained only from numerical method for 2-to-1 system.

We demonstrate a modified Viterbi algorithm for the common destination to efficiently decode all the data for *nonregenerative* braid coding at the destination. The key is a balance between performance gain (which may favor a larger m) and complexity (which favors a smaller m). Through free-distance analysis and computer simulations, we recommend $m = 2$ and 3, and demonstrate their performance advantages over the nonregenerative case and conventional superposition modulation cooperative schemes.

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II. Braid Coding Cooperative Scheme

The proposed braid code works for general M-to-1 cooperative systems, but for ease of proposition, our discussion below focuses on $M=2$.

Let S_1 and S_2 be the two sources taking turns to communicate to the common destination D. Half-duplex mode is assumed at both users. Suppose each communication session consists of N equal-length time slices, and each time slice consists of two equal halves assigned to S_1 and S_2 respectively. Since user cooperation is most useful where time diversity is hard to get, we consider slow fading such that all the channel state information (CSI) remains invariant within each communication session (but changes independently between sessions). We assume that all the CSIs are known to the respective receiver.

Let subscript $i \in \{1,2\}$ be the user index and subscript $k \in \{1,2,\cdots N\}$ be the time index. Let $s_{i,k} \in \{\pm 1\}$ and $x_{i,k} \in \mathcal{R}$ be the fresh data and the transmitted signal from S_i at time k, respectively, and let $y_{i,k}$ and $r_{i,k}$ be the corresponding reception at the other user and at the destination, respectively. The idea to achieve full rate is to have each user superposition its fresh data with the relay data, using appropriate braiding schemes and weights.

• **Nonregenerative Braid Coding** $1/(a_0+a_1D)$.

Here, each user takes in what it hears from the other user as is (without any decoding or signal processing), and blends it with its fresh data via superposition modulation [4]. Mathematically, we have

$$\begin{aligned}
k=1: \mathsf{S}_1 &: x_{1,1}=a_0 s_{1,1},\\
\mathsf{S}_2 &: x_{2,1}=a_0 s_{2,1}+a_1' y_{1,1}=a_0 s_{2,1}+a_1'(h_0 x_{1,1}+z_{1,1}),\\
k=2: \mathsf{S}_1 &: x_{1,2}=a_0 s_{1,2}+a_1' y_{2,1}=a_0 s_{1,2}+a_1'(h_0 x_{2,1}+z_{2,1}),\\
\mathsf{S}_2 &: x_{2,2}=a_0 s_{2,2}+a_1' y_{1,2}=a_0 s_{2,2}+a_1'(h_0 x_{1,2}+z_{1,2}),
\end{aligned}$$

and so on, where h_0 is the Rayleigh CSI for the inter-user channel, $z_{i,k} \sim \mathcal{N}(0,\sigma_0^2)$ is the additive white Gaussian noise (AWGN) for the inter-user channel (assuming channel reciprocity), and a_0 and a_1' are the weights or the power allocation assigned to the fresh data and the relay data, respectively.

Let $a_1 = a_1' h_0$ be the channel-adjusted weight for the relay data, N be the session size. The signals transmitted by the users, $\mathbf{x} = [x_{1,1}, x_{2,1}, \cdots, x_{1,N}, x_{2,N}]^T$, can then be rewritten in a compact matrix form as:

$$\mathbf{x} \approx a_0 \begin{bmatrix} 1, & 0, & 0, & \cdots & 0\\ a_1, & 1, & 0, & \cdots & 0\\ a_1^2, & a_1, & 1, & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ a_1^{2N-1}, & a_1^{2N-2}, & a_1^{2N-3}, & \cdots & 1 \end{bmatrix} \begin{bmatrix} s_{1,1}\\ s_{2,1}\\ s_{1,2}\\ \vdots\\ s_{2,2N} \end{bmatrix}, \quad (1)$$

where a_0 can function as transmission power scalar.

The corresponding reception at the destination becomes [1]:

$$\mathbf{r} \approx \mathbf{H}\mathbf{x} + \mathbf{n} \quad (2)$$

where $\mathbf{n}$ is the noise vector following $\mathbf{n} \sim \mathbf{N}(\mathbf{0},\mathbf{\Sigma})$, $\mathbf{\Sigma}$ and $\mathbf{H}$ are $2N$-by-$2N$ diagonal square matrix , with diagonal $(\sigma_1^2,\sigma_2^2,\sigma_1^2,\sigma_2^2...\sigma_1^2,\sigma_2^2)$ and $(h_1,h_2,h_1,h_2,...h_1,h_2)$.

From the coding perspective, the nonregenerative scheme is similar in spirit to the recursive code $1/(a_0+a_1D)$ followed by a cyclic-2 fading channel, see the linear shift register (LSR) representation in Fig. 1-left. It requires minimal effort on the user side, but since the resultant real-domain trellis has an growing number of states with time, the overall code is not linear-time decodable at the destination.

• **Regenerative Braid Coding** $(b_0+b_1D+\cdots+b_mD^m)$**.**

In the regenerative case, each user performs progressive decoding on what it hears from the system, and re-packs some of them together with its fresh data using appropriate power allocating. We consider an adaptive cooperation between the source nodes, which means one source node would only help another when it correctly decodes the new information sent by another node. Or a new session will start. For example, if the session size lasts to N and each user superposes its fresh data with m previous source data (of which $\lfloor \frac{m}{2} \rfloor$ belong to itself and $\lceil \frac{m}{2} \rceil$ belong to the other user), then the signals that is successively transmitted by the two user will take the following matrix form[2].

$$\mathbf{x}= \begin{bmatrix} b_0, & 0, & 0, & 0, & 0, & \cdots & 0\\ b_1, & b_0, & 0, & 0, & 0, & \cdots & 0\\ b_2, & b_1, & b_0, & 0, & 0, & \cdots & 0\\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & 0\\ b_m, & \cdots & b_2, & b_1, & b_0, & \cdots & 0\\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0\\ 0, & \cdots & b_m & \cdots & b_2 & b_1 & b_0 \end{bmatrix} \begin{bmatrix} s_{1,1}\\ s_{2,1}\\ s_{1,2}\\ \vdots\\ \cdot\\ \vdots\\ s_{2,2N} \end{bmatrix}. \quad (3)$$

In general, the braid code seen by the common destination takes the form of a real-domain nonrecursive convolutional code $(b_0 + b_1D + \cdots + b_mD^m)$. When $m=1$, the cooperative scheme using regenerative braid coding regresses to the conventional superposition modulation [5]. An example LSR for $m=2$ is shown in Fig. 1-right. Comparing to the nonregenerative case, here the code has a fixed number of states (2^m) in the trellis, and the destination can therefore resort to the modified Viterbi algorithm (will be detailed in next section) to decode all the data efficiently and optimally. (The nonregenerative code must use a higher-complexity algorithm such as the list decoding.) Further, although the users have also performed decoding in each of their cooperation stage, their decoding involves only data subtraction (signal cancellation) – provided that each user is provisioned with m memories to store the historic source data – and hence has an extremely low complexity.

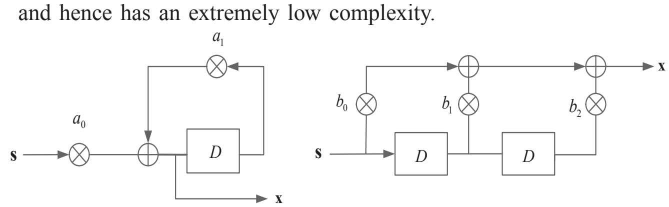

Fig. 1: LSR structure of nonregenerative (left) and regenerative $m=2$ (right) braid coding.

III. Analysis and Code Optimization

A. Optimal Weights a_i and b_i

The performance of the braid coding and hence the cooperative gain closely depend on the choice of the weights. [4] was the first to demonstrate an example of nonregenerative braid coding, and [5] [8] presented a regenerative case with memory $m=1$ for two-user system. These papers also suggested a few empirical weight choices, but lack analytical results. By formulation the signal-based cooperative schemes using braid coding, we provide a rigorous

[1]For nonregenerative braid coding, we calculate the average transmission power of each block $\mathbf{x}$ via stochastic averaging, and scale it to ensure that the overall transmission power is $2NP$.

[2]In regenerative braid coding, the transmission power is scaled to P for every transmission session.

derivation of the optimal weights that simultaneously achieve per-transmission optimality and per-session optimality below, where the optimality is measured in terms of the pairwise error probability of the two nearest neighbors in the signal constellation (worst-case PEP), and can be applied to multi-user system.

Theorem 1: Under a given power constraint, the optimal amplitude shift keying (ASK) that minimizes the worst-case PEP is one that has a uniform constellation.

Proof: This theorem can be easily proven by contradiction. The steps are omitted to save space. ∎

In our braid coding, each user essentially transmits an ASK-modulated signal – possibly with a different constellation size – every time. The question then is whether it is possible or how to find appropriate values of a_i's and b_i's such that the nonregenerative/regenerative code will achieve a uniform signal constellation *every time* of the transmission.

Theorem 2: Consider nonregenerative braid coding among users with negligible inter-user channel noise. The choice $a_1 = 1/2$ (and arbitrary nonzero a_0) will guarantee a uniform ASK constellation in every transmission.

Proof: In the nonregenerative case, the users take turns to transmit an ever-increasing ASK constellation – each time the size doubles that of the previous one. Specifically, the signals transmitted by S_1 and S_2 at time k are

$$x_{1,k} = a_0\big(s_{1,k} + a_1 s_{2,k-1} + a_1^2 s_{1,k-1} + ... + a_1^{2k-2} s_{1,1}\big), \quad (4)$$

$$x_{2,k} = a_0\big(s_{2,k} + a_1 s_{1,k} + a_1^2 s_{2,k-1} + ... + a_1^{2k-1} s_{1,1}\big), \quad (5)$$

where $s_{i,j} \in \{+1, -1\}$ for $i = 1, 2$ and $j = 1, 2, \cdots, k$. Clearly, a_0 is just a scalar that does not affect the signal spacing whatsoever. To show $a_1 = 1/2$ will consistently produce a uniform constellation, it is sufficient to show that the set $\mathcal{X}_n = \{s_0 + \frac{1}{2}s_1 + \frac{1}{2^2}s_2 + \cdots + \frac{1}{2^n}s_n : s_i \in \{+1, -1\}, i = 1, 2, \cdots, n\}$ is uniform for all non-negative integer n. ∎

Remark: Recall that $a_1 = a_1' H_0$, where H_0 is the inter-user channel fading coefficient. This suggests that it is enough for the respective receiving user (and no need for the common destination) to know the inter-user CSI H_0. For ease of discussion, we have assumed that the fading coefficients remain unchanged during a session; but H_0 does not have to be invariant (nor does H_1 or H_2. As long as the respective user compensates for H_0 by choosing the right weight $a_1' = \frac{1}{2H_0}$, the signals are bounded between $-2a_0$ and $2a_0$, and the common destination can guarantee to receive optimal signal every time throughout the session.

Theorem 3: Consider memory-m regenerative braid coding among users. The choice $b_i = \frac{1}{2}b_{i-1}$ ($i = 1, 2, \cdots, m$, and arbitrary positive b_0) will guarantee a uniform ASK constellation in every transmission.

Proof: In memory-m regenerative coding, the signals transmitted by S_1 and S_2 are given in (3). The choice $b_i = 1/2 b_{i-1}$ will lead to

$$\mathbf{x} = b_0 \begin{bmatrix} 1, & 0, & 0, & 0, & 0, & \cdots & 0 \\ \frac{1}{2}, & 1, & 0, & 0, & 0, & \cdots & 0 \\ \frac{1}{2^2}, & \frac{1}{2}, & 1, & 0, & 0, & \cdots & 0 \\ \vdots & \ddots & \ddots, & \ddots, & 0, & \vdots & 0 \\ \frac{1}{2^m}, & \cdots & \frac{1}{2^2}, & \frac{1}{2}, & 1, & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0, & \cdots, & \frac{1}{2^m}, & \cdots & \frac{1}{2^2}. & \frac{1}{2}. & 1 \end{bmatrix} \begin{bmatrix} s_{1,1} \\ s_{2,1} \\ s_{1,2} \\ \vdots \\ \cdot \\ \vdots \\ s_{2,2N} \end{bmatrix},$$

where each row of $\mathbf{x}$ constitutes a transmission. To see each transmission corresponds to a uniform ASK, it is enough to show $\mathcal{X}_n = \{s_0 + \frac{1}{2}s_1 + \frac{1}{2^2}s_2 + \cdots + \frac{1}{2^n}s_n : s_i \in \{+1, -1\}, i = 1, 2, \cdots, n\}$ is uniform for $n = 0, 1, \cdots, m$; and in the proof of Theorem 2, we have shown this for $n = 0, 1, \cdots, \infty$. ∎

B. Modified Viterbi Decoder of Regenerative Code

The finite memory in the regenerative code not only allows the users to perform simple cancellation-based decode-and-forward (and can therefore clean up the inter-user channel noise), but also allows the common destination to perform efficient Viterbi decoding on a trellis of 2^m states.

Fig. 2 demonstrates an example of such a trellis with $m = 2$, where each branch is associated with an input of ± 1 and an output of $c = \pm b_0 \pm b_1 \pm b_2$. The initial two time stages, where the branches are associated with output $\pm b_0$ and $\pm b_0 \pm b_1$, respectively, are not shown. The branch metric is calculated as

$$m_{i,k} = \frac{(r_{i,k} - h_i c)^2}{\sigma_i^2}. \quad (6)$$

The branch metric is accumulated to form the path/state metric. It is worth noting that the overall does not necessarily end in the all-zero state, and the code is therefore a "non-terminating" code. The complexity of the decoder is $O(2N2^m)$ for a communication session with N cooperative rounds.

A short summary of this ML Viterbi decoding process goes in Algorithm 1.

Algorithm 1 ML decoding algorithm I.

Input: Reception $\mathbf{r}$ from the relay-destination transmission.
Output: Binary decisions of the original source data $\mathbf{s}$.
Initialization:
A 4-state trellis corresponding to the $(1+D+D^2)$ analog convolutional code is constructed, as shown in Fig. 2.
In the first stage of the trellis, the two branches leaving the state (-1, -1) are marked with $-b_0$ (upper branch) and $+b_0$ (lower branch), respectively (and all the other branches can be ignored). In the second stage, the branches leaving the state (-1, -1) and (+1, -1) are marked with $-b_0-b_1$, b_0-b_1, $-b_o+b_1$, and $+b_0+b_1$ (from top down), respectively. Starting from the third time instant, the trellis is expanded in full and the branches labels are shown as Fig. 2.
All the state metrics are pre-set to zero.
Trellis Decoding:
Starting from the (-1,-1) state, the decoder performs the usual Viterbi algorithm along the trellis, where each branch metric is computed using (6).
The survival path leading into any state is the one having the smaller cumulative metric so far, and the other competing path with a larger cumulative metric is eliminated (random choice in case of a tie).
After all the state metrics are computed, the state at time $2N$ with the smallest state metric is declared as the final survivor, The binary input bits corresponding to this survival path are declared as the decoding decision.

C. Optimal Memory Size m

Not only do the weights b_i's, but the memory size m also directly affects the code performance (as well as the complexity). We now identify the optimal m that leads to the best overall regenerative braid code, and we do so by evaluating the free distance of the corresponding trellis.

Theorem 4: The (Euclidean) free distance for a regenerative braid code $(b_0 + b_1 D + \cdots + b_m D^m)$ is $d_{free}(m) = 2\sum_{j=0}^{m} b_j$.

Proof: Let $\mathbf{s} = [\cdots, s_t, s_{t+1}, \cdots]$ be the source sequence that was transmitted (the correct path); and let the competing path $\tilde{}= [\cdots, \tilde{s}_t, \tilde{s}_{t+1}, \cdots]$ diverge from at time stage t (i.e. $s_t \neq \tilde{s}_t$). Consider encoding $\mathbf{s}$ and $\tilde{\mathbf{s}}$ using the linear shift register. Let $v_1, v_2, \cdots, v_m$ be the values of the registers $D^1, D^2, \cdots, D^m$ at time t, respectively. We have the following codeword for $\mathbf{s}$ (starting at time k):

$$\mathbf{c}(\mathbf{s}) = \underbrace{\left[b_0, b_1 b_2, \cdots, b_m\right]}_{\mathbf{b}} \underbrace{\begin{bmatrix} s_t, & s_{t+1}, & s_{t+2}, & \cdots \\ v_1, & s_t, & s_{t+1}, & \cdots \\ v_2, & v_1, & s_t, & \cdots \\ v_3, & v_2, & v_1, & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ v_m, & v_{m-1}, & v_{m-2}, & \cdots \end{bmatrix}}_{\mathbf{S}}.$$

Similarly, we have the codeword $\mathbf{c}(\tilde{\mathbf{s}}) = \mathbf{b}\tilde{\mathbf{S}}$ for source sequence $\tilde{\mathbf{s}}$, and the Euclidean distance between them are:

$$d = |\mathbf{b}(\mathbf{S} - \tilde{\mathbf{S}})|,$$

$$= \mathbf{b} \begin{bmatrix} |s_t - \tilde{s}_t|, & |s_{t+1} - \tilde{s}_{t+1}|, & |s_{t+2} - \tilde{s}_{t+2}|, & \cdots \\ 0, & |s_t - \tilde{s}_t|, & |s_{t+1} - \tilde{s}_{t+1}|, & \cdots \\ 0, & 0, & |s_t - \tilde{s}_t|, & \cdots \\ 0, & 0, & 0, & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ 0, & 0, & 0, & \cdots \end{bmatrix}.$$

Clearly, d is minimized when $\tilde{s}_t \neq s_t$, but $\tilde{s}_j = s_j$, $\forall j \neq t$, in which case $(\mathbf{S}–\tilde{\mathbf{S}}) = [diag(|s_t - \tilde{s}_t|), \mathbf{0}] = [diag(2), \mathbf{0}]$, and the free distance becomes $2b_0 + 2b_1 + \cdots + 2b_m$. ■

Corollary 1: Consider a per-transmission power constraint of E for a memory-m braid code with optimal weights $b_i = 0.5 b_{i-1}$. We have $d_{free} = 2\sqrt{3P}\sqrt{1 - \frac{2}{2^{m+1}+1}}$.

As indicated by Theorem 4 and Corollary 1, a larger m leads to a larger d_{free} and hence a better P_{dfree}, but the gain quickly hits a diminishing return for $m \geq 3$. Considering the increasing decoding complexity, we therefore recommend $m = 2$ or 3 as the best choice for regenerative braid coding.

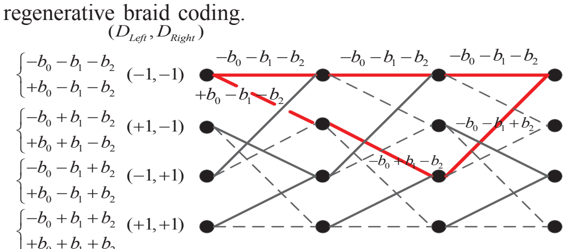

Fig. 2: Trellis for regenerative code $(b_0 + b_1 D + b_2 D^2)$ (solid lines are associated with input -1 and dashed lines are associated with input $+1$)

IV. Simulations

Our simulation employs quasi-static flat Rayleigh fading for all channels. BPSK modulation is adopted in both user. The proposed nonregenerative braid codes, the regenerative braid codes with $m = 1, 3$ and different weights, conventional scheme in [5], the non-cooperative scheme and the time-division cooperation (where each user uses 4ASK and spares half of its time to relay the other user's data) are compared in Fig. 3. Since channel CSIs remain constant in a session, time-division achieves the same diversity order of 2 as braid coding, but as we see from simulations, it falls short in power gain. All the braid coding schemes can reach full diversity gain 2. Fig. 3 shows that braid coding schemes with our optimal weight coefficients and larger constraint length performs better than the original proposed superposition modulation scheme in [5]. The power gain increases when the value of m increases.

The BER performance of braid coding cooperative scheme in 3-user cooperative system is depicted in Fig. 4. Assume all the inter-source channels have the same average SNR. Round robin scheduling scheme is used by the source nodes. It is shown that m=2 braid coding cooperative scheme is able to achieve more diversity than conventional m=1 schemes in [5]. In addition, we also evaluate several other choices of weights for braid coding, and in each case, it is clear that the optimal weights we derived outperform all the others.

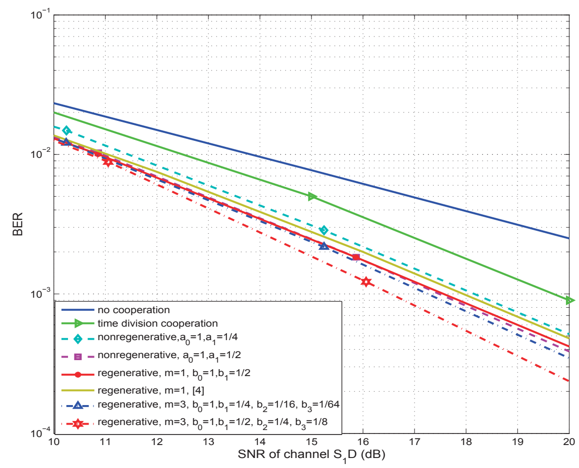

Fig. 3: BER vs. SNR (dB) of S_1-D channel for 2-user braid coding cooperative systems with different weights, $\mathrm{SNR}_{\mathsf{S}_1\mathsf{D}} = \mathrm{SNR}_{\mathsf{S}_2\mathsf{D}}$, $\mathrm{SNR}_{\mathsf{S}_1\mathsf{S}_2} = \mathrm{SNR}_{\mathsf{S}_1\mathsf{D}} + 20\mathrm{dB}$.

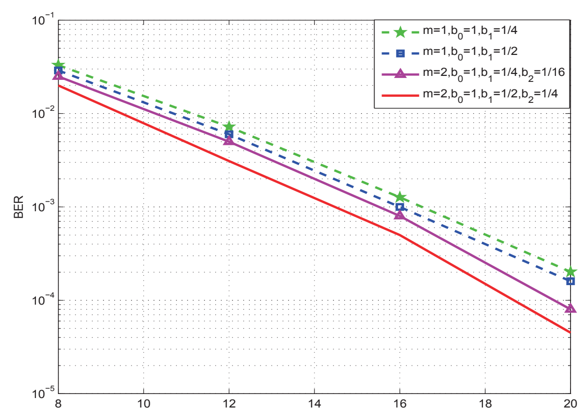

Fig. 4: BER vs. SNR (dB) of S_1-D channel for 3-user braid coding cooperative systems with different weights, $\mathrm{SNR}_{\mathsf{S}_1\mathsf{D}} = \mathrm{SNR}_{\mathsf{S}_2\mathsf{D}} = \mathrm{SNR}_{\mathsf{S}_3\mathsf{D}}$, $\mathrm{SNR}_{\mathsf{S}_1\mathsf{S}_2} = \mathrm{SNR}_{\mathsf{S}_1\mathsf{S}_3} = \mathrm{SNR}_{\mathsf{S}_2\mathsf{S}_3} = \mathrm{SNR}_{\mathsf{S}_1\mathsf{D}} + 15\mathrm{dB}$.

V. Conclusion

We have proposed a general class of signal-superposition-based cooperative schemes using braid coding for multi-user one destination system. Two subclasses, regenerative and nonregenerative braid coding, are considered, and the optimal weights and optimal memory-size are analyzed. By formulating the cooperative scheme using braid coding, a ML decoder with linear complexity to the message length is proposed. The proposed scheme easily generalizes to M users. Simulations corroborate the analysis.

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