APPROACH TO FRAME-MISALIGNMENT IN PHYSICAL-LAYER NETWORK CODING

B. Nguyen, D. Haley, Y. Chen and T. Chan

Institute for Telecommunications Research
University of South Australia
Email: nguqy007@mymail.unisa.edu.au, {David.Haley, Ying.Chen, Terence.Chan}@unisa.edu.au

ABSTRACT

By exploiting superimposed signals at relays, physical-layer network coding (PNC) could significantly improve the throughput of wireless communications systems. However, it is challenging to achieve perfect synchronization between superimposed signals at the relay. In this paper, we propose an approach to resolve the issue of frame-misalignment in PNC using type-I Euclidean geometry low-density parity-check (EG-LDPC) codes with cyclic prefix or zero padding. To estimate the arrival delay between two transmitted signals at the relay, Gold sequences are adopted as pilot sequences in our transmit frames. Simulation results show that our approach can effectively resolve the frame-misalignment issue in PNC.

***Index Terms*—** PNC, cyclic prefix, zero padding.

1. INTRODUCTION

The proposal of PNC [1] has attracted a surge of interest [2, 3, 4, 5]. The simplest system to which PNC can be applied is the two way relay channel (TWRC) shown in Fig. 1. In such a system, the end-nodes A and B exchange data via the relay node R. Considering a traditional time division multiple access (TDMA) transmission scheme, four time-slots are required for nodes A and B to exchange a total of two data frames via node R.

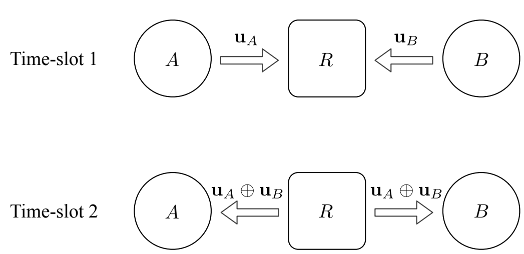

Fig. 1. A three-node TWRC applying the PNC concept.

In the PNC system, only two time-slots are required to exchange the same amount of data. In the first time-slot, referred to as the uplink phase, nodes A and B transmit their data frames simultaneously to the relay R. Using a special technique, referred to as PNC mapping [4], the relay R can map the superimposed signals into a network-coded sequence. In the second time-slot, referred to as the downlink phase, node R broadcasts the network-coded sequence to nodes A and B. From the received sequence, nodes A and B can remove their self-information to obtain the data sent to them. Hence, throughput of traditional wireless communication systems could be doubled if PNC is employed. To achieve that theoretical limit, it is often assumed that data frames and symbols are perfectly aligned, and there are also no carrier-frequency offset or carrier-phase offset when the signals overlap at the relay. However, achieving such synchronization is challenging.

Lu and Liew proposed using repeat-accumulate (RA) codes and a special decoding algorithm based on belief propagation (BP) to deal with phase and symbol asynchronies jointly [6]. With that approach, they claimed that symbol and phase asynchronies could bring positive effects to PNC systems. However, in their set-up, frame-misalignment was not considered. In 2009, Wang et al. employed convolutional codes, zero padding technique and a Viterbi decoder to resolve frame-misalignment in PNC [7]. Their approach will not be able to decode the data if the frame-misalignment exceeds the length of the zero-padding sequences inserted. Recently, the log-domain generalized sum-product-algorithm (Log-G-SPA) has been proposed for PNC systems [8]. By employing the Log-G-SPA algorithm, cyclic low-density parity-check (LDPC) codes and cyclic-redundancy-check (CRC), this system could decode the data in the presence of frame-misalignment, but will suffer a performance loss. Particularly, frame-misalignment can only be estimated and resolved after the superimposed signals are successfully decoded at the relay. In addition, multiple verifications of CRC also introduce more complexity to the system.

In this paper, we present a scheme using type-I EG-LDPC codes [9] with cyclic prefix (CP) or zero padding (ZP) in order to resolve frame-misalignment in PNC systems. We also propose using Gold sequences [10] as pilot sequences in the transmit frames to estimate arrival delay between the two superimposed signals at the relay. With knowledge of the delay and the protection from CP or ZP, the relay can employ a conventional sum-product decoder [11] with log-likelihood-ratios (LLR) tailored for PNC mapping as inputs. Hence, a

978-1-4673-6997-8/15/$31.00

network-coded codeword can be retrieved from the superimposed signals. The estimated delay and the network-coded codeword are then broadcast to the end-nodes. Using these information and their self-information, the end-nodes can decode the data sent to each other.

The remainder of this paper is organized as follows. Sec. 2 describes our system model. Our proposed approach to resolving frame-misalignment in PNC systems is described in Sec. 3 and Sec. 4. Simulation results are provided in Sec. 5. Finally, Sec. 6 concludes the paper.

2. SYSTEM MODEL

Considering the three-node TWRC described in Fig. 1. In the system, nodes A and B exchange data via node R because there is no direct link connecting them. The nodes cannot transmit and receive signals simultaneously. In a PNC system, during the uplink phase, nodes A and B will concurrently transmit their data frames to node R. In the downlink phase, node R will broadcast a network-coded sequence, which is produced using the superimposed signals. Then nodes A and B can use their self-information to extract the desired data from the network-coded codeword sent by node R.

We denote $\mathbf{u}_i$ where $i \in \{A, B\}$ as the binary information sequence of length k from node i. Let C be a (n, k) linear code employed at all nodes. After encoding $\mathbf{u}_i$ with respect to C, its corresponding binary codeword is denoted as $\mathbf{v}_i = (\mathbf{v}_i[1], \ldots, \mathbf{v}_i[n])$ where $\mathbf{v}_i[j]$, $j \in \{1, \ldots, n\}$, is the j-th bit in $\mathbf{v}_i$. The modulated signal[1] of $\mathbf{v}_i$ is denoted as $\mathbf{x}_i = (\mathbf{x}_i[1], \ldots, \mathbf{x}_i[n])$ where $\mathbf{x}_i[j]$ is the j-th symbol in $\mathbf{x}_i$.

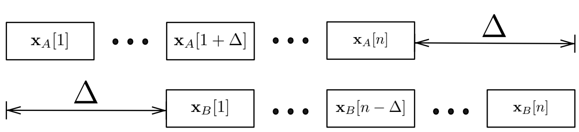

Fig. 2. Superimposed signals at the relay node.

Fig. 2 describes the situation when $\mathbf{x}_A$ and $\mathbf{x}_B$ overlap at node R. Assume without loss of generality that $\mathbf{x}_A$ is ahead of $\mathbf{x}_B$ by Δ symbols. Perfect symbol-alignment is also assumed. According to Fig. 2, the signal received at node R (denoted by $\mathbf{y}_R$) is a vector of length $n + \Delta$ such that

$$\begin{aligned}\mathbf{y}_R \;=\; & (\mathbf{x}_A[1], \ldots, \mathbf{x}_A[1+\Delta] + \mathbf{x}_B[1], \ldots, \\ & \mathbf{x}_A[n] + \mathbf{x}_B[n-\Delta], \ldots, \mathbf{x}_B[n]) + \mathbf{w}_R \qquad (1)\end{aligned}$$

where $\mathbf{w}_R$ is the additive white Gaussian noise (AWGN) at node R, considered across $\mathbf{y}_R$ with zero mean and variance of σ^2. Without noise, when $\Delta = 0$, we have $\mathbf{y}_R[j] = \mathbf{x}_A[j] + \mathbf{x}_B[j]$. Let $\mathbf{v}_R$ be the outcome of the PNC mapping such that $\mathbf{v}_R[j] = 1 - |\mathbf{y}_R[j]|/2$. Then, it can be seen immediately that $\mathbf{v}_R[j] = \mathbf{v}_A[j] \oplus \mathbf{v}_B[j]$ where $\oplus$ denotes the binary exclusive OR (XOR) operation. Since C is linear, $\mathbf{v}_R$ is still a codeword and can be decoded even in the presence of noise. When $\Delta \neq 0$, $\mathbf{y}_R[j] \neq \mathbf{x}_A[j] + \mathbf{x}_B[j]$ and $\mathbf{v}_R[j] \neq \mathbf{v}_A[j] \oplus \mathbf{v}_B[j]$. The vector produced by the PNC mapping, $\mathbf{v}_R$, is not a valid codeword in C. Thus, node R cannot decode $\mathbf{v}_R$ properly. Our approach to resolving this problem caused by frame-misalignment in PNC is presented in the next section.

[1] We assume without loss of generality binary phase shift keying (BPSK) modulation is adopted such that $\mathbf{x}_i[j] \in \{1, -1\}$.

3. PROPOSED APPROACH TO FRAME-MISALIGMENT

3.1. Cyclic LDPC Codes and Cyclic Prefixes

Our approach is based on the use of cyclic codes. Let C be a (n, k) type-I EG-LDPC code. A CP corresponding to the codeword $\mathbf{v}_i$ is the last l symbols of $\mathbf{v}_i$. A packet $\mathbf{p}_i$ is produced by appending $\mathbf{v}_i$ to its CP, i.e., $\mathbf{p}_i = (v_{n-l+1}, \ldots, v_n, v_1, \ldots, v_n)$. After that, BPSK modulation is applied to $\mathbf{p}_i$ to produce $\mathbf{x}_i = (x_{n-l+1}, \ldots, x_n, x_1, \ldots, x_n)$. When $\mathbf{x}_A$ and $\mathbf{x}_B$ are misaligned by Δ symbols, Fig. 3 illustrates our approach to resolving the frame-misalignment issue by using CP. Node R can obtain a vector of length n, $\mathbf{y}_R^{CP}$, from the received signals, $\mathbf{y}_R$, where

$$\begin{aligned}\mathbf{y}_R^{CP} \;=\; & (\mathbf{x}_A[1] + \mathbf{x}_B[n-\Delta+1], \ldots, \\ & \mathbf{x}_A[n] + \mathbf{x}_B[n-\Delta]) + \mathbf{w}_R^{CP} \qquad (2)\end{aligned}$$

and $\mathbf{w}_R^{CP}$ is the AWGN.

Let $\mathbf{v}_i^{(j)} = (\mathbf{v}_i[n-j+1], \ldots, \mathbf{v}_i[n], \mathbf{v}_i[1], \ldots, \mathbf{v}_i[n-j])$. Since $\mathbf{v}_A, \mathbf{v}_B^{(\Delta)} \in \mathsf{C}$ which is cyclic, $\mathbf{v}_R = \mathbf{v}_A \oplus \mathbf{v}_B^{(\Delta)}$ also belongs to C and hence can be properly decoded from $\mathbf{y}_R^{CP}$ at node R using the algorithm discussed in Sec. 3.3.

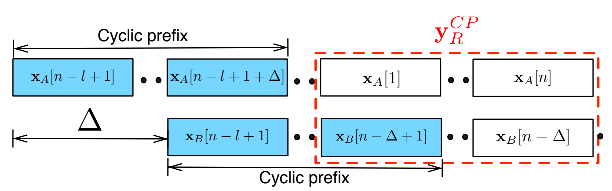

Fig. 3. PNC using cyclic prefixes.

So far, Δ is assumed to be known by node R. The estimation of Δ will be discussed in Sec. 4. In the downlink phase, $\mathbf{v}_R$ and Δ are broadcast to nodes A and B. Note that a CP is not required in the downlink phase. At node A, upon receiving $\mathbf{v}_R$ and Δ, $\mathbf{v}_A$ is extracted from $\mathbf{v}_R$ to obtain $\mathbf{v}_B^{(\Delta)}$. Node A can then retrieve $\mathbf{v}_B$ and decode it to obtain $\mathbf{u}_B$. Node B can follow a similar procedure to retrieve $\mathbf{u}_A$. The next section will propose using ZP as an alternative to CP.

3.2. Zero Padding

Considering a system similar to that described in the previous section, node i transmits $\mathbf{x}_i = (\mathbf{0}, x_1, \ldots, x_n)$, where $\mathbf{0}$ is a zero-vector of length l, and $x_1, \ldots, x_n$ are the BPSK modulated symbols of $\mathbf{v}_i$. Therefore, instead of transmitting CP, node i will keep 'silent' for l symbol periods. Note that, unlike the CP approach, this approach does not introduce overhead to the system in terms of transmit power. When $\mathbf{x}_A$ and $\mathbf{x}_B$ are misaligned by Δ symbols, Fig. 4 illustrates our approach to resolving the frame-misalignment issue by using ZP. With knowledge of Δ, node R can retrieve the non-overlapped symbols, i.e., $(\mathbf{x}_B[n-\Delta+1], \ldots, \mathbf{x}_B[n]) + \mathbf{w}'_R$, and add them to the start of the portion containing $\mathbf{v}_A$, i.e., $(\mathbf{x}_A[1], \ldots, \mathbf{x}_A[1+\Delta]+\mathbf{x}_B[1], \ldots, \mathbf{x}_A[n]+\mathbf{x}_B[n-\Delta])+\mathbf{w}''_R$, where $\mathbf{w}'_R$ and $\mathbf{w}''_R$ denote the AWGN corresponding to each of those portions respectively. The result of that addition is

$$\mathbf{y}_R^{ZP}[j] = \begin{cases} \mathbf{x}_A[j]+\mathbf{x}_B[n-\Delta+j]+\mathbf{w}'_R[j]+\mathbf{w}''_R[j] & 1 \leqslant j \leqslant \Delta \\ \mathbf{x}_A[j]+\mathbf{x}_B[j-\Delta]+\mathbf{w}''_R[j] & \Delta < j \leqslant n \end{cases} \quad (3)$$

Note that the extra amount of noise in $\mathbf{y}_R^{ZP}[j]$, for $1 \leqslant j \leqslant \Delta$, will make the system perform worse than that using CP. Node R can follow the algorithm described in Sec. 3.3 to decode $\mathbf{v}_R = \mathbf{v}_A \oplus \mathbf{v}_B^{(\Delta)}$ from $\mathbf{y}_R^{ZP}$.

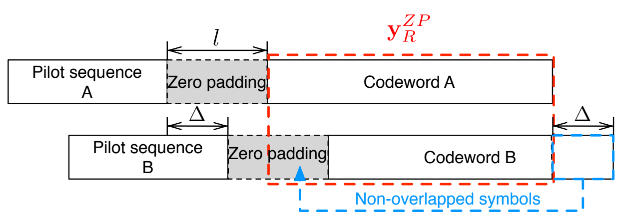

Fig. 4. PNC using zero padding.

In the downlink phase, by following the steps discussed in the previous section, nodes A and B can retrieve $\mathbf{u}_B$ and $\mathbf{u}_A$ respectively. The next section provides a general description of the method we used to decode the superimposed signals at node R.

3.3. Decoding Superimposed Signals

To decode the superimposed signals at node R, we employ a well-known technique in PNC systems, which is referred to as joint channel-decoding and network-coding (JCNC) in [8, 12], or XOR-CD in [4]. Hence, the LLR input of the decoder can be calculated from the received signal as [7]:

$$LLR = \ln\left\{\frac{1}{2}\left[\exp\left(\frac{-2y_R-2}{\sigma^2}\right)+\exp\left(\frac{2y_R-2}{\sigma^2}\right)\right]\right\} \quad (4)$$

A conventional sum-product decoder may be used at node R. Based on the LLR inputs calculated from (4), the decoder attempts to retrieve the network-coded codeword, i.e., $\mathbf{v}_R$, rather than individual codewords sent by nodes A and B. However, performance of JCNC is lower than that of other techniques, such as arithmetic sums channel-decoding-network-coding (AS-CNC) [4], generalized sum-product algorithm (G-SPA) [12] and Log-G-SPA [8]. These other techniques achieve higher performance at the expense of higher complexity at the relay. The next section will discuss the utilization of Gold sequences together with the approaches presented in Sec. 3.1 and Sec. 3.2.

4. FRAME STRUCTURE AND DELAY TOLERANCE

Our proposed method in Sec. 3 strictly requires the knowledge of Δ at node R. To estimate Δ, two distinct Gold sequences [10] of the same length are assigned to nodes A and B respectively. The pre-assigned Gold sequences are used as the pilot sequences described in Fig. 4. Upon receiving the superimposed signals, node R can perform two cross-correlations with respect to the Gold sequence of node A and that of node B respectively. Based on the cross-correlation outcomes, node R can estimate where each individual signal starts, and Δ can be determined. The results in [13] show that Δ can be estimated effectively under low-SNR regimes by employing this approach.

From Fig. 4, it is worth noticing that since pilot is not required for every codeword, overhead introduced by the pilot sequence can be ignored. It can be seen that, when $\Delta \leq l$, the impact of frame-misalignment can be effectively tolerated by utilization of CP or ZP. When $\Delta > l$, with respect to the decoding algorithms discussed in Sec. 3, there are additional $\Delta - l$ erroneous symbols introduced to $\mathbf{y}_R^{CP}$ and $\mathbf{y}_R^{ZP}$. On the contrary, if a cyclic code is used without CP or ZP, there will be an extra Δ erroneous symbols introduced to the decoding process at the relay. Since C is cyclic, $\mathbf{y}_R^{CP}$ and $\mathbf{y}_R^{ZP}$ are still decodable to produce $\mathbf{v}_R$; however, performance of the system degrades proportionally with Δ. Note that we only discuss the uplink phase in this section, the readers interested in the downlink phase can refer to [13] for more details. The next section will provide simulation results and discussions about the performance of our proposed approaches.

5. SIMULATION RESULTS

In our simulation, at all nodes, the type-I (1023,781) EG-LDPC code is adopted. At each transmitter, a 127-bit Gold sequence is used for synchronization. One packet consists of a 1023-bit codeword and its corresponding 50-bit CP (or ZP), as described in Fig. 4. The overhead introduced by the Gold sequence is assumed to be negligible. At the relay, we use a conventional sum-product decoder for a maximum of 50 iterations. The JCNC technique discussed in Sec. 3.3

is utilized to produce the network-coded codeword at the relay. In all simulations, symbol-alignment is assumed. Channel gains are also assumed to be unity for all links between the nodes, and BPSK is adopted for all transmissions. We only focus on the error probability when the relay reproduces the network-coded codeword from the superimposed signals, which significantly impacts the performance of the PNC system [4]. The performance of our system is measured over the AWGN channel for each value of E_b/N_0, i.e., energy per information bit over the received noise variance measured in dB. We assume the received noise at the relay has the variance of $\sigma^2 = N_0/(2RE_b)$, where R is the rate of the used code.

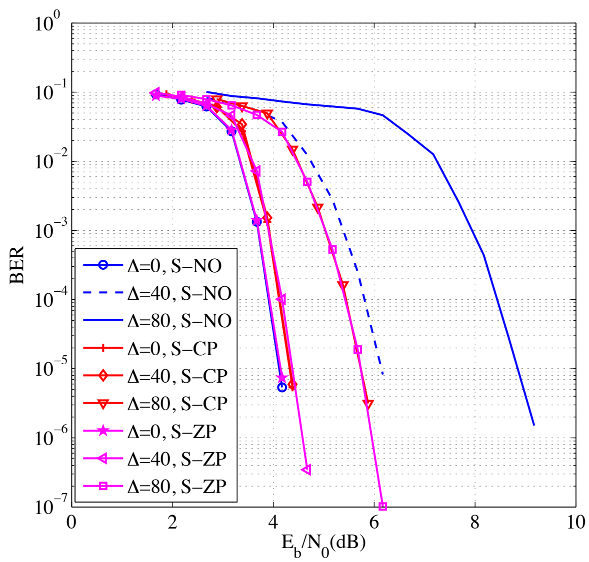

Fig. 5. BER performance of the system using no compensation, CP and ZP at different arrival delays.

Fig. 5 compares bit-error-rate (BER) performances of the systems using no compensation, CP and ZP, which are represented by the curves labeled S-NO, S-CP and S-ZP respectively. When $\Delta = 0$, the systems do not suffer from frame-misalignment. However, S-CP suffers a penalty of about $0.2dB$ due to the overhead introduced by CP. When $\Delta = 40$, at BER of 10^{-5}, S-NO degrades approximately $2dB$ because of additional 40 erroneous symbols introduced to the decoder (as discussed in Sec. 4). With CP, S-CP does not suffer from the frame-misalignment. Although Δ is smaller than length of the ZP, S-ZP degrades about $0.2dB$. That is because of the extra noise introduced to decoding process of the system using ZP (as discussed in Sec. 3.2). When $\Delta = 80$, there is an extra amount of 80 erroneous symbols introduced to S-NO, which causes a performance degradation in S-NO of about $4.7dB$ compared to that when $\Delta = 0$. In the CP or ZP systems, that amount is only 30 erroneous symbols. That makes S-CP and S-ZP degrade by approximately $1.4dB$ and $1.6dB$ respectively, compared to those when $\Delta = 0$. Overall, with utilization of 50-bit CP (or ZP), performance of the system has been improved by approximately $1.8dB$ and $3.1dB$ when $\Delta = 40$ and $\Delta = 80$ respectively.

As we expect, when the frames are aligned, the system using CP performs worse than other systems due to the overhead introduced by CP. It is worth noticing that although the ZP approach does not introduce overhead to the system in terms of E_b/N_0, it still reduces overall throughput of the system due to the 'silent' period corresponding to ZP. Moreover, greater lengths of CP (or ZP) allow the system to tolerate greater amounts of frame-misalignment. Thus, to select the optimal length of CP (or ZP), one has to consider the channel delay characteristics and the compromise on system throughput.

6. CONCLUSION

This paper presents a PNC system which can operate under the presence of frame-misalignment between the superimposed signals at the relay. The problem caused by frame-misalignment can be resolved by exploiting cyclic and linear properties of the channel codes. We proposed using CP or ZP as a design parameter that offers robustness to frame-misalignment. Particularly, the maximum arrival delay between the superimposed signals (measured in symbol periods) which is tolerable by the system is increased by an amount equal to length of the adopted CP or ZP. To estimate the relative frame-misalignment, we proposed using Gold sequences as pilot sequences for the frames transmitted to the relay. Due to utilization of the more powerful and longer channel code, performance of the system is significantly improved, and the overhead introduced by CP is considerably reduced, compared to the work presented in [13]. In this paper, we only considered the transmission of one packet. The proposed technique, however, is also applicable to the transmission of multiple concatenating packets. Overall, our system successfully illustrates the concept of using CP or ZP with cyclic LDPC codes to resolve the frame-misalignment issue in PNC. In future, we aim to tackle the problem caused by symbol-misalignment in PNC systems, which requires more advanced decoding algorithms at the relay.

Acknowledgment

We would like to thank Associate Professor Sarah Johnson from University of Newcastle for providing us with the parity-check matrices of the type-I EG-LDPC codes discussed throughout this paper. The paper is supported in part by ARC DP150103658.

7. REFERENCES

[1] Z. Shengli, S. Liew, and P. Lam, “Physical layer network coding,” *arXiv preprint arXiv:0704.2475*, 2007.

[2] P. Popovski and T. Koike-Akino, “Coded bidirectional relaying in wireless networks,” in *New Directions in Wireless Communications Research*, pp. 291–316, Springer US, 2009.

[3] B. Nazer and M. Gastpar, “Reliable physical layer network coding,” *Proceedings of the IEEE*, vol. 99, no. 3, pp. 438–460, 2011.

[4] S. Liew, S. Zhang, and L. Lu, “Physical-layer network coding: tutorial, survey, and beyond,” *Physical Communication*, vol. 6, pp. 4–42, 2013.

[5] L. Lu, T. Wang, S. C. Liew, and S. Zhang, “Implementation of physical-layer network coding,” *Physical Communication*, vol. 6, pp. 74–87, Mar. 2013.

[6] L. Lu and S. Liew, “Asynchronous physical-layer network coding,” *IEEE Transactions on Wireless Communications*, vol. 11, no. 2, pp. 819–831, 2012.

[7] D. Wang, S. Fu, and K. Lu, “Channel coding design to support asynchronous physical layer network coding,” *2009 IEEE Global Telecommunications Conference*, pp. 1–6, Nov. 2009.

[8] X. Wu, C. Zhao, and X. You, “Joint LDPC and physical-layer network coding for asynchronous bi-directional relaying,” *IEEE Journal on Selected Areas in Communications*, vol. 31, pp. 1446–1454, Aug. 2013.

[9] Y. Kou, S. Lin, and M. P. C. Fossorier, “Low-density parity-check codes based on finite geometries: a rediscovery and new results,” *IEEE Transactions on Information Theory*, vol. 47, no. 7, pp. 2711–2736, 2001.

[10] Z. Xinyu, “Analysis of M-sequence and Gold-sequence in CDMA system,” *2011 IEEE International Conference on Communication Software and Networks*, pp. 466–468, May 2011.

[11] F. Kschischang, “Factor graphs and the sum-product algorithm,” *IEEE Transactions on Information Theory*, vol. 47, no. 2, pp. 498–519, 2001.

[12] D. Wubben and Y. Lang, “Generalized sum-product algorithm for joint channel decoding and physical-layer network coding in two-way relay systems,” *2010 Global Telecommunications Conference*, pp. 1–5, Dec. 2010.

[13] B. Nguyen, D. Haley, and Y. Chen, “Frame-asynchronous physical-layer network coding using cyclic codes,” *2014 Australian Communications Theory Workshop (AusCTW)*, Feb. 2014.