MAXIMUM EXPECTED ACHIEVABLE RATE COMBINING FOR LIMITED FEEDBACK BLOCK-DIAGONALIZATION

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ABSTRACT

We consider downlink multi-user MIMO transmission based on block-diagonalization precoding with quantized channel state information at the transmitter, obtained through limited feedback. We assume that users are equipped with excess receive antennas, i.e., the number of receive antennas is larger than the number of data streams per user, and propose a novel receive antenna combining method that maximizes an estimate of the expected achievable user rate. By means of simulations, we validate our assumptions and demonstrate significant rate gains compared to existing combiners of similar complexity.

Index Terms— antenna combining, limited feedback, multi-user MIMO, block-diagonalization precoding

1. INTRODUCTION

Spatial multiplexing of multiple users in the multiple-input multiple-output (MIMO) broadcast channel, also known as downlink multi-user MIMO transmission in cellular networks, is a promising technique for achieving high spectral efficiencies [1], but still struggles in practical implementations with many obstacles preventing its widespread deployment. Capacity achieving strategies are based on highly complex techniques, such as, vector perturbation [2] and Tomlinson-Harashima precoding [3], which are hardly realizable with current technology. Practically feasible linear precoding methods, such as, block-diagonalization (BD) precoding [4] and its regularized variant [5], perform well provided the transmitter has sufficiently accurate channel state information (CSI) available [6, 7]. CSI at the transmitter, however, can be very hard to obtain especially in frequency division duplex systems. CSI requirements become less strict if user are provided with excess receive antennas, i.e., the number of receive antennas is larger than the number of data streams per user [8, 9].

In this paper, we consider BD precoding based multi-user MIMO, employing limited CSI feedback from users that are equipped with excess receive antennas. We propose an efficient antenna combining method that improves the throughput performance of BD precoding with limited feedback compared to existing techniques, bringing downlink multi-user MIMO another step closer to practical feasibility.

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Relation to prior work: Receive antenna combining strategies for the MIMO broadcast channel are proposed and investigated in [8, 10, 11] for zero forcing (ZF) beamforming and in [9, 12–14] for BD precoding. These methods are extended in [15] to the MIMO interference channel. In this work, we extend the maximum expected SINR combining (MESC) technique, proposed for single-stream ZF beamforming in [11], to multi-stream BD precoding, by maximizing an estimate of the expected achievable rate of a user under BD precoding. Correspondingly, we denote the proposed technique as maximum expected achievable rate combining (MERC). We furthermore derive the corresponding optimal CSI quantization metric for Grassmannian quantization, which can be evaluated ahead of antenna combining, thus facilitating efficient implementations.

Notation: Vectors and matrices are represented with lower- and upper-case bold-face letters, respectively. The conjugate-transpose of matrix $\mathbf{A}$ is $\mathbf{A}^{\mathrm{H}}$, the Moore-Penrose pseudo-inverse is $\mathbf{A}^{\#}$, the trace is $\mathrm{tr}\left(\mathbf{A}\right)$ and the ℓ_2-norm of vector $\mathbf{a}$ is $\|\mathbf{a}\|_2$. The space spanned by the columns of matrix $\mathbf{A}$ is written as $\mathrm{span}(\mathbf{A})$ and the left null space is $\mathrm{null}\left(\mathbf{A}\right)$. The complex-valued Gaussian distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ is denoted as $\mathcal{CN}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$.

2. SYSTEM MODEL

We consider downlink multi-user MIMO transmission, where U users are served in parallel through spatial multiplexing by a single transmitter equipped with N_t transmit antennas. For simplicity we assume that all users are provided with N_r receive antennas and are served over $L \leq N_r$ streams each. We are interested in the practically important case that the number of receive antennas is smaller than the number of transmit antennas: $N_r < N_t$. This assumption is reasonable for cellular networks, as base stations are commonly equipped with far more antennas than users, for reasons of complexity and available space. Furthermore, we assume that the spatial multiplexing capabilities of the transmitter are fully exploited, that is, the total number of streams is equal to the number of transmit antennas: $N_t = UL$. This simplifying assumption

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allows us to exclude scheduling issues from the present work and to focus on the study of the performance of the proposed antenna combiner. The input-output relationship of user u, excluding the antenna combiner, is

$$\mathbf{y}_u = \mathbf{H}_u^{\mathrm{H}} \mathbf{F}_u \mathbf{x}_u + \mathbf{H}_u^{\mathrm{H}} \sum_{\substack{j=1 \\ j \neq u}}^{U} \mathbf{F}_j \mathbf{x}_j + \mathbf{z}_u \in \mathbb{C}^{N_r \times 1}, \quad (1)$$

with $\mathbf{x}_u \in \mathbb{C}^{L \times 1}$, $\|\mathbf{x}_u\|_2 = 1$ denoting the symbol vector intended for user u. The transmit symbols are mapped onto the transmit antennas employing the linear precoder $\mathbf{F}_u \in \mathbb{C}^{N_t \times L}$, $\operatorname{tr}\left(\mathbf{F}_u \mathbf{F}_u^{\mathrm{H}}\right) = P_t/U$, with P_t representing the total transmit power, which is assumed to be equally distributed among users. The transmit signal is received over channel matrix $\mathbf{H}_u \in \mathbb{C}^{N_t \times N_r}$; notice, we employ the conjugate-transpose of $\mathbf{H}_u$ in (1) to simplify later notations. The additive complex-valued Gaussian receiver noise is captured in vector $\mathbf{z}_u \sim \mathcal{CN}\left(\mathbf{0}, \sigma_z^2 \mathbf{I}_{N_r}\right)$. To separate the intended signal from multi-user interference, antenna combiner $\mathbf{G}_u \in \mathbb{C}^{N_r \times L}$ is applied by the user: $\mathbf{r}_u = \mathbf{G}_u^{\mathrm{H}} \mathbf{y}_u \in \mathbb{C}^{L \times 1}$. We denote the product of channel matrix and antenna combiner as the effective user channel: $\mathbf{H}_u^{(\mathrm{e})} = \mathbf{H}_u \mathbf{G}_u \in \mathbb{C}^{N_t \times L}$.

We assume that BD precoding [16] is applied by the transmitter to ideally achieve interference-free transmission to the U users in parallel. BD precoding was originally proposed for the transmission of N_r streams per user; however, as we are interested in the transmission of $L \leq N_r$ streams, it is sufficient if only an L dimensional subspace of the channel matrix $\mathbf{H}_u$ is kept free of interference, because this subspace can be filtered-out by the antenna combiner $\mathbf{G}_u$.

Given the antenna combiners $\mathbf{G}_u, \forall u$, the precoding matrices of all users $u \in \{1, \ldots, U\}$ are obtained from

$$\mathbf{F}_u = \sqrt{\frac{P_t}{UL}} \tilde{\mathbf{F}}_u, \quad \tilde{\mathbf{F}}_u \triangleq \operatorname{span}\left(\mathbf{B}_u \mathbf{B}_u^{\mathrm{H}} \mathbf{H}_u^{(\mathrm{e})}\right), \quad (2)$$
$$\mathbf{B}_u \triangleq \operatorname{null}\left(\overline{\mathbf{H}}_u\right), \ \overline{\mathbf{H}}_u = \left[\mathbf{H}_1^{(\mathrm{e})}, \ldots, \mathbf{H}_{u-1}^{(\mathrm{e})}, \mathbf{H}_{u+1}^{(\mathrm{e})}, \ldots, \mathbf{H}_U^{(\mathrm{e})}\right].$$

with $\triangleq$ defining an orthonormal basis for the term on the right-hand side; e.g., matrix $\mathbf{B}_u \in \mathbb{C}^{N_t \times (N_t - (U-1)L)}$ is an orthonormal basis for the left null space of the other users' channels. The best L dimensional subspace within $\operatorname{span}(\mathbf{B}_u)$, in terms of maximizing the achievable transmission rate of user u, is represented by matrix $\tilde{\mathbf{F}}_u$. As we consider a fully loaded system $N_t = UL$, however, $\operatorname{span}(\tilde{\mathbf{F}}_u) = \operatorname{span}(\mathbf{B}_u)$. The product $\left(\mathbf{H}_u^{(\mathrm{e})}\right)^{\mathrm{H}} \mathbf{F}_u$ is in general not diagonal; hence, unequal power-loading over the L transmit streams of user u is not reasonable and we thus consider *equal power allocation*.

On the other hand, if we consider the precoders $\mathbf{F}_u, \forall u$ as given, optimal interference aware antenna combiners can be determined by the users. There hence exists an interdependency between precoders and antenna combiners, which can be resolved with a joint optimization at a central entity or employing iterative approaches [17, 18], requiring a substantial amount of signaling overhead between users and transmitter. In this work, however, we follow the practically feasible approach proposed in [12], in which the users select the effective channels $\mathbf{H}_u^{(\mathrm{e})}$ already beforehand, reducing the dimensionality of the required CSI feedback from $N_t \times N_r$ to $N_t \times L$. Also, with this approach Grassmannian quantization can be applied to enable efficient limited feedback operation [19, 20]. This is because each matrix $\mathbf{H}_j^{(\mathrm{e})}$ in (2) can be replaced with any arbitrary other matrix that spans the same space [9].

3. ANTENNA COMBINING

3.1. Review of Existing Combiners

Below, we briefly review for the readers' convenience two established antenna combining techniques, that is, maximum eigenmode transmission (MET) and subspace quantization based combining (SQBC), which are comparable to the proposed method in terms of complexity and feedback overhead; please see [9] for detailed proofs and derivations.

The goal of MET antenna combining is to generate an L dimensional effective channel $\mathbf{H}_u^{(\mathrm{e})}$ that maximizes the achievable transmission rate of a user in the absence of multi-user interference [9, 10]. This is achieved with

$$\mathbf{G}_u^{(\mathrm{MET})} = \mathbf{V}_u^{(L)} = [\mathbf{V}_u]_{:,1:L}, \quad \mathbf{H}_u = \mathbf{U}_u \boldsymbol{\Sigma}_u \mathbf{V}_u^{\mathrm{H}}, \quad (3)$$

with $[\mathbf{V}_u]_{:,1:L}$ denoting the matrix of the L right singular vectors of $\mathbf{H}_u$, corresponding to the largest singular values. The resulting effective channel is $\mathbf{H}_u^{(\mathrm{e})} = \mathbf{U}_u^{(L)} \boldsymbol{\Sigma}_u^{(L)}$, with $\mathbf{U}_u^{(L)} = [\mathbf{U}_u]_{:,1:L}$ and $\boldsymbol{\Sigma}_u^{(L)} = [\boldsymbol{\Sigma}_u]_{:,1:L}$. To calculate the BD precoder for $\mathbf{H}_u^{(\mathrm{e})}$, the space spanned by matrix $\mathbf{U}_u^{(L)}$ must be provided to the transmitter. Because $\operatorname{span}(\mathbf{U}_u^{(L)})$ represents a point on the Grassmann manifold of L dimensional subspaces in the N_t dimensional Euclidean space, Grassmannian quantization is applicable to efficiently provide the CSI to the transmitter over limited capacity feedback channels. As proposed in [7], the chordal distance is the appropriate quantization metric for quantization of $\mathbf{U}_u^{(L)}$

$$\hat{\mathbf{H}}_u = \underset{\mathbf{Q}_i \in \mathcal{Q}_L^{(N_t)}}{\arg\min} \ \mathrm{d}_c^2\left(\mathbf{Q}_i, \mathbf{U}_u^{(L)}\right), \quad (4)$$

$$\mathrm{d}_c^2\left(\mathbf{Q}_i, \mathbf{U}_u^{(L)}\right) = L - \operatorname{tr}\left(\mathbf{Q}_i^{\mathrm{H}} \mathbf{U}_u^{(L)} \left(\mathbf{U}_u^{(L)}\right)^{\mathrm{H}} \mathbf{Q}_i\right), \quad (5)$$

$$\mathcal{Q}_L^{(N_t)} = \left\{\mathbf{Q}_i \in \mathbb{C}^{N_t \times L} \,\middle|\, \mathbf{Q}_i^{\mathrm{H}} \mathbf{Q}_i = \mathbf{I}_L, i \in \{1, \ldots, 2^B\}\right\}, \ (6)$$

with $\mathcal{Q}_L^{(N_t)}$ denoting the Grassmannian quantization codebook, consisting of 2^B orthonormal bases $\mathbf{Q}_i$ that span L dimensional subspaces in the N_t dimensional Euclidean space. The BD precoder is then calculated by replacing $\mathbf{H}_j^{(\mathrm{e})}$ in (2) with $\hat{\mathbf{H}}_j$. Employing random vector quantization (RVQ) and assuming i.i.d. Rayleigh fading, it is shown in [9] that the number of feedback bits B must grow linearly with the logarithmic signal to noise ratio (SNR) with a slope of $L(N_t - L)$, to achieve the same multiplexing gain as with perfect CSI at the transmitter.

If N_r is larger than L, this required feedback overhead can be substantially reduced by applying SQBC instead of MET. With this method, the antenna combiner $\mathbf{G}_u$ and the quantized effective channel subspace $\hat{\mathbf{H}}_u$ are jointly determined such as to minimize the resulting quantization error [8, 9]

$$\left\{\hat{\mathbf{H}}_u, \mathbf{G}_u^{(\text{SQBC})}\right\} = \underset{\mathbf{Q}_i \in \mathcal{Q}_L^{(N_t)}, \mathbf{G} \in \mathbb{C}^{N_r \times L}}{\arg\min} \mathrm{d}_c^2\left(\mathbf{Q}_i, \tilde{\mathbf{H}}_u^{(e)}\right), \quad (7)$$

$$\tilde{\mathbf{H}}_u^{(e)} \triangleq \mathrm{span}\left(\mathbf{H}_u \mathbf{G}\right).$$

As shown in [9], to solve problem (7) quantization can be performed independently of antenna combining by minimizing first the chordal distance with respect to the full channel matrix

$$\hat{\mathbf{H}}_u = \underset{\mathbf{Q}_i \in \mathcal{Q}_L^{(N_t)}}{\arg\min} \mathrm{d}_c^2\left(\mathbf{Q}_i, \mathbf{U}_u\right). \quad (8)$$

The corresponding antenna combiner is then obtained as

$$\mathbf{G}_u^{(\text{SQBC})} = \mathbf{H}_u^{\#} \hat{\mathbf{H}}_u. \quad (9)$$

Eq. (8) and (9) thus represent the solutions of (7). In that way, the slope of the feedback bit scaling law to achieve the same multiplexing gain as with perfect CSI at the transmitter is reduced from $L(N_t - L)$ to $L(N_t - N_r)$ [9].

MET combining is appropriate whenever the noise dominates the residual multi-user interference, that is, at low SNR and in case the CSI quantization is very accurate, and SQBC vice versa. To tradeoff between these two methods, MESC has been proposed in [11] for single-stream transmission per user, i.e., ZF beamforming. For BD precoding, SQBC with dimensionality adaptation [14] is able to obtain the maximum of MET and SQBC. This, however, is not the best possible performance, as demonstrated below.

3.2. Maximum Expected Achievable Rate Combining

In this section, we propose a blind antenna combining method that maximizes the achievable rate of a user under BD precoding; blind hereby refers to the users not knowing the actual precoders that are applied by the transmitter during transmission. However, the structure of the BD precoder is exploited to estimate the achievable rate based on local CSI. According to [21], the instantaneous achievable rate of user u is

$$R_u = \log_2 \det\Big(\mathbf{I}_L + \mathbf{G}_u^{\mathrm{H}} \mathbf{H}_u^{\mathrm{H}} \mathbf{S}_u \mathbf{H}_u \mathbf{G}_u \left(\mathbf{G}_u^{\mathrm{H}}\left(\sigma_z^2 \mathbf{I}_{N_r} + \mathbf{H}_u^{\mathrm{H}} \mathbf{C}_u \mathbf{H}_u\right) \mathbf{G}_u\right)^{-1}\Big), \quad (10)$$

$$\mathbf{S}_u = \mathbb{E}\left(\mathbf{F}_u \mathbf{x}_u \mathbf{x}_u^{\mathrm{H}} \mathbf{F}_u^{\mathrm{H}}\right), \ \mathbf{C}_u = \sum_{j \neq u} \mathbb{E}\left(\mathbf{F}_j \mathbf{x}_j \mathbf{x}_j^{\mathrm{H}} \mathbf{F}_j^{\mathrm{H}}\right), \quad (11)$$

with $\mathbf{S}_u$ denoting the covariance matrix of the intended signal and $\mathbf{C}_u$ being the interference covariance matrix, both unknown to user u. To estimate $\mathbf{S}_u$, we consider two sets of assumptions: assuming that the set of U users has been selected by a scheduling algorithm such as semi-orthogonal user selection (SUS) [14, 22] from a very large pool of users, the quantized channel subspaces $\hat{\mathbf{H}}_u$ of the served users are close to orthogonal: $\hat{\mathbf{H}}_u^{\mathrm{H}} \hat{\mathbf{H}}_j \approx \mathbf{0}, \forall u \neq j$. Correspondingly, precoder $\mathbf{F}_u$ lies approximately within $\mathrm{span}(\hat{\mathbf{H}}_u)$ and the input covariance matrix is thus obtained as

$$\mathbf{S}_u \approx \hat{\mathbf{S}}_u^{(1)} = \frac{P_t}{UL} \hat{\mathbf{H}}_u \hat{\mathbf{H}}_u^{\mathrm{H}}. \quad (12)$$

The second assumption that we consider is that the user pool only contains U users. Then, we cannot restrict $\mathrm{span}(\mathbf{F}_u)$ further, but have to assume that $\mathrm{span}(\mathbf{F}_u)$ is isotropically distributed in the N_t dimensional Euclidean space, leading to

$$\mathbf{S}_u \approx \hat{\mathbf{S}}_u^{(2)} = \frac{P_t}{UL} \mathbb{E}\left(\tilde{\mathbf{F}}_u \tilde{\mathbf{F}}_u^{\mathrm{H}}\right) = \frac{P_t}{UL} \frac{L}{N_t} \mathbf{I}_{N_t}. \quad (13)$$

To estimate the interference covariance matrix, we exploit knowledge of the precoder construction. Due to BD precoding, the precoders $\mathbf{F}_j$ of the other users are restricted to the left null space of $\hat{\mathbf{H}}_u$. For a fully loaded system ($N_t = UL$), the $N_t - L = (U-1)L$ dimensional left null space of $\hat{\mathbf{H}}_u$ is completely occupied by interference. Correspondingly, the interference covariance matrix can be estimated as

$$\mathbf{C}_u = \sum_{j=1,\, j \neq u}^{U} \mathbf{F}_j \mathbf{F}_j^{\mathrm{H}} \approx \hat{\mathbf{C}}_u = \frac{P_t}{N_t}(\mathbf{I}_{N_t} - \hat{\mathbf{H}}_u \hat{\mathbf{H}}_u^{\mathrm{H}}). \quad (14)$$

Notice that $\hat{\mathbf{S}}_u^{(1)}$ and $\hat{\mathbf{C}}_u$ are accurate in case users with orthogonal quantized channel subspaces are scheduled.

Given the estimated covariance matrices, we now define the MERC optimization and quantization problem

$$\left\{\hat{\mathbf{H}}_u, \mathbf{G}_u^{(\text{MERC})}\right\} = \underset{\mathbf{Q}_i \in \mathcal{Q}_L^{(N_t)}, \mathbf{G} \in \mathbb{C}^{N_r \times L}}{\arg\max} \hat{R}_u(\mathbf{Q}_i, \mathbf{G}), \quad (15)$$

with $\hat{R}_u(\mathbf{Q}_i, \mathbf{G})$ being the estimated achievable rate with quantized channel subspace $\mathbf{Q}_i$ and antenna combiner $\mathbf{G}$. For fixed $\mathbf{Q}_i$, the rate maximizing antenna combiner is obtained as

$$\mathbf{G}^{(\text{MERC})}(\mathbf{Q}_i) = \underset{\mathbf{G} \in \mathbb{C}^{N_r \times L}}{\arg\max} \frac{\det\Big(\mathbf{G}^{\mathrm{H}}(\mathbf{A}^{(i)} + \mathbf{B})\mathbf{G}\Big)}{\det\Big(\mathbf{G}^{\mathrm{H}} \mathbf{B} \mathbf{G}\Big)}, \quad (16)$$

$$\mathbf{A}^{(1)} = \frac{P_t}{UL} \mathbf{H}_u^{\mathrm{H}} \mathbf{Q}_i \mathbf{Q}_i^{\mathrm{H}} \mathbf{H}_u \ \text{ and } \ \mathbf{A}^{(2)} = \frac{P_t}{UN_t} \mathbf{H}_u^{\mathrm{H}} \mathbf{H}_u, \quad (17)$$

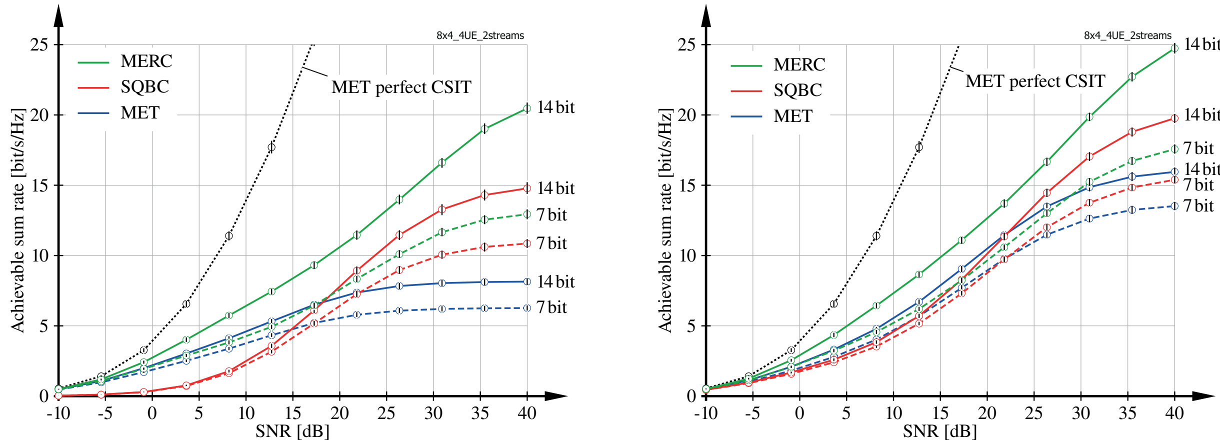
$$\mathbf{B} = \sigma_z^2 \mathbf{I}_{N_r} + \frac{1}{N_t} \mathbf{H}_u^{\mathrm{H}}(\mathbf{I}_{N_t} - \mathbf{Q}_i \mathbf{Q}_i^{\mathrm{H}})\mathbf{H}_u. \quad (18)$$

This multidimensional generalized Rayleigh quotient is maximized by setting $\mathbf{G}$ equal to the stacked L eigenvectors of $\mathbf{B}^{-1}\mathbf{A}^{(i)}$ corresponding to the maximum eigenvalues [23]. However, any other L dimensional $\mathbf{G}$ that lies in the subspace spanned by these L eigenvectors can equivalently be used.

Applying the first set of assumptions mentioned above, i.e., employing matrix $\mathbf{A}^{(1)}$, one specific closed-form solution is

$$\mathbf{G}^{(\text{MERC},1)}(\mathbf{Q}_i) = \mathbf{B}^{-1} \mathbf{H}_u^{\mathrm{H}} \mathbf{Q}_i. \quad (19)$$

Notice that matrix $\mathbf{C} = \mathbf{B}^{-1}\mathbf{A}^{(1)}$ is of rank L and thus the



(a) Interference unaware antenna combining.

(b) Interference aware antenna combining.

Fig. 1: Comparison of the achievable sum rate of the considered antenna combiners, assuming $N_t \times N_r = 8 \times 4$, $L = 2$ and $U = 4$.

space spanned by the L eigenvectors corresponding to the only non-zero eigenvalues is equal to $\mathrm{span}(\mathbf{C})$. With this result it can easily be shown that (19) is a solution of (16), by projecting $\mathbf{G}^{(\mathrm{MERC},1)}(\mathbf{Q}_i)$ onto $\mathrm{span}(\mathbf{C})$

$$\left(\mathbf{C}\left(\mathbf{C}^{\mathrm{H}}\mathbf{C}\right)^{\#}\mathbf{C}^{\mathrm{H}}\right)\mathbf{B}^{-1}\mathbf{H}_u^{\mathrm{H}}\mathbf{Q}_i = \left(\mathbf{C}\left(\mathbf{C}^{\mathrm{H}}\mathbf{C}\right)^{\#}\mathbf{C}^{\mathrm{H}}\right)\mathbf{C}\left(\mathbf{Q}_i^{\mathrm{H}}\mathbf{H}_u\right)^{\#} = \mathbf{B}^{-1}\mathbf{H}_u^{\mathrm{H}}\mathbf{Q}_i. \quad (20)$$

Thus, the same space is spanned by $\mathbf{G}^{(\mathrm{MERC},1)}(\mathbf{Q}_i)$ and $\mathbf{C}$. Plugging (19) back into (15), the quantization metric simplifies

$$\hat{\mathbf{H}}_u = \underset{\mathbf{Q}_i \in \mathcal{Q}_L^{(N_t)}}{\arg\min}\ \log_2\det\left(\sigma_z^2\mathbf{I}_{N_r} + \frac{1}{N_t}\mathbf{H}_u^{\mathrm{H}}(\mathbf{I}_{N_t} - \mathbf{Q}_i\mathbf{Q}_i^{\mathrm{H}})\mathbf{H}_u\right), \quad (21)$$

which can be determined without calculating the antenna combiner for each $\mathbf{Q}_i$, thus significantly reducing computational complexity. The corresponding antenna combiner simply is

$$\mathbf{G}_u^{(\mathrm{MERC},1)} = \left(\sigma_z^2\mathbf{I}_{N_r} + \frac{1}{N_t}\mathbf{H}_u^{\mathrm{H}}(\mathbf{I}_{N_t} - \hat{\mathbf{H}}_u\hat{\mathbf{H}}_u^{\mathrm{H}})\mathbf{H}_u\right)^{-1}\mathbf{H}_u^{\mathrm{H}}\hat{\mathbf{H}}_u. \quad (22)$$

Comparing this results to the SQBC combiner in (9), we observe that the simple pseudo-inverse is now replaced with a term that resembles very much a minimum mean-squared error (MMSE) solution. Notice also that this combiner reduces to MESC in case of single-stream transmission per user $L = 1$.

For the second set of assumptions, i.e., employing matrix $\mathbf{A}^{(2)}$, we have not found a closed-form solution yet, because matrix $\mathbf{B}^{-1}\mathbf{A}^{(2)}$ is of rank N_r and thus requires explicit calculation of the eigenvectors. Quantization is therefore much harder, as the antenna combiner must be calculated for each $\mathbf{Q}_i$. We thus do not apply this practically infeasible method in the following, even though we have observed that it performs slightly better in the simulated case with only U users present, which correspond to the second set of assumptions.

4. PERFORMANCE INVESTIGATION

In Figure 1, we demonstrate the performance of the proposed antenna combiner compared to MET and SQBC in terms of achievable rate. We consider i.i.d. Rayleigh fading channels with receive-side correlation, applying a Kronecker correlation model as defined in [24]. RVQ is applied to quantize the channel subspace [9]. The code for reproduction of the presented results is available at [25]. In Figure 1a, the presented antenna combiners are applied to determine the CSI feedback as well as to separate the intended signal from the interference. We observe substantial rate gains with MERC for both considered quantization codebook sizes $B \in \{7, 14\}$. In Figure 1b, MERC is only applied to determine the CSI feedback, while an interference aware filter is applied to detect the data. This filter is calculated by maximizing the achievable rate (10), employing the actual covariance matrices after precoding. The solution is similar to MERC, just with different covariance matrices. With this interference aware receiver, all methods perform better while MERC still outperforms the others. Notice that in case of single-stream transmission per user $L = 1$, MERC = MESC performs equal to MET at low SNR and equal to SQBC at high SNR. This, however, is not the case with $L > 1$; here, MERC strictly outperforms SQBC, because it explicitly accounts for the interference on the individual streams, instead of minimizing the average interference.

5. CONCLUSIONS

We proposed a novel antenna combining method for block-diagonalization based multi-user MIMO transmission with limited feedback and excess receive antennas. The proposed antenna combiner enables efficient limited feedback operation of block-diagonalization precoding, significantly outperforming existing antenna combiners. The obtained channel state information quantization metric can be evaluated without calculation of the antenna combiner and thus enables practical implementation with reasonable complexity.

References

[1] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. on Info. Theory*, vol. 52, no. 9, pp. 3936 –3964, Sept. 2006.

[2] B. Hochwald, C. Peel, and A. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication-part II: perturbation," *IEEE Transactions on Communications*, vol. 53, no. 3, pp. 537–544, March 2005.

[3] A. Mezghani, R. Hunger, M. Joham, and W. Utschick, "Iterative THP transceiver optimization for multi-user MIMOsystems based on weighted sum-MSE minimization," in *IEEE 7th Workshop on Signal Processing Advances in Wireless Communications*, July 2006, pp. 1–5.

[4] C. Peel, B. Hochwald, and A. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication-part I: channel inversion and regularization," *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 195 – 202, Jan. 2005.

[5] V. Stankovic and M. Haardt, "Generalized design of multi-user MIMO precoding matrices," *IEEE Transactions on Wireless Communications*, vol. 7, no. 3, pp. 953–961, 2008.

[6] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Transactions on Information Theory*, vol. 52, no. 11, p. 5, Nov. 2006.

[7] N. Ravindran and N. Jindal, "Limited feedback-based block diagonalization for the MIMO broadcast channel," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 8, pp. 1473 –1482, Oct. 2008.

[8] N. Jindal, "Antenna combining for the MIMO downlink channel," *IEEE Trans. on Wireless Comm.*, vol. 7, no. 10, pp. 3834 –3844, Oct. 2008.

[9] S. Schwarz and M. Rupp, "Subspace quantization based combining for limited feedback block-diagonalization," *IEEE Transactions on Wireless Communications*, vol. 12, no. 11, pp. 5868–5879, 2013.

[10] M. Trivellato, F. Boccardi, and F. Tosate, "User selection schemes for MIMO broadcast channels with limited feedback," in *65th IEEE Vehicular Technology Conference, Spring 2007*, Dublin, April 2007.

[11] M. Trivellato, F. Boccardi, and H. Huang, "On transceiver design and channel quantization for downlink multiuser MIMO system with limited feedback," *IEEE Journal on selected areas in communications*, vol. 26, No. 8, Oct. 2008.

[12] S. Schwarz and M. Rupp, "Antenna combiners for block-diagonalization based multi-user MIMO with limited feedback," in *IEEE International Conf. on Communications, WS: Beyond LTE-A*, Budapest, June 2013, pp. 127–132.

[13] E. Bjornson, M. Kountouris, M. Bengtsson, and B. Ottersten, "Receive combining vs. multi-stream multiplexing in downlink systems with multi-antenna users," *IEEE Transactions on Signal Processing*, vol. 61, no. 13, pp. 3431–3446, July 2013.

[14] S. Schwarz and M. Rupp, "Evaluation of distributed multi-user MIMO-OFDM with limited feedback," *IEEE Transactions on Wireless Communications*, vol. 13, no. 11, pp. 6081–6094, Aug. 2014.

[15] ——, "Exploring coordinated multipoint beamforming strategies for 5G cellular," *IEEE Access*, vol. 2, pp. 930–946, 2014.

[16] Q. Spencer, A. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. on Signal Processing*, vol. 52, no. 2, pp. 461 – 471, Feb. 2004.

[17] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE transceiver optimization for multiuser MIMO systems: MMSE balancing," *IEEE Transactions on Signal Processing*, vol. 56, no. 8, pp. 3702–3712, Aug 2008.

[18] J. Park, B. Lee, and B. Shim, "A MMSE vector precoding with block diagonalization for multiuser MIMO downlink," *IEEE Transactions on Communications*, vol. 60, no. 2, pp. 569–577, 2012.

[19] S. Schwarz, R. Heath, Jr., and M. Rupp, "Adaptive quantization on a Grassmann-manifold for limited feedback beamforming systems," *IEEE Transactions on Signal Processing*, vol. 61, no. 18, pp. 4450–4462, 2013.

[20] ——, "Adaptive quantization on the Grassmann-manifold for limited feedback multi-user MIMO systems," in *38th International Conference on Acoustics, Speech and Signal Processing*, Vancouver, Canada, May 2013, pp. 5021 – 5025.

[21] R. Blum, "MIMO capacity with interference," *IEEE Journal on Sel. Areas in Comm.*, vol. 21, no. 5, pp. 793–801, 2003.

[22] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-antenna broadcast channels with limited feedback and user selection," *IEEE Journal Sel. Areas in Communications*, 2007.

[23] R. Prieto, "A general solution to the maximization of the multidimensional generalized Rayleigh quotient used in linear discriminant analysis for signal classification," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 6, April 2003, pp. 157–161.

[24] 3GPP, "TSG RAN; Evolved Universal Terrestrial Radio Access (E-UTRA); user equipment (UE) radio transmission and reception," http://www.3gpp.org/ftp/Specs/html-info/36101.htm, Dec. 2010.

[25] "Maximum expected achievable rate combining for limited feedback block-diagonalization," [Online] http://www.nt.tuwien.ac.at/about-us/staff/stefan-schwarz/reproducible-research/.