A BEAMFORMED ALAMOUTI AMPLIFY-AND-FORWARD SCHEME IN MULTIGROUP MULTICAST CLOUD-RELAY NETWORKS

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ABSTRACT

In this paper, we consider a cloud relay network (C-RN) which provides reliable communication between long-distance users. Specifically, we study the amplify-and-forward (AF) schemes in C-RNs. In our scenario setting, with the cloud processor units fully coordinating in the network, the C-RN can be treated as an MIMO relay system. We therefore propose the beamformed (BF) Alamouti AF scheme to provide multigroup multicast information delivery in this network. By applying an Alamouti space-time code structure, the relays adopt two rank-one weights to AF the received signals in two time slots. Then, one more degree of freedom is available compared to the traditional BF AF scheme, and a new fractional semidefinite relaxation (SDR) is obtained from a max-min-fair quality-of-service (QoS) perspective. We prove that the Gaussian randomization algorithm based on the new fractional SDR has the same approximation quality–i.e., on the order of $\sqrt{M}$–as the traditional rank-two SDR approximation in multigroup multicast networks without relays, where M is the number of users served in the network. This result is verified by our numerical experiments.

Index Terms— Cloud relay network (C-RN), multigroup multicast, amplify-and-forward (AF), SDR, rank approximation.

1. INTRODUCTION

In recent years, the cloud radio access network (C-RAN) [1–3] has been considered as a promising network architecture to offer a 1000x increase in capacity to support broadband applications. The key enabling technologies in C-RANs are the cloud processors pool and fronthaul and backhaul links, which coordinate all the base-stations in different cells as a cloud base-station, so that users can be served in a jointly optimized way. This idea can be extended to relay networks to facilitate information delivery between long-distance users, and we call this type of networks a *cloud relay network* (C-RN).

The C-RN we consider in this paper is a typical one-way relay network including transmitter nodes, receiver nodes and relay nodes, where each node has a single antenna. Similar to C-RANs, a distinguishing characteristic for C-RNs is that the relay nodes are connected via a cloud central processing pool with fronthaul and backhaul links. In this work, we particularly consider the scenarios where the cloud relays enable a reliable information delivery by means of amplifying-and-forwarding (AF) received signals.[1] In general, there are various scenario settings for C-RNs. The specific C-RN we consider here is essentially an MIMO relay system, since the cloud relays in our problem can fully cooperate with each other and share information within the cloud. In the literature, there are many works involving MIMO relay AF designs [4–12]. Herein we consider the multigroup multicast transmission from a max-min-fair (MMF) quality-of-service (QoS) perspective. Assume that the channel state information is perfectly known in the C-RNs. The classic approach in this context is to apply a rank-one beamformed (BF) AF scheme [5]. The resulting design problem can be formulated as a fractional quadratically constrained quadratic problem (QCQP), which is NP-hard in general [13, 14]. By applying the semidefinite relaxation (SDR) technique [15], the MMF signal-to-interference-plus-noise ratio (SINR) associated with the SDR solution is at least on the order of $1/M$ times that associated with the optimal solution to the fractional QCQP [13, 14], where M is the number of users served in the network. To further improve system performance, we propose an Alamouti code in the AF structure such that two independent rank-one weights are adopted to AF the received signals in two time slots. We call such a scheme *the BF Alamouti AF*. The performance of the BF Alamouti AF scheme is provably no worse than the BF AF scheme. Moreover, by applying SDR, a new fractional SDP is obtained. Our analysis shows that the SDR-based BF Alamouti AF scheme is guaranteed to be optimal when $M \leq 4$, which is better than the BF AF scheme, for which optimality is guaranteed only when $M \leq 3$. Moreover, when $M > 4$, the MMF SINR of the SDR solution to the BF Alamouti AF scheme scales on the order of $1/\sqrt{M}$ times that of the optimal solution, which is also better than the BF AF scheme.

It is worth mentioning that in [16], the authors proposed a rank-two BF AF scheme in relay networks. However, the scenario they focused on is the distributed relay AF in a single-group multicasting network, while our work deals with a more general multigroup multicasting setting for both the distributed relay AF and MIMO relay AF (we present the MIMO relay case as an example in this paper). Moreover, we provide a provable result for the proposed scheme. From a theoretical perspective, our contribution in this paper is to generalize the rank-two approximation bounds for fractional SDR problems in our previous work [13]. In particular, we prove that for fractional SDRs, the approximation obtained by two independent rank-one random vectors can exhibit the same quality as a rank-two approximation. These results are verified by numerical simulations.

2. THE MULTIGROUP MULTICAST C-RN MODEL AND THE BEAMFORMING AF SCHEME

Consider a C-RN system shown in Figure 1, where we assume that the relay network architecture consists of three components: 1) the centralized processors units (PUs) pool with backhauls, 2) the optical transport network, i.e., the fronthaul links, and 3) relays in the network. The relays can acquire the channel state information (CSI)

[1] The cloud relays can also decode-and-forward (DF) the received signals, but this is not in the scope of this paper.

of its own links and pass them to the PUs pool via fronthaul. The PUs pool is the centralized processing center in the network. Throughout this paper, the scenario we consider is that the capacity of the fronthaul-backhaul links is unlimited and the relays fully cooperate and share received signals with each other. Under this setting, it is easy to see that the C-RN can be treated as an MIMO relay system.

We therefore consider multigroup multicast information delivery in the target MIMO relay system. Specifically, assume that G single-antenna transmitters aim for sending G independent information to G groups of single-antenna users. There are m_k users in group-k that request the same information, while users in different groups request different information. Thus, we have in total $\sum_{k=1}^{G} m_k = M$ users in this network. Since the transmitters and receivers are far apart, there is no direct link between them and reliable information delivery is enabled by L distributively-located single-antenna relays, which AF the received signals from sources to destinations. In this way, the information can be delivered in two time phases:
1) Phase I: *the transmitters send information to relays.* The receive model for the source-to-relay link is described as

$$\mathbf{r}(t) = \sum_{j=1}^{G} \boldsymbol{f}_j s_j(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{r}(t) = [r^1(t), ...r^\ell(t), ..., r^L(t)]^T$ with $r^\ell(t) = \sum_{j=1}^{G} f_j^\ell s_j(t) + n^\ell(t)$ being the received signal at relay-ℓ; $s_j(t)$ is the common information for group-j with $\mathbb{E}[|s_j(t)|^2] = P_j$, where P_j is the transmit power at transmitter-j; $\boldsymbol{f}_j = [f_j^1, ..., f_j^\ell, ..., f_j^L]^T$ is the channel from source-j to the relays, where f_j^ℓ is the channel from source-j to relay-ℓ; $\mathbf{n}(t) = [n^1(t), ..., n^\ell(t), ..., n^L(t)]^T$ with $n^\ell(t)$ being the white noise at relay-ℓ with variance σ_ℓ^2.
2) Phase II: *relays process the received signals and then forward them to receivers.* In the literature, a popular AF scheme in this context is to implement rank-one beamforming [5]. Since the relays fully cooperate and share information with each other, the AF process can be given by

$$\mathbf{x}(t) = \mathbf{V}\mathbf{r}(t) = \sum_{\ell=1}^{L} \mathbf{v}^\ell r^\ell(t), \tag{2}$$

where $\mathbf{V} = [\mathbf{v}^1, ..., \mathbf{v}^\ell, ..., \mathbf{v}^L]$ is an AF weighting matrix with $\mathbf{v}^\ell \in \mathbb{C}^L$. Then, the received signal for user-k in group-i is expressed as

$$\begin{aligned} y_{k,i}(t) &= \boldsymbol{g}_{k,i}^H \mathbf{x}(t) + v_{k,i}(t), \quad i = 1, ..., m_k, \quad k = 1, ..., G \quad (3) \\ &= \textstyle\sum_{j=1}^{G} \sum_{\ell=1}^{L} \boldsymbol{g}_{k,i}^H \mathbf{v}^\ell f_j^\ell s_j(t) + \sum_{\ell=1}^{L} \boldsymbol{g}_{k,i}^H \mathbf{v}^\ell n^\ell(t) + v_{k,i}(t), \end{aligned}$$

where $\boldsymbol{g}_{k,i} = [g_{k,i}^1, ..., g_{k,i}^\ell, ..., g_{k,i}^L]^T$ is the channel from the relays to user-(k,i) with $g_{k,i}^\ell$ being the channel from relay-ℓ to user-(k,i) and $v_{k,i}(t)$ is the white noise at user-(k,i) with variance $\sigma_{k,i}^2$. Therefore, the SINR for user-(k,i) can be expressed as

$$\gamma_{k,i} = \frac{P_k \left| \sum_{\ell=1}^{L} \boldsymbol{g}_{k,i}^H \mathbf{v}^\ell f_k^\ell \right|^2}{\sum_{m \neq k} P_m \left| \sum_{\ell=1}^{L} \boldsymbol{g}_{k,i}^H \mathbf{v}^\ell f_m^\ell \right|^2 + \sum_{\ell=1}^{L} \left| \sigma_\ell \boldsymbol{g}_{k,i}^H \mathbf{v}^\ell \right|^2 + \sigma_{k,i}^2}.$$

Assuming that all the channel state information (i.e., $\boldsymbol{f}_k$ and $\boldsymbol{g}_{k,i}$) are perfectly known at the cloud PUs pool and denoting $\mathbf{w} = \mathrm{vec}(\mathbf{V})$, we arrive at the following MMF SINR design problem:

$$\begin{aligned} &(\text{R1BF}) \quad \mathbf{w}^\star = \arg \max_{\mathbf{w} \in \mathbb{C}^{L^2}} \min_{\substack{i=1,...,m_k \\ k=1,...,G}} \gamma_{k,i} = \frac{\mathbf{w}^H \mathbf{A}_{k,i} \mathbf{w}}{\mathbf{w}^H \mathbf{C}_{k,i} \mathbf{w} + 1} \\ &\text{subject to} \qquad \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P, \end{aligned}$$

where P is a given power limit to the AF signal $\mathbf{x}(t)$ such that $\mathbb{E}[\|\mathbf{x}(t)\|^2] \leq P$ and $\mathbf{A}_{k,i}, \mathbf{C}_{k,i}, \mathbf{D}$ are defined as

$$\mathbf{A}_{k,i} = P_k (\boldsymbol{f}_k \otimes \boldsymbol{g}_{k,i})(\boldsymbol{f}_k \otimes \boldsymbol{g}_{k,i})^H / \sigma_{k,i}^2, \tag{4}$$

$$\mathbf{C}_{k,i} = \textstyle\sum_{m \neq k} P_m (\boldsymbol{f}_m \otimes \boldsymbol{g}_{k,i})(\boldsymbol{f}_m \otimes \boldsymbol{g}_{k,i})^H / \sigma_{k,i}^2 + \boldsymbol{\Sigma}_L \otimes (\boldsymbol{g}_{k,i} \boldsymbol{g}_{k,i}^H) \sigma_{v,i}^2, \tag{5}$$

$$\mathbf{D} = (\textstyle\sum_{j=1}^{G} \boldsymbol{f}_j \boldsymbol{f}_j^H + \boldsymbol{\Sigma}_L) \otimes \mathbf{I}_L, \tag{6}$$

where $\boldsymbol{\Sigma}_L = \mathrm{Diag}(\sigma_1^2, ..., \sigma_\ell^2, ..., \sigma_L^2)$ and $\otimes$ denotes the kronecker product. In general, (R1BF) is an NP-hard fractional QCQP [13,14]. We can approximate it via the SDR technique [15]. That is, by letting $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ and then dropping the non-convex rank constraint, a fractional SDR of (R1BF) is obtained as

$$\begin{aligned} &(\text{R1SDR}) \quad \mathbf{W}^\star = \arg \max_{\mathbf{W} \in \mathbb{H}_+^{L^2}} \gamma(\mathbf{W}) \\ &\text{subject to} \qquad \mathbf{D} \cdot \mathbf{W} \leq P, \end{aligned}$$

where $\mathbb{H}_+^N$ is the set of all $N \times N$ positive semidefinite matrices and

$$\gamma(\mathbf{W}) = \min_{\substack{k=1,...,G; \\ i=1,...,m_k}} \frac{\mathbf{A}_{k,i} \cdot \mathbf{W}}{\mathbf{C}_{k,i} \cdot \mathbf{W} + 1}$$

with $\cdot$ being the matrix inner product operator. Problem (R1SDR) serves as a convex relaxation of (R1BF) and it always admits that $\gamma(\mathbf{W}^\star) \geq \gamma(\mathbf{w}^\star \mathbf{w}^{\star H})$, where equality holds whenever (R1SDR) has a rank-one solution. If we have $\mathrm{rank}(\mathbf{W}^\star) > 1$, a Gaussian randomization algorithm in [13, 14] can be applied to generate an approximate rank-one solution $\hat{\mathbf{w}}$. In Proposition 1, we provide SDR approximation bounds for (R1SDR).

Proposition 1. *Let M denote the number of users in relay networks. When $M \leq 3$, SDR can always produce an optimal solution to Problem (R1BF). When $M > 3$, let $\hat{\mathbf{w}}$ be the solution returned by Gaussian randomization algorithm and N be the number of randomizations. Then, with probability at least $1 - (5/6)^N$, we have*

$$\gamma\left(\hat{\mathbf{w}}\hat{\mathbf{w}}^H\right) \geq \frac{\gamma(\mathbf{W}^\star)}{8M(6\log(3)+1)} \geq \frac{\gamma(\mathbf{w}^\star \mathbf{w}^{\star H})}{8M(6\log(3)+1)}.$$

Proposition 1 is the rank-one version of Theorem 1 in [13]. A direct consequence of this proposition is that, SDR-based BF AF scheme works well when $M \leq 3$; otherwise, it may experience an SINR performance degradation and the worst-case degradation rate is on the order of $1/M$. In other words, the BF AF scheme may not work well when the number of users in the system is large.

3. THE RANK-TWO BEAMFORMED ALAMOUTI AF SCHEME FOR CLOUD-RELAY NETWORKS

Previous discussions reveal that the BF AF scheme may not be effective for large scale systems. This motivates us to propose the BF Alamouti AF scheme in the C-RN. The key idea here is to adopt the Alamouti space-time code structure in the AF process. Intuitively, we expect that with the aid of an Alamouti code, one more degree of freedom can be exploited so that we may enhance performance in terms of the worst user's SINR. In this section, we introduce the transmit structure of the BF Alamouti AF scheme and moreover, as a main contribution of this paper, we provide a theoretical analysis for the proposed AF scheme.

To describe the BF Alamouti AF scheme, we parse the transmit signal in two time slots as $\mathbf{s}(m) = [\ s(2m) \ \ s(2m+1)\ ]^T$ and modify the AF signal model at Phase-II (Phase-I remains the same as before) as follows

$$\mathbf{X}(m) = \sum_{\ell=1}^{L} [\mathbf{v}_1^\ell, \mathbf{v}_2^\ell] \mathbf{C}(\mathbf{r}_\ell(m)), \tag{7}$$

where $\mathbf{C} : \mathbb{C}^2 \to \mathbb{C}^{2\times 2}$ is the Alamouti space-time code, $\mathbf{r}_\ell(m) = [\, r^\ell(2m) \;\; r^\ell(2m+1) \,]^T$ and $\mathbf{v}_p^\ell \in \mathbb{C}^L$ is defined in such a way that $\mathbf{V}_p = [\mathbf{v}_p^1, ..., \mathbf{v}_p^\ell, ..., \mathbf{v}_p^L]$ is the AF weighting matrix for time slot p ($p = 1, 2$). Then, the received signal for user-(k, i) is expressed as

$$\begin{aligned} \mathbf{y}_{k,i}(m) &= [y_{k,i}(2m), y_{k,i}(2m+1)] \qquad (8) \\ &= \sum_{\ell=1}^{L} \boldsymbol{g}_{k,i}^H [\mathbf{v}_1^\ell, \mathbf{v}_2^\ell] \mathbf{C}(\mathbf{r}_\ell(m)) + [v_{k,i}(2m), v_{k,i}(2m+1)]. \end{aligned}$$

Let $\mathbf{w}_1 = \mathrm{vec}(\mathbf{V}_1)$, $\mathbf{w}_2 = \mathrm{vec}(\mathbf{V}_2)$. Similar to (R1BF), the MMF design problem for the BF Alamouti AF scheme is formulated as

$$\begin{aligned} \text{(R2BF)} \quad & \max_{\mathbf{w}_1, \mathbf{w}_2 \in \mathbb{C}^{L^2}} \; \min_{\substack{k=1,\dots,G;\\ i=1,\dots,m_k}} \frac{\mathbf{w}_1^H \mathbf{A}_{k,i} \mathbf{w}_1 + \mathbf{w}_2^H \bar{\mathbf{A}}_{k,i} \mathbf{w}_2}{\mathbf{w}_1^H \mathbf{C}_{k,i} \mathbf{w}_1 + \mathbf{w}_2^H \bar{\mathbf{C}}_{k,i} \mathbf{w}_2 + 1} \\ & \text{subject to} \quad \mathbf{w}_1^H \mathbf{D} \mathbf{w}_1 + \mathbf{w}_2^H \mathbf{D} \mathbf{w}_2 \le P, \end{aligned}$$

where $\mathbf{A}_{k,i}, \mathbf{C}_{k,i}$ and $\mathbf{D}$ are defined in (4), (5) and (6), and

$$\bar{\mathbf{A}}_{k,i} = P_k (\boldsymbol{f}_k^* \otimes \boldsymbol{g}_{k,i})(\boldsymbol{f}_k^* \otimes \boldsymbol{g}_{k,i})^H / \sigma_{k,i}^2, \tag{9}$$

$$\bar{\mathbf{C}}_{k,i} = \textstyle\sum_{m \neq k} P_m (\boldsymbol{f}_m^* \otimes \boldsymbol{g}_{k,i})(\boldsymbol{f}_m^* \otimes \boldsymbol{g}_{k,i})^H / \sigma_{k,i}^2 + \boldsymbol{\Sigma}_L \otimes (\boldsymbol{g}_{k,i} \boldsymbol{g}_{k,i}^H) / \sigma_{k,i}^2. \tag{10}$$

We therefore obtain the SDR of (3) as follows:

$$\begin{aligned} \text{(R2SDR)} \quad & (\mathbf{W}_1^\star, \mathbf{W}_2^\star) = \arg \max_{\mathbf{W}_1, \mathbf{W}_2 \in \mathbb{H}_+^{L^2}} \theta(\mathbf{W}_1, \mathbf{W}_2) \\ & \text{subject to} \quad \mathbf{D} \cdot \mathbf{W}_1 + \mathbf{D} \cdot \mathbf{W}_2 \le P, \end{aligned}$$

where

$$\theta(\mathbf{W}_1, \mathbf{W}_2) = \min_{\substack{k=1,\dots,G;\\ i=1,\dots,m_k}} \frac{\mathbf{A}_{k,i} \cdot \mathbf{W}_1 + \bar{\mathbf{A}}_{k,i} \cdot \mathbf{W}_2}{\mathbf{C}_{k,i} \cdot \mathbf{W}_1 + \bar{\mathbf{C}}_{k,i} \cdot \mathbf{W}_2 + 1}.$$

Let $\mathbf{w}_1^\star, \mathbf{w}_2^\star$ be the optimal solution to (R2BF). Clearly, we have $\theta(\mathbf{W}_1^\star, \mathbf{W}_2^\star) \ge \theta(\mathbf{w}_1^\star \mathbf{w}_1^{\star H}, \mathbf{w}_2^\star \mathbf{w}_2^{\star H})$, where equality holds whenever (R2SDR) has rank-one solutions. In the sequel, we will analyze the performance of the BF Alamouti AF scheme by answering three questions.

Question 1: What is the relationship between the convex relaxations (R2SDR) and (R1SDR)?

In our recent paper [13, 17], we show that for traditional multigroup multicasting without relays, rank-one beamforming and the BF Alamouti scheme result in the same SDR problem. The only difference is that rank-one beamforming (BF Alamouti) admits a rank-one (rank-two) approximate solution. This situation is quite different in relay networks. Herein, we actually arrive at two different SDR problems; i.e., (R1SDR) and (R2SDR). It is easy to see that

Proposition 2. *Any feasible solution of (R1SDR) must be feasible solutions of (R2SDR).*

The proof is straightforward: Let $\hat{\mathbf{W}}$ be the any feasible solution to (R1SDR). Then, $(\hat{\mathbf{W}}, \mathbf{0})$ must be a feasible solution to (R2SDR).

Question 2: When is (R2SDR) equivalent to (R2BF)?

Since the proposed BF Alamouti AF scheme increases one degree of freedom in the AF weights, similar to Proposition 1, by exploiting the results in [18], we provide a sufficient condition for the existence of rank-one solutions to (R2SDR) in Proposition 3.

Proposition 3. *Problem (R2SDR) can always produce an optimal solution to Problem (R2BF) when the number of users $M \le 4$.*

This, together with Proposition 2, implies that the BF Alamouti AF scheme is at least no worse than the BF AF scheme when $M \le 4$. While when $M > 4$, non-rank-one $\mathbf{W}_1^\star, \mathbf{W}_2^\star$ may exist. For those non-rank-one cases, we adopt the following algorithm to produce rank-one approximate solutions $\hat{\mathbf{W}}_1$ and $\hat{\mathbf{W}}_2$.

Algorithm 1 Gaussian Randomization Procedure for (R2SDR)

1: **for** $n = 1, 2, \dots, N$ **do**
2: For each $p = 1, 2$, if $\mathrm{rank}(\mathbf{W}_p^\star) = 1$, $\boldsymbol{\xi}_p^n = \mathbf{w}_p^\star$ where $\mathbf{w}_p^\star \mathbf{w}_p^{\star H} = \mathbf{W}_p^\star$; if $\mathrm{rank}(\mathbf{W}_p^\star) > 1$, generate an independent random vector $\boldsymbol{\xi}_p^n \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}_p^\star)$;
3: Scale randomized $\boldsymbol{\xi}_p^n$ to satisfy the power constraint and set $\theta^n = \theta(\boldsymbol{\xi}_1^n \boldsymbol{\xi}_1^{nH}, \boldsymbol{\xi}_2^n \boldsymbol{\xi}_2^{nH})$
4: **end for**
5: Set $n^\star := \arg\max_{n=1,\dots,N} \theta^n$ and output: $\hat{\mathbf{W}}_1 = \boldsymbol{\xi}_1^{n^\star} \boldsymbol{\xi}_1^{n^\star H}$ and $\hat{\mathbf{W}}_2 = \boldsymbol{\xi}_2^{n^\star} \boldsymbol{\xi}_2^{n^\star H}$.

Question 3: What is the approximation quality of (R2SDR)?

Based on Algorithm 1, we have Theorem 1 to identify the approximation bounds for the BF Alamouti AF scheme.

Theorem 1. *For the cases where $\mathrm{rank}(\mathbf{W}_p^\star) > 1$ for some $p = 1, 2$ in (R2SDR), let $\hat{\mathbf{W}}_1, \hat{\mathbf{W}}_2$ be the solutions returned by Algorithm 1. Then, with probability at least $1 - (7/8)^N$, we have*

$$\theta(\hat{\mathbf{W}}_1, \hat{\mathbf{W}}_2) \ge \frac{\theta(\mathbf{W}_1^\star, \mathbf{W}_2^\star)}{16\sqrt{M}(2\log 16)} \ge \frac{\theta(\mathbf{w}_1^\star \mathbf{w}_1^{\star H}, \mathbf{w}_2^\star \mathbf{w}_2^{\star H})}{16\sqrt{M}(2\log 16)},$$

where N is the number of randomizations.

We relegate the proof in the Appendix. This theorem provides the SDR approximation quality of the BF Alamouti AF scheme in the multigroup multicast one-way relay network. It says that if (R2SDR) does not return optimal rank-one solutions, the Gaussian randomization algorithm can generate approximate solutions such that in the worst case, the corresponding objective of (R2BF) scales on the order of $1/\sqrt{M}$ with respect to (w.r.t.) its optimal objective. Obviously, this result is better than that in Proposition 1. This implies that the BF Alamouti AF scheme can outperform the BF AF counterpart.

Remark 1: One may observe that (R2BF) can otherwise be written in the form of (R1BF) by letting $\mathbf{w} = [\mathbf{w}_1; \mathbf{w}_2]$ and $\tilde{\mathbf{A}}_{k,i} = [\mathbf{A}_{k,i}, \mathbf{0}; \mathbf{0}, \bar{\mathbf{A}}_{k,i}]$, and we can get the same approximation bounds as that in Proposition 1. Apparently, this bound is inferior to our result in Theorem 1.

Remark 2: If we set $\bar{\mathbf{A}} = \mathbf{A}$ and $\bar{\mathbf{C}} = \mathbf{C}$ in (R2SDR), Theorem 1 is indeed a generalization of our previous work [13, Theorem 1], where we consider the rank-two approximation quality of the SDR problem (R1SDR) in a multigroup multicast network without relays.

4. SIMULATION RESULTS AND CONCLUSIONS

In this section, we show numerical simulations in C-RNs. Without loss of generality, we assume that each group has an equal number of users, i.e., $m_k = \frac{M}{G}$; the channels are independently generated by $\boldsymbol{f}_k, \boldsymbol{g}_{k,i} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$; the noise power at relays and users are both set to be 0.25; the signal power at each transmitter is 0dB. For each AF scheme, we averaged 100 channel realizations to get the plots. The number of randomizations is set to be $1,000$.

In Figure 2, we vary the power allowed at relays to see the worst user's SINR for different problem formulations. Herein, the number of relays is $L = 4$ and two scenario settings are shown respectively. One is the single group multicasting with $G = 1$ and $M = 16$; the other is multigroup multicasting with $G = 2$ and $M = 12$. For both scenarios, we see that (R2SDR) indeed has a better objective (obj.) value than (R1SDR). Moreover, the BF Alamouti AF scheme based on (R2SDR) can outperform the BF AF scheme based on (R1SDR) in all power regions. The worst user's SINR scaling w.r.t. the number of users is shown in Figure 3 when $P = 10$dB. From the figure, we see that (R2SDR) serves as an upper bound of (R1SDR), which is consistent with Proposition 2. Moreover, it shows that the BF Alamouti AF scheme has a better scaling than the BF AF scheme w.r.t. number of users, which verifies the results in Theorem 1.

To conclude, in this paper we have studied the relay AF schemes in C-RNs. We show that, with a cloud PUs pool fully coordinating in the relay network, the C-RN we consider here can be seen as an MIMO relay system. A novel BF Alamouti AF scheme is therefore proposed for this system, which has a better SINR scaling w.r.t. the number of users, compared to the traditional BF AF counterpart. Our main theoretical result implies that the approximation for Problem (R2SDR) obtained by two independent rank-one vectors exhibits the same quality as a rank-two approximation for Problem (R1SDR). This generalizes the approximation bounds for the existing SDR rank-two approximations.

5. APPENDIX

To prove Theorem 1, we follow the same flow as the proof of Theorem 1 in [13] by replacing Lemmas 1 and 2 in [13] as follows

Lemma 1. *Given Hermitian positive semidefinite matrices* $\mathbf{A}$, $\bar{\mathbf{A}}$, $\mathbf{C}$, $\bar{\mathbf{C}}$, *let* $\boldsymbol{\xi} \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}_1^\star)$, $\boldsymbol{\eta} \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}_2^\star)$ *be independent random vectors. Consider the matrix* $\mathbf{W}_1 = \boldsymbol{\xi}\boldsymbol{\xi}^H$ *and* $\mathbf{W}_2 = \boldsymbol{\eta}\boldsymbol{\eta}^H$. *Then, for any* $\rho \le 1$,

$$\Pr\left(\frac{\mathbf{A}\cdot\hat{\mathbf{W}}_1+\bar{\mathbf{A}}\cdot\hat{\mathbf{W}}_2}{\mathbf{C}\cdot\hat{\mathbf{W}}_1+\bar{\mathbf{C}}\cdot\hat{\mathbf{W}}_2+1} \le \rho\frac{\mathbf{A}\cdot\mathbf{W}_1^\star+\bar{\mathbf{A}}\cdot\mathbf{W}_2^\star}{\mathbf{C}\cdot\mathbf{W}_1^\star+\bar{\mathbf{C}}\cdot\mathbf{W}_2^\star+1}\right) \le \left(\frac{5\rho}{\omega-2\rho}\right)^2,$$

where $\omega \le 1/(\mathrm{rank}((\mathbf{W}_1^\star)^{\frac{1}{2}}\mathbf{A}(\mathbf{W}_1^\star)^{\frac{1}{2}})+\mathrm{rank}((\mathbf{W}_2^\star)^{\frac{1}{2}}\bar{\mathbf{A}}(\mathbf{W}_2^\star)^{\frac{1}{2}}))$ *and* $0 < \rho < \frac{\omega}{2}$. *Moreover, given that* $\mathbf{D}$ *is a Hermitian positive semidefinite matrix, for any* $v \ge 2$, *we have*

$$\Pr\left(\mathbf{D}\cdot\hat{\mathbf{W}}_1 + \mathbf{D}\cdot\hat{\mathbf{W}}_2 \ge v(\mathbf{D}\cdot\mathbf{W}_1^\star + \mathbf{D}\cdot\mathbf{W}_2^\star)\right) \le 2\exp(-\frac{v}{2}).$$

Note that Lemma 1 is a non-trivial extension of Lemmas 1 and 2 in [13]. We omit the proof here due to page limit and will defer it to the full version of this paper.

Now, consider a fixed n in Algorithm 1. Let $\hat{\mathbf{W}}_1 = \boldsymbol{\xi}_1^n\boldsymbol{\xi}_1^{nH}$, $\hat{\mathbf{W}}_2 = \boldsymbol{\xi}_2^n\boldsymbol{\xi}_2^{nH}$. For any $\rho, v > 0$, consider the events

$$F_{k,i} = \left\{\frac{\mathbf{A}_{k,i}\cdot\hat{\mathbf{W}}_1+\bar{\mathbf{A}}_{k,i}\cdot\hat{\mathbf{W}}_2}{\mathbf{C}_{k,i}\cdot\hat{\mathbf{W}}_1+\bar{\mathbf{C}}_{k,i}\cdot\hat{\mathbf{W}}_2+1} \le \rho\frac{\mathbf{A}_{k,i}\cdot\mathbf{W}_1^\star+\bar{\mathbf{A}}_{k,i}\cdot\mathbf{W}_2^\star}{\mathbf{C}_{k,i}\cdot\mathbf{W}_1^\star+\bar{\mathbf{C}}_{k,i}\cdot\mathbf{W}_2^\star+1}\right\},$$

$$E = \left\{\mathbf{D}\cdot\hat{\mathbf{W}}_1 + \mathbf{D}\cdot\hat{\mathbf{W}}_2 \ge v(\mathbf{D}\cdot\mathbf{W}_1^\star + \mathbf{D}\cdot\mathbf{W}_2^\star)\right\}.$$

As in [13, Theorem 1], we bound $\Pr(F)$ and $\Pr(E)$, where $F = \bigcup_{m=1}^M F_{k,i}$. Armed with Lemma 1, by choosing $\omega = 0.5$, $\rho = 1/(16\sqrt{M})$, $v = 2\log 16$, we obtain

$$\Pr(F) \le \sum_{m=1}^M \Pr(F_{k,i}) \le M\left(\frac{5\rho}{0.5-2\rho}\right)^2 < \frac{3}{4}, \quad (11)$$

$$\Pr(E) \le 2\exp(-v/2) = 1/8, \quad (12)$$

where (12) comes from the remarks after the proof of [19, Proposition 2.1]. Thus, we have

$$\Pr\left(\left\{\theta\left(\hat{\mathbf{W}}_1, \hat{\mathbf{W}}_2\right) \ge \rho\theta\left(\mathbf{W}_1^\star, \mathbf{W}_2^\star\right)\right\} \wedge \left\{\mathbf{D}\cdot\hat{\mathbf{W}}_1 + \mathbf{D}\cdot\hat{\mathbf{W}}_2 \le v(\mathbf{D}\cdot\mathbf{W}_1^\star + \mathbf{D}\cdot\mathbf{W}_2^\star)\right\}\right) = 1 - \Pr(E) - \Pr(F) > 1/8.$$

This implies that, with probability at least $1/8$, we have

$$\theta\left(\hat{\mathbf{W}}_1, \hat{\mathbf{W}}_2\right) \ge \frac{\rho}{v}\theta\left(\mathbf{W}_1^\star, \mathbf{W}_2^\star\right) \ge \frac{\theta\left(\mathbf{W}_1^\star, \mathbf{W}_2^\star\right)}{16\sqrt{M}(2\log 16)}.$$

This, together with $\theta(\mathbf{W}_1^\star, \mathbf{W}_2^\star) \ge \theta(\mathbf{w}_1^\star\mathbf{w}_1^{\star H}, \mathbf{w}_2^\star\mathbf{w}_2^{\star H})$, completes the proof.

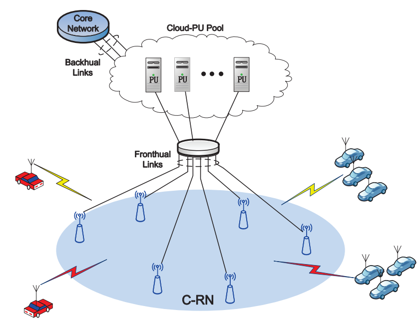

Fig. 1. An example of the cloud relay network.

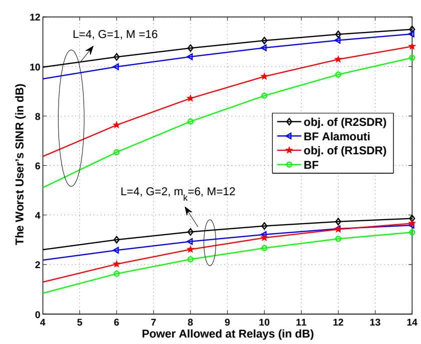

Fig. 2. The worst user's SINR versus power allowed at relays.

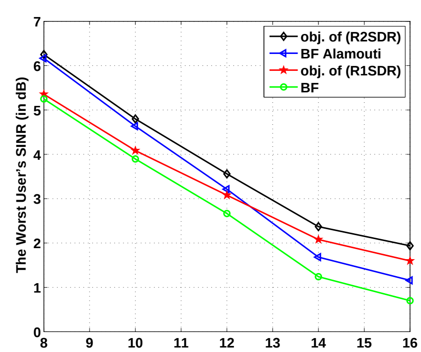

Fig. 3. The worst user's SINR w.r.t. number of users in the C-RN.

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