AN ITERATIVE REWEIGHTED MINIMIZATION FRAMEWORK FOR JOINT CHANNEL AND POWER ALLOCATION IN THE OFDMA SYSTEM

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ABSTRACT

We consider the joint channel and power allocation problem for the OFDMA system. The problem is to find a joint channel and power allocation strategy to minimize the total transmission power subject to quality of service constraints and the OFDMA constraint (i.e, at most one user is allowed to access each channel). Since the problem is generally NP-hard, the idea of the existing algorithms is to heuristically allocate the channel and power resources separately. In this paper, we propose a novel iterative reweighted minimization framework based on an effective relaxation, which is beneficial by reformulating the combinatorial OFDMA constraint as an equivalent continuous optimization problem. The proposed framework simultaneously allocates the channel and power resources, and thus is sharply different from the existing ones. Simulation results show the proposed iterative reweighted minimization methods significantly outperform the existing algorithms.

1. INTRODUCTION

The Orthogonal Frequency Division Multiple Access (OFDMA) technique has been popularly used in modern wireless communications due to its merit of mitigating the frequency selective fading. In the OFDMA system, multiple users share multiple orthogonal channels, but at most one user is allowed to transmit power on each channel. A fundamental question for the OFDMA system is to optimally allocate channels to users in a nonoverlapping way and at the same time to determine the transmit powers on the allocated channels. The joint channel and power allocation problem is often formulated as two versions: the first is the one of maximizing a system utility function subject to power budget constraints, and the second is the one of minimizing the total transmit power subject to rate requirement constraints. The later is usually called the margin adaptive (MA) problem [1]. Recently, the MA problem has been re-addressed in the sense of energy efficiency and inter-cell interference avoidance [2]. This paper focuses on the MA problem.

The MA problem is generally NP-hard except in some special cases where the number of users and channels are equal [3, 4, 5]. The hardness of the MA problem is mainly due to the combinatorial OFDMA constraint, which requires that at most one user is allowed to access each channel. Therefore, various heuristic approaches have been proposed for solving the MA problem [2, 6, 7, 8]. The basic idea of the existing approaches is to allocate channel and power resources separately to leverage the difficulty of the simultaneous allocation. For instance, one typical method in [6] (called Method A in the sequel) first relaxes the binary channel indicator variables to the continuous variables of $[0, 1]$, solves the relaxed problem, and rounds the obtained solutions to obtain an approximate channel allocation strategy. Given the channel allocation, the transmit power is regulated to satisfy the rate requirements. The other typical method in [2] (called Method B in the sequel) determines the modulation and code level based on average channel conditions and adjusts the power to meet the signal-to-interference-plus-noise ratio (SINR) of the corresponding modulation and code level. Given the power, the remaining is to allocate channels to users. Another practical approach is to allocate the channels to the users with maximum channel gains and then calculate the power by the water-filling algorithm.

In this paper, we propose a novel iterative reweighted minimization framework for solving the MA problem, where a new reweighting strategy is designed to connect channel and power allocation variables. More specifically, we first rewrite the unfriendly OFDMA constraint as an equivalent continuous minimization problem with equality constraints. Then, we penalize the objective of the above equivalent problem in the objective of the MA problem and obtain an effective relaxation of the MA problem. Furthermore, we propose a novel iterative reweighted minimization framework for solving the relaxed MA problem. In the proposed iterative reweighted minimization framework, the variables associated with channel and power allocation are simultaneously updated and thus channel and power are simultaneously allocated. Thanks to jointly allocating channel and power, our proposed framework outperforms the aforementioned methods.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a multi-user single-cell OFDMA system where there are K users sharing N channels and the base station (BS) implements the resource allocation in a central way. Let $\mathcal{K} = \{1, 2, ..., K\}$ and $\mathcal{N} = \{1, 2, ..., N\}$ denote the set of users and the set of channels, respectively. Throughout the paper, we assume that $N \geq K$ (i.e., the number of channels is greater than or equal to the number of users). The other case of $N < K$ is associated with admission

This work is supported by the National Natural Science Foundation, Grants 11301516, 11331012 and 61231008.

control [10, 11], which is out of scope of this paper. The cell is assumed but not restricted to the downlink transmission mode.

Let p_k^n be the transmit power to user k on channel n. The received power of user k on channel n is given by $\alpha_k^n p_k^n + \eta_k^n$, where $\alpha_k^n := |h_k^n|^2$ stands for the channel gain between the BS and user k on channel n and $h_k^n \in \mathbb{C}$ is the channel coefficient of this link. Then, we can write the SINR of user k on channel n as $\text{SINR}_k^n = \dfrac{\alpha_k^n p_k^n}{\eta_k^n}$, and user k's achievable data rate R_k (bits/sec) as

$$R_k = \sum\nolimits_{n\in\mathcal{N}} \log_2\left(1+\text{SINR}_k^n\right),\ k\in\mathcal{K}. \tag{1}$$

In this paper, we consider the joint channel and power allocation problem for the OFDMA system, which requires that at most one user is allowed to access each channel. Mathematically, the OFDMA constraint can be formulated as

$$0 \le p_k^n \le y_k^n P^{\max},\ y_k^n \in \{0,1\},\ k\in\mathcal{K},\ n\in\mathcal{N} \tag{2}$$

and

$$\sum\nolimits_{k\in\mathcal{K}} y_k^n \le 1,\ n\in\mathcal{N}. \tag{3}$$

In the above, the binary variable $y_k^n = 1$ if user k occupies channel n and $y_k^n = 0$ otherwise; $P^{\max}$ is the maximum transmit power. From (2), we see that if $y_k^n = 0$ then $p_k^n = 0$; while if $y_k^n = 1$, then p_k^n can be any value in $[0,\ P^{\max}]$.

The MA problem is to minimize the total transmit power subject to rate requirement and OFDMA constraints, which is expressed as

$$\begin{aligned}\text{MA:}\ \min_{\{p_k^n, y_k^n\}}\quad & \sum\nolimits_{k\in\mathcal{K}}\sum\nolimits_{n\in\mathcal{N}} p_k^n\\ \text{s.t.}\quad & R_k \ge \gamma_k,\ \forall\, k\in\mathcal{K},\\ & \text{(2) and (3).}\end{aligned}$$

In the above, γ_k represents the desired rate demand of user k.

3. AN ITERATIVE REWEIGHTED MINIMIZATION FRAMEWORK

In this section, we first derive a relaxation of the MA problem and then propose a novel iterative reweighted minimization framework for solving the MA problem.

3.1. Relaxation

Letting $x_k^n = \dfrac{p_k^n}{P^{\max}}$ and $\beta_k^n = \dfrac{\alpha_k^n P_k^n}{\eta_k^n}$, the MA problem can be equivalently rewritten as problem (4) in the following:

$$\begin{aligned}\min_{\{x_k^n,\, y_k^n\}}\ & \sum\nolimits_{k\in\mathcal{K}}\sum\nolimits_{n\in\mathcal{N}} P^{\max} x_k^n\\ \text{s.t.}\quad & \sum\nolimits_{n\in\mathcal{N}} \log_2\left(1+\beta_k^n x_k^n\right) \ge \gamma_k,\ k\in\mathcal{K}, && \text{(4a)}\\ & y_k^n \ge x_k^n \ge 0,\ k\in\mathcal{K},\ n\in\mathcal{N}, && \text{(4b)}\\ & \sum\nolimits_{k\in\mathcal{K}} y_k^n = 1,\ n\in\mathcal{N}. && \text{(4c)}\\ & y_k^n \in \{0,1\},\ n\in\mathcal{N}. && \text{(4d)}\end{aligned}$$

Notice that (4c) in the above is different from (3). In fact, the MA problem and problem (4) are equivalent to each other. On one hand, any feasible point of problem (4) is feasible to the MA problem. On the other hand, given any feasible point $\{\hat{x}_k^n,\ \hat{y}_k^n\}$ of the MA problem, we can construct a feasible point $\{\bar{x}_k^n,\ \bar{y}_k^n\}$ of problem (4) such that the two problems have the same objective value as follows:

$$\bar{x}_k^n = \hat{x}_k^n,\ k\in\mathcal{K},\ n\in\mathcal{N}$$

and

$$\bar{y}_k^n = \begin{cases} \max\left\{\hat{y}_k^n,\ 1-\sum\nolimits_{k\in\mathcal{K}} \hat{y}_k^n\right\} & \text{if } k=1;\\ \hat{y}_k^n & \text{if } k\neq 1,\end{cases}\ , n\in\mathcal{N}. \tag{5}$$

The distinctive advantage of rewriting (3) into (4c) is that it allows for a simple (optimization) reformulation of the OFDMA constraint. In particular, $\{y_k^n\}$ satisfies the constraints (4c) and (4d) if and only if $\{y_k^n\}$ solves the following minimization problem

$$\begin{aligned}\min_{\{y_k^n\}}\quad & \sum\nolimits_{k\in\mathcal{K}}\sum\nolimits_{n\in\mathcal{N}} (y_k^n+\epsilon)^q\\ \text{s.t.}\quad & \text{(4c) and } y_k^n \ge 0,\ k\in\mathcal{K},\ n\in\mathcal{N},\end{aligned} \tag{6}$$

where $q \in (0,1)$ and ϵ can be any nonnegative value. We remark that the above equivalence does not hold if $q = 1$.

Based on the above reformulation (6), we can relax problem (4) to

$$\begin{aligned}\min_{\{x_k^n, y_k^n\}}\quad & \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} P^{\max} x_k^n + \lambda \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} (y_k^n+\epsilon)^q\\ \text{s.t.}\quad & (4a),\ (4b),\ \text{and } (4c),\end{aligned} \tag{7}$$

where λ is a positive penalty parameter. We have the following theorem, which can be shown in a similar way as [16, Theorem 17.1].

Theorem 3.1 *The solution of problem* (7) *converges to the one of problem (4) (with any* $\epsilon \ge 0$*) as the parameter* λ *goes to infinity.*

In practical computations, it is enough to solve problem (7) with a relatively large λ. This is because the solution of problem (7) with a relatively large λ must be close to the point satisfying (4c) and (4d) according to Theorem 3.1. Therefore, we can round the solution of problem (7), and obtain a feasible solution to problem (4). The parameter λ can be chosen adaptively as in [16, Framework 17.1]. Moreover, the parameter ϵ should not be very small, since a large ϵ will smooth out local minimizers of problem (7). Therefore, adaptively updating ϵ allows one to get close to the global minimizer of problem (7).

3.2. A Novel Iterative Reweighted Minimization (IRM) Framework

In this subsection, we propose a novel iterative reweighted minimization framework for solving the relaxed problem (7); see Algorithm 1. The idea is to solve a series of convex subproblems (8) wherein the weights $\{w_k^n(t)\}$ at the t-th iteration are calculated based on its solution at the last iteration. The proposed iterative

Algorithm 1 The IRM Framework for Solving MA

Initialization:
$\lambda = NP^{\max}$, $q \in (0,1)$, $\sigma_1 \in (0,1)$, $\sigma_2 \in (0,1)$, and $\tau > 1$; $w_k^n(0) = 1$ for all $k \in \mathcal{K}$ and $n \in \mathcal{N}$ and $\epsilon(0) = 1$.
while true **do**
  for t=1, 2,..., MaxItr **do**
    Solve the subproblem for $\{x_k^n(t+1), y_k^n(t+1)\}$:

$$\begin{aligned} \min_{\{x_k^n, y_k^n\}} \quad & \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} P^{\max} x_k^n + \lambda q \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} w_k^n(t) y_k^n \\ \text{s.t.} \quad & (4a),\ (4b),\ \text{and}\ (4c). \end{aligned} \tag{8}$$

    Update

$$w_k^n(t+1) = \left[(x_k^n(t+1)) + \epsilon(t+1)\right]^{q-1} \tag{9}$$

    and $\epsilon(t+1)$ for all $k \in \mathcal{K}$ and $n \in \mathcal{N}$.
    if $\sum_k \sum_n |x_k^n(t+1) - x_k^n(t)| < \sigma_1$ **then**
      break;
    end if
  end for
  if $f(\{y_k^n(t+1)\}) < \sigma_2$ **then**
    Stop.
  else
    $\lambda = \tau\lambda$
  end if
end while

reweighted minimization framework is given in Algorithm 1 as follows. Several remarks on Algorithm 1 are in order.

First, the subproblem (8) in Algorithm 1 is jointly convex with respect to $\{x_k^n\}$ and $\{y_k^n\}$ and thus it can be efficiently solved to global optimality by general-purpose solvers like CVX [13].

Second, according to (9), the weights $w_k^n(t+1)$ in Algorithm 1 is updated based on $x_k^n(t+1)$, which is in sharp contrast to the existing ones [9, 12] based on $y_k^n(t+1)$, i.e., the reweighted strategy

$$w_k^n(t+1) = \left[(y_k^n(t+1)) + \epsilon(t+1)\right]^{q-1}. \tag{10}$$

At first glance, one might say that it is not intuitive. Recall that our goal is to find $\{x_k^n\}$ that minimizes the objective of problem (4) and at the same time satisfies (4a) and

$$0 \le x_k^n \le 1,\ x_k^n x_j^n = 0,\ \forall\, k \ne j,\ k, j \in \mathcal{K},\ n \in \mathcal{N}. \tag{11}$$

However, this problem is hard to deal with and we thus relax it to problem (7) by introducing the variables $\{y_k^n\}$. The constraint (4b) connects the "original" variables $\{x_k^n\}$ and the "auxiliary" variables $\{y_k^n\}$. Then, we solve problem (7) with a large λ to obtain a sparse $\{y_k^n\}$ (that approximately satisfies (4c) and (4d)) and thus a sparse $\{x_k^n\}$ (that satisfies (11)) by (4b). Therefore, our primary goal is to find a sparse $\{x_k^n\}$ and to achieve this goal we replace the sparsity conditions of $\{x_k^n\}$ with the ones of $\{y_k^n\}$. However, a gap of the sparsity of $\{y_k^n\}$ and $\{x_k^n\}$ occurs when the iterative reweighted minimization framework is used to solve problem (8). More specifically, although the solution $\{\hat{x}_k^n\}$ of problem (8) with $w_k^n(t) = 1$ is unique, its solution $\{\hat{y}_k^n\}$ is not, which can be arbitrary ones in the set

$$\left\{ \{y_k^n\} \mid y_k^n \ge \hat{x}_k^n, \sum_{k\in\mathcal{K}} y_k^n = 1,\ k \in \mathcal{K},\ n \in \mathcal{N} \right\}.$$

The update strategy in (9) is actually to push the sparsity of $\{y_k^n\}$ to agree with that of $\{x_k^n\}$. This is the idea behind the proposed reweighted strategy (9).

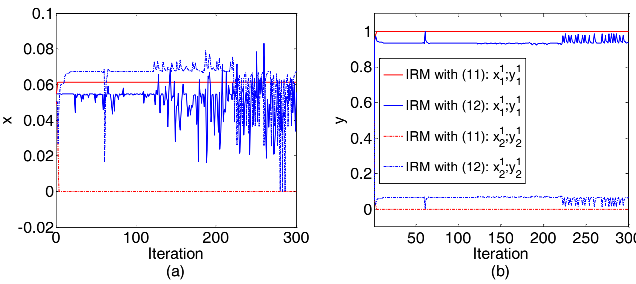

Fig. 1. Comparison of IRM based on the proposed reweighted strategy (9) and the reweighted strategy (10) for solving problem (7) where there are 2 users and 4 channels. (a) $x_1^1,\ x_2^1$. (b) $y_1^1,\ y_2^1$.

The effectiveness of the proposed reweighted strategy (9) is illustrated in Fig.1. Fig.1 shows that the oscillation (unconvergence) phenomenon occurs for Algorithm 1 based on the existing reweighted strategy (10). However, Algorithm 1 equipped with the proposed reweighted strategy (9) converges very fast (albeit its convergence is not rigorously proved), taking only 4 iterations to converge. Algorithm 1 with the reweighted strategy (10), even after about 300 iterations, returns positive x_1^1 and x_2^1, which implies both user 1 and 2 transmit positive power on channel 1. Obviously, this is not we want, since it does not satisfy the OFDMA constraint. In contrast to this, Algorithm 1 equipped with the new reweighted strategy (9) returns an OFDMA solution, i.e., only user 1 transmits positive power on channel 1, since y_2^1 is very close to 0 and so x_2^1; see Fig.1 (b).

Third, the parameters $\{\epsilon(t)\}$ in Algorithm 1 must be positive to guarantee (9) well defined. There are some possible ways of updating $\epsilon(t+1)$: (a) fix it to be a positive constant (e.g., $\epsilon(t) = 1e{-}4$); (b) set it to be a positive decreasing sequence (e.g., $\epsilon(t) = \epsilon(0)\gamma^t$, where $\gamma \in (0,1)$); (c) update it adaptively [9] according to

$$\epsilon(t+1) = \min\left\{\epsilon(t), \gamma \cdot f(\{x_k^n(t)\})\right\}, \tag{12}$$

where $f(\{x_k^n(t)\}) = \max_{n\in\mathcal{N}} \left(\{x_k^n(t)\}_{k\in\mathcal{K}}\right)_2$ and $\left(\{x_k^n(t)\}_{k\in\mathcal{K}}\right)_2$ is the second largest element of $\{x_k^n(t)\}_{k\in\mathcal{K}}$. Our simulation results show that the updating strategy (c) performs the best, (b) the second, and (c) the last; see Section 4.

Finally, the termination criterion of Algorithm 1 is $f(\{y_k^n(t+1)\}) < \sigma_2$. Therefore, Algorithm 1, when terminated, returns an approximate OFDMA solution. The quality of the returned solution depends on the tolerance σ_2. The less the tolerance is, the better the returned solution. In particular, if $\sigma_2 = 0$, then Algorithm 1 returns an OFDMA solution.

4. NUMERICAL RESULTS

In this section, we do numerical simulations to evaluate the effectiveness of the proposed iterative reweighted minimization framework (Algorithm 1) for solving the MA problem. We employ an OFDMA small cell network as our simulation scenario [14, 15], i.e., the cell radius is 20 m; the location of each user is uniformly generated in the cell; channel gains are given by

$$\alpha_k^n = 10^{L_k^n/10}\xi_k^n,\ L_k^n = 38.48 + 20\log(d_k),$$

where d_k is the Euclidean distance from the BS to user k and $\xi_k^n \sim \mathcal{CN}(0,1)$ accounts the Rayleigh fading. The maximum transmit power per channel is set to be $P^{\max} = 5$ mW, and without loss of generality the number of channels is set to be three times larger than the number of users, i.e., $N = 3K$. To guarantee the feasibility in the simulation, we assume that each user has a channel demand of two and its rate target ($\{\gamma_k\}$) is calculated by Method B. Other parameters in Algorithm 1 are MaxItr$=100$, $\tau = 10$, $\sigma_1 = 1e-4$ and $\sigma_2 = 1e-3$. All of the following results are obtained by averaging over 100 channel realizations.

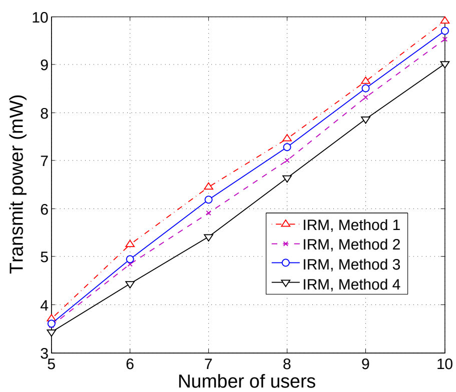

Fig. 2. IRM with different updating strategies.

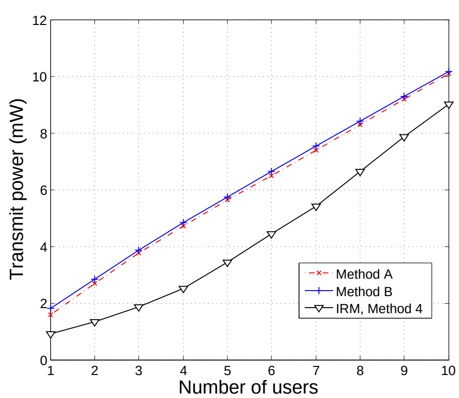

Fig. 3. Comparison of Method 4 with existing methods.

We first test the performance of the proposed framework equipped with different update strategies of $\{w_k^n(t)\}$ and $\epsilon(t)$, since different choices of these parameters lead to different methods:

Method 1, where $w(t+1) = 1/(x(t) + \epsilon(t+1))$ and $\epsilon(t+1)$ is updated by (12);

Method 2, where $w(t+1)$ is updated by (9) and $\epsilon(t+1) = 0.5\epsilon(t)$;

Method 3, where $w(t+1)$ is updated by (9) and $\epsilon(t) = 1e-4$;

Method 4, where $w(t+1)$ is updated by (9) and $\epsilon(t+1)$ is updated by (12).

In our simulation, q is set to be 0.8. Fig 2 plots the total transmit power performance of the above four methods. We can see from Fig 2 that Method 4 outperforms the others. This is consistent with our analysis that both $\{w_k^n(t)\}$ and $\epsilon(t)$ should be updated adaptively.

Next, we compare Method 4 with the other two existing methods in [6] and [2]. Fig. 3 shows the performance comparison of Method 4 with Method A in [6] and Method B in [2]. As we can see in Fig. 3, the total transmit power increases as the number of total users in the network increases for all of the three methods. However, the proposed IRM framework with adaptive update strategies (i.e., Method 4) exhibits a significant better performance than Method A and Method B in saving the total transmit power. The performance loss of Method A is due to its loose relaxation and the one of Method B is due to its fixed transmission mode.

Finally, we check the gaps between the solution returned by the proposed Method 4 and the global solution. Generally there do not exist polynomial time algorithms which can solve the MA problem to global optimality, since the problem is NP-hard. However, the problem with the same number of users and subcarriers (i.e., $N = K$) can be solved by the Hungarian method in polynomial time [3]. Therefore, we apply the proposed Method 4 to solve the MA problem with $N = K$ and compute its optimality gap in this special case. For each channel realization, denote the objective values of the MA problem at the solution returned by Method 4 and at the global solution by $\hat{P}$ and P^*, respectively. We use $(\hat{P} - P^*)/P^*$ to measure the optimality gap of Method 4. Table 1 summarizes the average optimality gap over 100 channel realizations. It can be seen from Table 1 that the average optimality gap between the solution obtained by the proposed method and the global solution is smaller than 11%, which means that the proposed Method 4 can find a solution close to the global solution.

Table 1. Average optimality gap when $N = K$

N	2	4	6	8	10
gap	10.63%	7.56%	0.82%	6.18%	7.67%

5. CONCLUDING REMARKS

In this paper, we consider the NP-hard joint channel and power allocation problem in the OFDMA system. We propose a novel iterative reweighted minimization framework for solving the problem. Simulations results show the proposed framework with adaptive weight update strategies is very effective.

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