TOPOLOGICAL INTERFERENCE MANAGEMENT FOR TWO CELL INTERFERENCE BROADCAST CHANNELS WITH ALTERNATING CONNECTIVITY

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ABSTRACT

Topological interference management refers to the study of achievable rates within communication networks with no channel state information at the transmitter (CSIT) beyond knowledge of the network structure itself. In this work we consider the topological interference management problem within the context of a two cell two user interference broadcast channel (IBC) with alternating connectivity. Topological information, even though minimal, allows the transmitters to track the changing network structure and then exploit the varying connectivity states to obtain a degrees of freedom (DoF) gain. Thus, the main result of this work is the derivation of a novel outer bound on the DoF achievable by the two cell two user IBC in an alternating connectivity scenario. Additionally, we propose a scheme based on joint coding across states that achieves this outer bound for the case where all alternating connectivity states are equiprobable.

Index Terms— Degrees of freedom, interference broadcast channel, topological interference management.

1. INTRODUCTION

In recent years major advances have been made in terms of understanding the information-theoretic capacity limits of interference limited networks. The new results indicate that the maximum achievable capacity is higher than what is currently obtained via the use of conventional techniques, although primarily under the assumption of abundant and accurate channel state information at the transmitter (CSIT). This in turn has given rise to the development of a number of innovative ways on how to exploit various CSIT aspects for interference management purposes.

The perfect CSIT assumption is highly idealistic and cannot be achieved in practice, making it quite difficult to translate theoretical gains into practical ones. Therefore moving on from the initial perfect CSIT studies [1, 2], the current research direction is to focus on more relaxed CSIT assumptions, for example compound channels [3] or scenarios where the available CSIT is delayed [4, 5], mixed [6] or partial [7].

A new but complementary perspective to interference management was introduced in [8]. Instead of starting with abundant CSIT and then moving into more relaxed scenarios, the issue is approached from the opposite end of the spectrum with no CSIT except for knowledge of the networks' topological structure. One main advantage is the minimal CSIT requirement; a single bit per transmitter/receiver link is enough to indicate whether a link is present or not. This approach is known as topological interference management and provides a unified view of wired and wireless networks. The study in [8] shows that capacity in a wired system and degrees of freedom (DoF) for the corresponding wireless network are equivalent in their normalised forms, such that analysis for one scenario can be easily translated to the other.

Throughout the work in [8] it is assumed that network topology remains fixed for the duration of communication, even in cases where the channel is time-varying. For the purpose of this work we move beyond this limitation and allow an alternating network topology, in order to analyse the DoF gains that may be achieved. A similar setting was considered for the two user interference channel (IC) and X channel in [9]; however here we place our focus on the more complex two cell two user interference broadcast channel (IBC).

Therefore the main contribution of this paper is the derivation of a novel DoF outer bound for the two cell two user IBC with alternating connectivity. Our focus is on wireless networks; however derivations start as achievable rate for the wired case and are then translated into DoF for the corresponding wireless scenario, using the equivalence established in [8]. Additionally, we also propose a scheme which achieves the derived outer bound when all alternating connectivity states are equally probable.

2. SYSTEM MODEL

Consider two adjacent cells in a wireless network. The first cell, A, consists of base station (BS) A and receivers $a1$, $a2$. Similarly, the second cell, B, consists of BS B and receivers $b1$, $b2$. The basic network structure is shown in Fig. 1 overleaf, where inter-cell interference links are omitted.

Within the cells themselves spatial multiplexing is applied. This allows each BS to simultaneously deliver one

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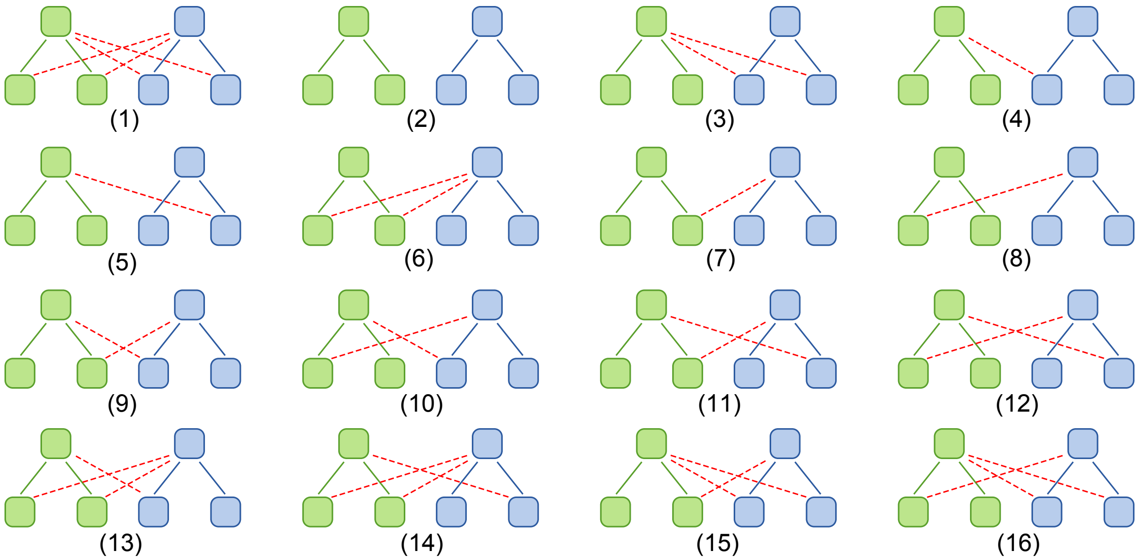

Fig. 2. All possible connectivity scenarios for the two cell two user IBC. Cell A transmitters and receivers are in green, while cell B elements are in blue. The solid green/blue are the desired links, while dashed red lines represent the interference links.

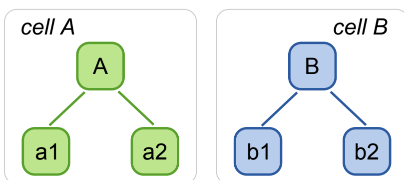

Fig. 1. Two cell two user wireless network, with inter-cell interference links omitted.

symbol to its corresponding two users, thereby achieving 2 DoF per cell provided no inter-cell interference is present. For each cell in Fig. 1, let us define M as the number of antennas at the BS and N_1, N_2 as the number of antennas at each of the receivers respectively. If local CSIT feedback within the cell is allowed, the sum DoF is characterised as $\min\{M, \sum_{i=1}^{K} N_i\}$ [10]. Thus the required 2 DoF can be obtained using a multiple-input single-output (MISO) structure with $M = 2$ and $N_1 = N_2 = 1$. Alternatively, if local CSIT is not available, the sum DoF is given by $\min\{M, \max\{N_1, N_2\}\}$ [11], therefore a multiple-input multiple-output (MIMO) structure with $M = N_1 = N_2 = 2$, is needed to obtain the required DoF of 2.

However, once inter-cell interference comes into play, cells A and B are no longer able to achieve 2 DoF each. This interference can arise between any of the users and the non-corresponding BS and may vary in wireless networks due to user movement or change in frequency/time resource allocated. For the system considered in this work, it may take the form of any of the states in Fig. 2. Each state has a probability of occurrence λ_i for $i = 1, \ldots, 16$, where $\sum_{i=1}^{16} \lambda_i = 1$.

If no information is available with respect to the changing network topology, both transmitters have to assume full connectivity at all times, i.e. State 1 in Fig. 2. This allows only for one possible transmission strategy, where BS A and BS B are provided with non-overlapping transmission opportunities, leading to a sum DoF of 2 over the entire network.

However, looking at Fig. 2 it is clear that assuming full connectivity throughout is wasteful in terms of network resource use. States 2 to 16 have a smaller amount of inter-cell interference and are able to achieve higher DoF than the fully connected scenario in State 1. Thus if the transmitters are able to track changes in network structure, they can adapt their transmission strategies accordingly to improve the achievable sum DoF. Our interest lies in exploiting this opportunity, while keeping the CSIT requirement to a minimum; therefore similar to the setup in [8] we assume that CSIT only consists of topological information. This requires just a single CSIT bit per transmitter/receiver link, used to indicate whether interference may be experienced over the link or not. A zero implies that interference received over that specific link is below the noise floor, i.e. the link is very weak and effectively non-existent, while a one represents a strong link over which considerable interference is experienced.

3. DERIVATION OF DOF OUTER BOUND

Here we derive a novel DoF outer bound for the two cell two user IBC with alternating connectivity. Using the equivalence established in [8], we first consider rate bounds for the wired case and then translate them into DoF results for the corresponding wireless network, which ultimately enables the characterisation of the sum DOF as in Theorem 1.

Theorem 1. *For the two cell two user IBC with alternating connectivity, where intra-cell interference is handled via spatial multiplexing, the achievable sum DoF, denoted by d_Σ, can*

$$
\begin{aligned}
nR_{a1} &\leq H(Y_1^{a1}, Y_2^{a1}, \ldots, Y_{16}^{a1}) - H(Y_1^{a1}, \ldots, Y_{16}^{a1} \mid W^A) + n\epsilon \\
&= H(Y_1^{a1}, Y_2^{a1}, \ldots, Y_{16}^{a1}) - H(Y_2^{a1}, \ldots, Y_{15}^{a1} \mid W^A) - \underbrace{H(Y_1^{a1}, Y_{16}^{a1} \mid W^A, Y_2^{a1}, \ldots, Y_{15}^{a1})}_{\geq 0} + n\epsilon \\
&\overset{(a)}{\leq} H(Y_1^{a1}, Y_2^{a1}, \ldots, Y_{16}^{a1}) - H(Y_2^{a1}, \ldots, Y_{15}^{a1} \mid W^A, X_1^A, \ldots, X_{16}^A) + n\epsilon && (2) \\
&= H(Y_1^{a1}, Y_2^{a1}, \ldots, Y_{16}^{a1}) - H(Y_\Delta^{a1} \mid W^{a1}, X_1^A, \ldots, X_{16}^A) + n\epsilon && (3) \\
&\overset{(b)}{=} H(Y_1^{a1}, Y_2^{a1}, \ldots, Y_{16}^{a1}) - H(h_{a1,A}X_6^A + h_{a1,B}X_6^B, \ldots, h_{a1,A}X_{14}^A + h_{a1,B}X_{14}^B \mid W^A, X_1^A, \ldots, X_{16}^A) + n\epsilon \\
&\overset{(c)}{=} H(Y_1^{a1}, Y_2^{a1}, \ldots, Y_{16}^{a1}) - H(h_{a1,B}X_6^B, h_{a1,B}X_8^B, h_{a1,B}X_{10}^B, h_{a1,B}X_{12}^B, h_{a1,B}X_{13}^B, h_{a1,B}X_{14}^B) + n\epsilon \\
&\overset{(d)}{=} H(Y_1^{a1}, Y_2^{a1}, \ldots, Y_{16}^{a1}) - H(h_{b1,B}X_6^B, h_{b2,B}X_8^B, h_{b1,B}X_{10}^B, h_{b2,B}X_{12}^B, h_{b1,B}X_{13}^B, h_{b2,B}X_{14}^B) + n\epsilon \\
&\leq H(Y_1^{a1}) + H(Y_2^{a1}) + \cdots + H(Y_{16}^{a1}) - H(Y_6^{b2}) - H(Y_8^{b1}) - H(Y_{10}^{b2}) - H(Y_{12}^{b1}) - H(Y_{13}^{b2}) - H(Y_{14}^{b1}) + n\epsilon && (4)
\end{aligned}
$$

$$nR_{a1} \leq n\log|\mathbb{GF}| \left[(\lambda_1 + \lambda_2 + \cdots + \lambda_{16}) - (\lambda_6 + \lambda_8 + \lambda_{10} + \lambda_{12} + \lambda_{13} + \lambda_{14})\right] + n\epsilon \quad (5)$$

$$nR_{a2} \leq n\log|\mathbb{GF}| \left[(\lambda_1 + \lambda_2 + \cdots + \lambda_{16}) - (\lambda_6 + \lambda_7 + \lambda_9 + \lambda_{11} + \lambda_{13} + \lambda_{14})\right] + n\epsilon \quad (6)$$

$$nR_{b1} \leq n\log|\mathbb{GF}| \left[(\lambda_1 + \lambda_2 + \cdots + \lambda_{16}) - (\lambda_3 + \lambda_4 + \lambda_9 + \lambda_{10} + \lambda_{15} + \lambda_{16})\right] + n\epsilon \quad (7)$$

$$nR_{b2} \leq n\log|\mathbb{GF}| \left[(\lambda_1 + \lambda_2 + \cdots + \lambda_{16}) - (\lambda_3 + \lambda_5 + \lambda_{11} + \lambda_{12} + \lambda_{15} + \lambda_{16})\right] + n\epsilon \quad (8)$$

be characterised as

$$d_\Sigma \leq 2 + 2\lambda_2 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \Phi$$

where

$$\Phi = \min \begin{cases} 2\lambda_1 \\ 2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{15} + \lambda_{16} \\ 2\lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14} . \end{cases}$$

Proof. The overall outer bound consists of merging together bounds originating from different sources; one comes from the summation of the achievable rate at each user and an additional set arises from the derivation of genie aided bounds.

Considering the summation bound, we first obtain expressions for the rate achievable at each user separately and then combine them. Starting with receiver $a1$, using Fano's inequality, we have

$$nR_{a1} \leq I(W^A; Y_1^{a1}, \ldots, Y_{16}^{a1}) + n\epsilon . \quad (1)$$

After exploiting various entropy properties this can be expressed as in (2), where (a) follows from the fact that W^A, is a function of $X_1^A, \ldots, X_{16}^A$. Next, it can be observed that $\{Y_2^{a1}, \ldots, Y_{15}^{a1}\}$ is divisible into two sets,

$\Omega = \{2, 3, 4, 5, 7, 9, 11, 15\}$ and $\Delta = \{6, 8, 10, 12, 13, 14\}$

where for Ω states the signals at receiver $a1$ consist only of a scaled version of X^A, implying terms belonging to this set have no effect on entropy, and thus can be removed; and for the Δ set, the data received at $a1$ is a combination of both X^A and X^B. This allows us to obtain (3), where (b) follows by expressing each received signal in terms of the original components X^A and X^B. Also (c) follows by noticing that X^A terms are negligible with respect to entropy and X^B terms are independent of $\{W^A, X_1^A, \ldots, X_{16}^A\}$.

Next (d) follows by replacing channel coefficients originating from BS A with ones originating from BS B, since scalar multiplication has no effect on entropy [12]. The final rate expression at $a1$ is thus given by (4). This can be expressed in terms of the probability of occurrence of each state as in (5), which follows from the fact that all random variables come from a $\mathbb{GF}$ for the wired equivalent network.

Using a similar process for the remaining receivers, we obtain expressions (6) through to (8); combining these with (5) and using the fact that $\sum_{i=1}^{16} \lambda_i = 1$, results in

$$
\begin{aligned}
nR_{\Sigma(SB)} &= n(R_{a1} + R_{a2} + R_{b1} + R_{b2}) \\
&\leq n\log|\mathbb{GF}|[2 + 2\lambda_1 + 2\lambda_2 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8] + n\epsilon . \quad (9)
\end{aligned}
$$

Normalising by $\log|\mathbb{GF}|$, gives the desired DoF sum bound

$$d_{\Sigma(SB)} \leq 2 + 2\lambda_1 + 2\lambda_2 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 . \quad (10)$$

Having obtained the summation bound, we now shift our attention to the genie aided ones. These are obtained by finding the rate outer bound achieved within each cell after providing it with enough extra information, i.e. genies, such that the total data required across the two cells can be decoded within that single cell. Starting with cell A, we have

$$nR_{\Sigma(GA)} \leq I(W^A, W^B; Y_1^{a1}, \ldots, Y_{16}^{a1}, Y_1^{a2}, \ldots, Y_{16}^{a2}, G) + n\epsilon \quad (11)$$

where G represents the additional set of genies required such that cell B data may be reconstructed within cell A.

Cell B has two users, therefore in cases where cell A has less than two interfering signals, additional genies need to be provided. The number of genies required is either one or two depending on the the number of interfering signals to cell A receivers. Looking at all the possible topologies in Fig. 2, the following genie set is required

$$G = \{2Y_2^B, 2Y_3^B, 2Y_4^B, 2Y_5^B, Y_7^B, Y_8^B, Y_9^B, Y_{10}^B, Y_{11}^B, Y_{12}^B, Y_{15}^B Y_{16}^B\}$$

where B represents either $b1$ or $b2$. Therefore (11) can be

$$
\begin{aligned}
nR_{\Sigma(GA)} &\leq H(Y_1^{a1},\ldots,Y_{16}^{a1},Y_1^{a2},\ldots,Y_{16}^{a2},G) + H(Y_1^{a1},\ldots,Y_{16}^{a1},Y_1^{a2},\ldots,Y_{16}^{a2},G \mid W^A,W^B) + n\epsilon \\
&\leq H(Y_1^{a1}) + \cdots + H(Y_{16}^{a1}) + H(Y_1^{a2}) + \cdots + H(Y_{16}^{a2}) + 2H(Y_2^B) + 2H(Y_3^B) + 2H(Y_4^B) + 2H(Y_5^B) \\
&\quad + H(Y_7^B) + H(Y_8^B) + H(Y_9^B) + H(Y_{10}^B) + H(Y_{11}^B) + H(Y_{12}^B) + H(Y_{15}^B) + H(Y_{16}^B) + n\epsilon \\
&\leq n\log|\mathbb{GF}|\left[2 + 2\lambda_2 + 2\lambda_3 + 2\lambda_4 + 2\lambda_5 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{15} + \lambda_{16}\right] + n\epsilon \qquad (12)
\end{aligned}
$$

expressed as (12) above, which in terms of DoF becomes

$$
\begin{aligned}
d_{\Sigma(GA)} \leq\ & 2 + 2\lambda_2 + 2\lambda_3 + 2\lambda_4 + 2\lambda_5 + \lambda_7 + \lambda_8 + \lambda_9 \\
& + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{15} + \lambda_{16}\ . \qquad (13)
\end{aligned}
$$

Going through a similar approach for cell B, we obtain

$$
\begin{aligned}
d_{\Sigma(GB)} \leq\ & 2 + 2\lambda_2 + \lambda_4 + \lambda_5 + 2\lambda_6 + 2\lambda_7 + 2\lambda_8 + \lambda_9 \\
& + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14}\ . \qquad (14)
\end{aligned}
$$

Finally, the result in Theorem 1 can be achieved by combining all the separate bounds from (10), (13) and (14). □

Remark 1. *Some similarities can be observed between the outer bound obtained here and the two user IC bound from [9]. This is expected since the IC is a subset of the IBC having only one user per cell. Both results can be summarised as*

$$
d_\Sigma \leq d_c + \lambda_\eta + \Theta \qquad (15)
$$

where d_c is the achievable DoF per cell when no external interference is present, equal to 1 for the two user IC and to 2 for the scenario considered in this paper; λ_η is solely a function of the probability of those topologies where the sum DoF is larger than d_c and its presence reflects the extra DoF that are achieved in these states; and finally Θ, for both IC and IBC, is a combination of the probabilities of the alternating connectivity states whose value depends on which bound is the most restrictive. For the sum bound, the Θ component for both IC and IBC is exclusively a function of the fully connected topology, while for the genie aided bounds it is a function of the remaining partially connected states.

4. ACHIEVABILITY OF DOF OUTER BOUND FOR EQUIPROBABLE STATES

When all states are equiprobable i.e. $\lambda_1 = \cdots = \lambda_{16} = \frac{1}{16}$, we can establish the following corollary from Theorem 1.

Corollary 1. *For the two cell two user IBC with alternating connectivity and equiprobable states, where intra-cell interference is handled via spatial multiplexing, $d_\Sigma \leq 2 + \frac{1}{2}$.*

Without topological information, a fully connected scenario is assumed at all times, achieving a sum DoF of 2 across the whole network. However once topological information is available, transmission strategies which exploit this knowledge can be applied to gain DoF from the partially connected scenarios in Fig. 2. Thus it is possible to obtain: 4 DoF in state 2; 3 DoF in states $4, 5, 7$ and 8; while the remaining 11 states achieve 2 DoF. With equiprobable states this implies 38 symbols are transmitted in 16 channel uses on average, leading to an average achievable DoF of $2\frac{3}{8}$. While this is an improvement over the no topological information case, it still does not achieve the outer bound established in Corollary 1. Next, we propose a scheme that achieves it, allowing us to establish the following theorem.

Theorem 2. *For the two cell two user IBC with alternating connectivity and equiprobable states, where intra-cell interference is handled via spatial multiplexing, the DoF outer bound equal to $2\frac{1}{2}$ can be achieved.*

Proof. This DoF outer bound can be achieved by using a scheme which applies joint coding across states. Looking at the states in Fig. 2, it can be noticed that the interference links present in states 3 and 6 are all contained within state 1; thus state 1 can be used to resolve them. New symbols are transmitted to each of the four receivers during states 3 and 6. For state 1, BS A retransmits the same symbols it transmitted in state 3, while BS B retransmits the state 6 symbols. This allows for interference cancellation decoding, thereby 2 symbols are transmitted to each of the 4 users across 3 channel uses, leading to an average DoF per state of $\frac{8}{3}$. Therefore, the achievable DoF across all states can be characterised as

$$
\text{DoF} = \begin{cases} 4 \text{ for state } 2 \\ 3 \text{ for states } 4, 5, 7, 8 \\ \frac{8}{3} \text{ for states } 1 \cup 3 \cup 6 \\ 2 \text{ for the remaining } 8 \text{ states}\ . \end{cases}
$$

With equiprobable states this implies 40 symbols can be transmitted in 16 channel uses on average, equivalent to $2\frac{1}{2}$ DoF, as originally stated in Theorem 2. □

Remark 2. *When equiprobable states are considered, the gain in achievable DoF for the two user IC from [9] is equal to $\frac{1}{2}$. This is equivalent to the gain achieved by the IBC system considered here. Moreover, for both scenarios the DoF gain is obtained via joint coding across states, further confirming compatibility between the two results.*

5. CONCLUSION

In this work we study topological interference management for the two user two cell IBC with alternating connectivity and present a novel outer bound in terms of the achievable sum DoF. We also provide a scheme that achieves the derived outer bound when the alternating connectivity states are equiprobable. Additionally, we show that the IBC results obtained here are compatible with the two user IC results from prior literature.

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