SUM RATE MAXIMIZATION MODEL OF NON-REGENERATIVE MULTI-STREAM MULTI-PAIR MULTI-RELAY NETWORK

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ABSTRACT

We consider the general MIMO relay network with K user pairs and R relays. The model to maximize the total signal to total interference plus noise ratio, with individual relay transmit power constraints, is set up. We constrain the precoding matrices to have orthogonal columns to guarantee multiple stream transmission. By alternating iteration method, we decompose the problem into several subproblems. For the precoding subproblem, which is a nonconvex matrix optimization problem, we apply the projected gradient method to its dual problem and prove there is zero duality gap between the primal and the dual problems. Simulation results show our proposed multiple stream model is efficient to achieve high sum rate and outperform the existing model in medium to high SNR scenarios.

Index Terms— MIMO relay, sum rate maximization, multiple stream transmission, projected gradient, zero duality gap

1. INTRODUCTION

Relays are widely used in wireless communications. Aided by relays, the network capacity is improved, and the stability is enhanced. The non-regenerative relay, known as the amplify-and-forward (AF) scheme, is especially in popular research for its simplicity.

There are many works related to MIMO relay AF networks, discussing the design of the precoding, decoding and the relay AF matrices [1–12]. Focusing on single antenna users and relays with direct links between user pairs, [2] and [3] proposed algorithms to maximize the sum rate and minimize the mean square error, respectively. Considering the network with one multi-antenna relay, the authors in [5] provided a tutorial of varies optimization problems and the practical implementations. For the network with one multi-antenna user pair and one multi-antenna relay, Tang et. al [6] studied the upper and lower bound of the system capacity; Zhang et. al [7] approximated the sum rate maximization problem by maximizing its lower bound. Extending the network to that with multiple relays, Zhao et. al [9] minimized the system mean square error with both total and individual relay power constraints. For the general relay networks with multiple links and multiple relays, Truong et. al [10] proposed a weighted mean square error minimization (WMMSE) model to solve the source and relay AF matrices with MMSE receiving filter, discussing both total and individual relay transmit power constraints. Sun et. al [11, 12] proposed the models to maximize the total signal to total interference plus noise ratio (TSTINR), to transmit single and multiple data streams, respectively, both with total relay transmit power constraints. The multiple stream TSTINR model in [12] guarantees to transmit multiple data streams, which achieves higher sum rate than the WMMSE model and the single stream TSTINR model in medium to high SNR. Moreover, the number of transmitted data streams has close relationship with the Degrees of Freedom (DoFs) of the network [12]. However, the total relay transmit power constraint in [12] is not quite practical. With such constraint, the power allocation among different relays might be unfair. In this paper, we propose the multiple TSTINR model with individual relay transmit power constraints. The system model is presented in Section 2. Different from the model in [12], we will have several quadratic constraints in the precoding subproblem, which makes the problem more difficult to analysis. In Section 3, we will propose a new algorithm to solve the dual problem of the new subproblem and guarantee zero duality gap. The simulation results in section 4 show the superior performance of our proposed model compared to the WMMSE model in [10] in medium to high SNR regimes.

Notation: $\mathbb{C}$ represents the complex domain. $(\cdot)^H$ means the Hermitian. $\mathrm{tr}(\mathbf{A})$ and $\|\mathbf{A}\|_F$ are the trace and the Frobenius norm of matrix $\mathbf{A}$, respectively. $\mathbf{I}_d$ represents the $d \times d$ identity matrix. $\mathcal{K}$ and $\mathcal{R}$ represent the set of the user indices $\{1, 2, \dots, K\}$ and that of relay indices $\{1, 2, \dots, R\}$, respectively. $\mathrm{vec}(\mathbf{A})$ means to compose a long vector by the columns of matrix $\mathbf{A}$. $\nu_{\min}^{d}(\mathbf{A})$ is composed of the eigenvectors of $\mathbf{A}$ corresponding to its d smallest eigenvalues.

2. SYSTEM MODEL

A two-hop half-duplex interference channel consisting of K user pairs and R relays is considered. Suppose all the users and relays have multiple antennas. For any $k \in \mathcal{K}$ and $r \in \mathcal{R}$, Transmitter k, Receiver k and Relay r have M_k, N_k and L_r antennas, respectively. User k wants to transmit d_k parallel data streams, and $\mathbf{s}_k \in \mathbb{C}^{d_k \times 1}$ denotes the transmit signal vector, where $\mathbb{E}(\mathbf{s}_k \mathbf{s}_k^H) = \mathbf{I}_{d_k}$. In the paper, we assume there is no direct links among users and perfect channel state information (CSI) is available at a central controller.

Transmission process has two time phases, from transmitters to relays and from relays to receivers, respectively. First, each relay receives precoded signals from all transmitters. Relay r receives $\mathbf{x}_r = \sum_{k \in \mathcal{K}} \mathbf{G}_{rk} \mathbf{U}_k \mathbf{s}_k + \mathbf{n}_r$, where $\mathbf{U}_k \in \mathbb{C}^{M_k \times d_k}$ is the precoding matrix of User k, $\mathbf{G}_{rk} \in \mathbb{C}^{L_r \times M_k}$ is the channel matrix between the Transmitter k and Relay r, and $\mathbf{n}_r$ with zero mean and variance matrix $\sigma_r^2 \mathbf{I}_{L_r}$ is the noise at Relay r. Next, Relay r multiplies the received signal with AF matrix $\mathbf{W}_r \in \mathbb{C}^{L_r \times L_r}$ as $\mathbf{t}_r = \mathbf{W}_r \mathbf{x}_r$, for all $r \in \mathcal{R}$. By decoding the received signal at Receiver k with decoding matrix $\mathbf{V}_k \in \mathbb{C}^{N_k \times d_k}$, it finally achieves

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$$\tilde{\mathbf{y}}_k = \underbrace{\mathbf{V}_k^H \mathbf{T}_{kk}\mathbf{s}_k}_{\text{desired signal}} + \underbrace{\sum_{q\in\mathcal{K},q\neq k} \mathbf{V}_k^H \mathbf{T}_{kq}\mathbf{s}_q}_{\text{interference}} + \underbrace{\sum_{r\in\mathcal{R}} \mathbf{V}_k^H \mathbf{H}_{kr}\mathbf{W}_r\mathbf{n}_r + \mathbf{V}_k^H \mathbf{z}_k}_{\text{noise}}.$$

This signal consists of three terms: the desired signal, the interference from other users and the noise including relay enhanced noise and the local noise. Here $\mathbf{H}_{kr} \in \mathbb{C}^{N_k \times L_r}$ is the channel matrix between Relay r and Receiver k, and $\mathbf{z}_k$ with zero mean and variance matrix $\mu_k^2 \mathbf{I}_{N_k}$ is the noise at Receiver k. The effective channel from Transmitter q to Receiver k is given by $\mathbf{T}_{kq} = \sum_{r\in\mathcal{R}} \mathbf{H}_{kr}\mathbf{W}_r\mathbf{G}_{rq}\mathbf{U}_q$. Suppose all the transmit signals and noise in the system are independent of each other. The transmit power at Relay r is $P_r^R = \mathbb{E}(\|\mathbf{t}_r\|_F^2) = \sum_{k\in\mathcal{K}} \|\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2 + \sigma_r^2\|\mathbf{W}_r\|_F^2$, for all $r \in \mathcal{R}$.

For the sake of expression simplicity, we predefine some symbols here: precoded and decoded effective channel from Transmitter k to Relay r as $\bar{\mathbf{G}}_{rk} = \mathbf{G}_{rk}\mathbf{U}_k$ and $\bar{\mathbf{W}}_{rk} = \mathbf{W}_r\mathbf{G}_{rk}$, respectively; precoded and decoded effective channel from Relay r to Receiver k as $\bar{\mathbf{H}}_{kr} = \mathbf{H}_{kr}\mathbf{W}_r$ and $\bar{\mathbf{V}}_{kr} = \mathbf{V}_k^H\mathbf{H}_{kr}$, respectively, for all $k \in \mathcal{K}$ and $r \in \mathcal{R}$.

3. MULTIPLE STREAM MODEL

Under individual relay transmit power constraints, our aim is to design precoding matrices $\{\mathbf{U}\} = \{\mathbf{U}_k, k \in \mathcal{K}\}$, decoding matrices $\{\mathbf{V}\} = \{\mathbf{V}_k, k \in \mathcal{K}\}$ and relay AF matrices $\{\mathbf{W}\} = \{\mathbf{W}_r, r \in \mathcal{R}\}$ to maximize the system sum rate as

$$R_{\text{sum}} = \frac{1}{2}\sum_{k\in\mathcal{K}} \log_2\det(\mathbf{I}_{N_k} + \mathbf{F}_k^{-1}\mathbf{T}_{kk}\mathbf{T}_{kk}^H), \quad (1)$$

with $\mathbf{F}_k = \sum_{q\neq k,q\in\mathcal{K}} \mathbf{T}_{kq}\mathbf{T}_{kq}^H + \sum_{r\in\mathcal{R}} \sigma_r^2 \bar{\mathbf{H}}_{kr}\bar{\mathbf{H}}_{kr}^H + \mu_k^2\mathbf{I}_{N_k}$. However it is quite complicated to optimize the system sum rate directly. There are several approaches to approximate the sum rate maximization model. Here we adopt the Total Signal to Total Interference plus Noise Ratio (TSTINR) maximization model, since it works well in medium to high SNR scenarios [11]. We have

$$\text{TSTINR} = \frac{P^S}{P^I + P^N} = \frac{\sum_{k\in\mathcal{K}} P_k^S}{\sum_{k\in\mathcal{K}}(P_k^I + P_k^N)},$$

and the desired signal power, the leakage interference and the noise power at Receiver k are expressed as below, respectively:

$$P_k^S = \|\mathbf{V}_k^H \sum_{r\in\mathcal{R}} \mathbf{H}_{kr}\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2,$$

$$P_k^I = \sum_{q\in\mathcal{K},q\neq k} \|\mathbf{V}_k^H \sum_{r\in\mathcal{R}} \mathbf{H}_{kr}\mathbf{W}_r\mathbf{G}_{rq}\mathbf{U}_q\|_F^2,$$

$$P_k^N = \sum_{r\in\mathcal{R}} \sigma_r^2 \|\mathbf{V}_k^H\mathbf{H}_{kr}\mathbf{W}_r\|_F^2 + \mu_k^2\|\mathbf{V}_k\|_F^2.$$

It is proved that maximizing TSTINR guarantees to achieve the lower bound of the sum rate maximization [11].

With individual relay power constraints, the multiple stream model to maximize TSTINR is as follows:

$$\begin{aligned} \max_{\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\}} \quad & \text{TSTINR} = \frac{\sum_{k\in\mathcal{K}} P_k^S}{\sum_{k\in\mathcal{K}}(P_k^I + P_k^N)} \\ \text{s.t.} \quad & \mathbf{U}_k^H\mathbf{U}_k = \frac{p_k^U}{d_k}\mathbf{I}_{d_k}, \mathbf{V}_k^H\mathbf{V}_k = \mathbf{I}_{d_k}, k \in \mathcal{K}, \\ & \sum_{k\in\mathcal{K}} \|\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2 + \sigma_r^2\|\mathbf{W}_r\|_F^2 \leq p_r^R, r \in \mathcal{R}. \quad (2) \end{aligned}$$

Here p_k^U and p_r^R are the power budgets for User k and Relay r, respectively. Similar to [12], in our model we assume each user transmit with the full power p_k^U, and require equal power allocation among parallel data streams for each user. This accords with the optimal power allocation scheme to maximize the system sum rate in the high SNR scenarios [13]. The precoding matrices are required to have orthogonal columns, to guarantee multiple data stream transmission. The orthogonality constraints for the decoding matrices are added to well define the problem. The optimal number of data stream d_k for all $k \in \mathcal{K}$ may be found by some heuristic selection, which is left for our future work.

To deal with the fraction objective function TSTINR, we use the parameter C to reformulate the objective function It is updated as

$$C = \frac{P^S(\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\})}{P^I(\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\}) + P^N(\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\})}. \quad (3)$$

The reformulated optimization problem is presented as (4). And the two problems (2) and (4) share the same stationary points [11].

$$\begin{aligned} \min_{\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\}} \quad & f(\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\};C) = C(P^I + P^N) - P^S \\ \text{s.t.} \quad & \mathbf{U}_k^H\mathbf{U}_k = \frac{p_k^U}{d_k}\mathbf{I}_{d_k}, \mathbf{V}_k^H\mathbf{V}_k = \mathbf{I}_{d_k}, k \in \mathcal{K}, \\ & \sum_{k\in\mathcal{K}} \|\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2 + \sigma_r^2\|\mathbf{W}_r\|_F^2 \leq p_r^R, r \in \mathcal{R}. \quad (4) \end{aligned}$$

As (4) is nonconvex and nonlinear, we apply the alternating iteration method to solve the precoders, decoders and relay AF matrices. Efficient algorithms are developed for the subproblems.

3.1. Subproblem for decoding matrix

Firstly, we fix $\{\mathbf{U}\}$ and $\{\mathbf{W}\}$, then all $\mathbf{V}_k, k \in \mathcal{K}$ are independent of each other. For any $k \in \mathcal{K}$, the subproblem for $\mathbf{V}_k$ becomes:

$$\begin{aligned} \min_{\mathbf{X}\in\mathbb{C}^{N_k\times d_k}} \quad & \text{tr}(\mathbf{X}^H\mathbf{A}_0\mathbf{X}) \\ \text{s.t.} \quad & \mathbf{X}^H\mathbf{X} = \mathbf{I}_{d_k}, \quad (5) \end{aligned}$$

where $\mathbf{X}$ represents the variable $\mathbf{V}_k$, and $\mathbf{A}_0 = C\mathbf{F}_k - \mathbf{T}_{kk}\mathbf{T}_{kk}^H$. Since $\mathbf{A}_0$ is Hermitian, we obtain the closed form solution of (5) from the eigenvalue decomposition of $\mathbf{A}_0$, as the eigenvectors corresponding to the d_k smallest eigenvalues, $\mathbf{X} = \nu_{\min}^{d_k}(\mathbf{A}_0)$.

3.2. Subproblem for relay AF matrix

Next, we consider the subproblem for $\mathbf{W}_r$. Given a certain index $r \in \mathcal{R}$, we fix all the variables except $\mathbf{W}_r$. Then the optimization subproblem for $\mathbf{W}_r$ is equivalent to:

$$\min_{\mathbf{x}\in\mathbb{C}^{L_r^2\times 1}} \quad \bar{f}(\mathbf{x}) = \mathbf{x}^H\mathbf{B}_1\mathbf{x} + \mathbf{b}^H\mathbf{x} + \mathbf{x}^H\mathbf{b} \quad (6a)$$

$$\text{s.t.} \quad \mathbf{x}^H\mathbf{B}_2\mathbf{x} \leq p_r^R. \quad (6b)$$

Here $\mathbf{x} = \text{vec}(\mathbf{W}_r)$, $\mathbf{B}_1 = \sum_{k\in\mathcal{K}}(\mathbf{P}_{rr}^k + C\sigma_r^2\mathbf{I}_{L_r})^T \otimes (\bar{\mathbf{V}}_{kr}^H\bar{\mathbf{V}}_{kr})$, and $\mathbf{P}_{rl}^k = C\sum_{q\neq k,q\in\mathcal{K}} \bar{\mathbf{G}}_{rq}\bar{\mathbf{G}}_{lq}^H - \bar{\mathbf{G}}_{rk}\bar{\mathbf{G}}_{lk}^H$, for any $k \in \mathcal{K}$ and $r, l \in \mathcal{R}$; $\mathbf{B}_2 = (\sum_{k\in\mathcal{K}} \bar{\mathbf{G}}_{rk}\bar{\mathbf{G}}_{rk}^H + \sigma_r^2\mathbf{I}_{L_r})^T \otimes \mathbf{I}_L$ and $\mathbf{b} = \text{vec}(\sum_{k\in\mathcal{K}}\sum_{l\neq r,l\in\mathcal{R}} \bar{\mathbf{V}}_{kr}^H\bar{\mathbf{V}}_{kl}\mathbf{W}_l\mathbf{P}_{rl}^k)$.

As $\mathbf{B}_2$ is positive definite, problem (6) is equivalent to the typical trust region (TR) subproblem in trust region optimization method. [14, Chapter 6.1.1] provides an efficient algorithm to achieve its optimal solution.

3.3. Subproblem for precoding matrix

For any $k \in \mathcal{K}$, fix all the variables other than $\mathbf{U}_k$. The subproblem for the precoding matrix $\mathbf{U}_k$ has the following expression.

$$\begin{aligned} \min_{\mathbf{X}\in\mathbb{C}^{M_k\times d_k}} \quad & \mathrm{tr}(\mathbf{X}^H\mathbf{D}_0\mathbf{X}) && \text{(7a)}\\ \text{s.t.} \quad & \mathbf{X}^H\mathbf{X} = \rho\mathbf{I}_{d_k}, && \text{(7b)}\\ & \mathrm{tr}(\mathbf{X}^H\mathbf{D}_r\mathbf{X}) \le \eta_r, r\in\mathcal{R}, && \text{(7c)} \end{aligned}$$

Here $\mathbf{X}$ represents $\mathbf{U}_k$, and

$$\begin{aligned} \mathbf{D}_0 &= \sum_{r\in\mathcal{R}}\sum_{l\in\mathcal{R}}\bar{\mathbf{W}}_{rk}^H\left(C\sum_{q\neq k,q\in\mathcal{K}}\bar{\mathbf{V}}_{qr}\bar{\mathbf{V}}_{ql}-\bar{\mathbf{V}}_{kr}\bar{\mathbf{V}}_{kl}\right)\bar{\mathbf{W}}_{lk},\\ \mathbf{D}_r &= \bar{\mathbf{W}}_{rk}^H\bar{\mathbf{W}}_{rk},\\ \eta_r &= p_r^R-\sum_{q\neq k,q\in\mathcal{K}}\|\bar{\mathbf{W}}_{rq}\mathbf{U}_q\|_F^2-\sigma_r^2\|\mathbf{W}_r\|_F^2. \end{aligned}$$

Although $\mathbf{D}_r$ are positive semi-definite for all $r\in\mathcal{R}$, the orthogonality constraint (7b) makes the problem (7) nonconvex. Besides, the matrix $\mathbf{D}_0$ is indefinite in general.

Different from [12], there are R quadratic constraints in (7), which is much more difficult to analyze. The method proposed in [12] is not applicable here. In the following we apply the dual method to deal with this nonconvex nonlinear matrix problem (7). First we present its dual problem. Suppose θ_r for all $r\in\mathcal{R}$ are the Lagrange multipliers for the R constraints (7c). Analyzed by the duality theory [16], the dual problem of (7) is expressed as follows[1]:

$$\begin{aligned} \min_{\theta_r,r\in\mathcal{R}} \quad & h(\theta_1,\theta_2,\ldots,\theta_R)\\ & =\sum_{r\in\mathcal{R}}\theta_r\eta_r-\min_{\mathbf{X}^H\mathbf{X}=\rho\mathbf{I}_{d_k}}\mathrm{tr}[\mathbf{X}^H(\mathbf{D}_0+\sum_{r\in\mathcal{R}}\theta_r\mathbf{D}_r)\mathbf{X}]\\ \text{s.t.} \quad & \theta_r\ge 0, r\in\mathcal{R}. \qquad (8) \end{aligned}$$

Since (8) simply has bound constraints, we apply the projected gradient method to solve it. Without loss of generality, we suppose $\rho=1$ in the following discussion and algorithm. For the case $\rho\neq 1$ ($\rho>0$), we can scale the other parameters to satisfy $\rho=1$.

Let $\boldsymbol{\theta}=(\theta_1,\theta_2,\ldots,\theta_R)^T$. Given $\boldsymbol{\theta}$,

$$g(\boldsymbol{\theta})=\min_{\mathbf{X}^H\mathbf{X}=\mathbf{I}_{d_k}}\mathrm{tr}[\mathbf{X}^H(\mathbf{D}_0+\sum_{r\in\mathcal{R}}\theta_r\mathbf{D}_r)\mathbf{X}]$$

is the sum of the smallest d_k eigenvalues of matrix $\mathbf{D}_0+\sum_{r\in\mathcal{R}}\theta_r\mathbf{D}_r$. Then the gradient of $g(\boldsymbol{\theta})$ with respect to θ_r should be [17]:

$$\frac{\partial g}{\partial\theta_r}=\sum_{i=1}^{d_k}\frac{\mathbf{x}_i^H\mathbf{D}_r\mathbf{x}_i}{\mathbf{x}_i^H\mathbf{x}_i},$$

for any $r\in\mathcal{R}$. Here $\mathbf{x}_i$ is the eigenvector corresponding to the ith smallest eigenvalue of $\mathbf{D}_0+\sum_{r\in\mathcal{R}}\theta_r\mathbf{D}_r$. If we require these eigenvectors to be unified and orthogonal to each other[2], then $\frac{\partial g}{\partial\theta_r}=\mathrm{tr}(\mathbf{X}^H\mathbf{D}_r\mathbf{X})$ with $\mathbf{X}^H\mathbf{X}=\mathbf{I}_{d_k}$, where the columns of $\mathbf{X}$ are consisted of $\mathbf{x}_i$, for $i=1,\ldots,d_k$. Thus we have the gradient of the objective function $\mathbf{y}=(y_1,y_2,\ldots,y_R)^T$, where $y_r=\frac{\partial h}{\partial\theta_r}=\eta_r-\mathrm{tr}(\mathbf{X}^H\mathbf{D}_r\mathbf{X})$.

Let $\boldsymbol{\theta}^j$ and $\mathbf{y}^j$ be the iterative point and its gradient in the jth iteration, respectively. By the projected gradient method, in the jth iteration the dual variables should be updated as $\boldsymbol{\theta}^{j+1}=(\boldsymbol{\theta}^j-\alpha_j\mathbf{y}^j)_+$, where $(\mathbf{a})_+$ means $\max(\mathbf{a},0)$ componentwisely. Here we take α_j as the Barzilai-Borwein stepsize [18], to accelerate the algorithm: $\alpha_j=\frac{\mathbf{s}_j^T\mathbf{t}_j}{\mathbf{t}_j^T\mathbf{t}_j}$, where $\mathbf{s}_j=\boldsymbol{\theta}^j-\boldsymbol{\theta}^{j-1}$ and $\mathbf{t}_j=\mathbf{y}^j-\mathbf{y}^{j-1}$.

From the above analysis, we summarize the algorithm to solve problem (7) as follows:

[1]Originally, it should be "maximization" in the dual problem. Here we have converted the maximization into minimization by multiplying the objective function with -1 for analysis convenience.

[2]This can be done because the matrices $\mathbf{D}_0$ and $\mathbf{D}_r$ are all symmetric, for all $r\in\mathcal{R}$.

input : Random initial point $\boldsymbol{\theta}\ge\mathbf{0}$, $\epsilon\ge 0$, $j=1$
output: the solution of (8) $\boldsymbol{\theta}^*=\boldsymbol{\theta}^j$, the solution of (7) $\mathbf{X}^*=\nu_{\min}^{d_k}(\mathbf{D}_0+\sum_{r\in\mathcal{R}}\theta_r^*\mathbf{D}_r)$
repeat
  Update the iterative point: $\boldsymbol{\theta}^{j+1}=(\boldsymbol{\theta}^j-\alpha_j\mathbf{y}^j)_+$;
  $j:=j+1$;
until $\|\mathbf{s}_j\|_2<\epsilon$;

Algorithm 1: Algorithm for subproblem (7)

The projected gradient method guarantees to achieve the stationary point of the dual problem (8) [14, Theorem 11.5.5]. As the dual problem of (7), (8) is convex [16]. Its stationary point is actually the optimal solution. Thus we achieve the optimal solution of (8) as $\boldsymbol{\theta}^*$. However, as we analyzed above, the primal problem (7) is nonconvex, and there might be a positive dual gap between the two problems (7) and (8). That is, the solution $\mathbf{X}^*$ achieved from Algorithm 1 might not be the optimal solution of (7). Fortunately, in Theorem 1 we are able to prove there is zero duality gap between (7) and (8), that is, $\mathbf{X}^*$ is the optimal solution of (7).

Theorem 1 *Given $\boldsymbol{\theta}^*$ as the optimal solution of the dual problem (8), $\mathbf{X}^*$ is the optimal solution of (7), where $\mathbf{X}^*$ is constructed by $\boldsymbol{\theta}^*$ as $\mathbf{X}^*=\nu_{\min}^{d_k}(\mathbf{D}_0+\sum_{r\in\mathcal{R}}\theta_r^*\mathbf{D}_r)$.*

Proof: As the optimal solution, $\boldsymbol{\theta}^*$ satisfies the KKT conditions of (8). Let t_r^* be the corresponding Lagrange multiplier for the rth bound constraint "$\theta_r\ge 0$", for any $r\in\mathcal{R}$. Then the KKT conditions are listed below.

KKT1. The gradient of the Lagrangian function being 0:
$\eta_r-\mathrm{tr}[(\mathbf{X}^*)^H\mathbf{D}_r\mathbf{X}^*]-t_r^*=0.$

KKT2. Complementary conditions: for all $r\in\mathcal{R}$, $\theta_r^*t_r^*=0$.

KKT3. Feasibility conditions: for all $r\in\mathcal{R}$, $\theta_r^*\ge 0$, $t_r^*\ge 0$.

From condition **KKT1** and **KKT3**, it is easy to deduce the inequalities $\eta_r-\mathrm{tr}[(\mathbf{X}^*)^H\mathbf{D}_r\mathbf{X}^*]\ge 0$ hold for all $r\in\mathcal{R}$. This proves $\mathbf{X}^*$ is a feasible solution of (7).

Noticing the dual problem (8) is to minimize $h(\boldsymbol{\theta})$, we have the duality gap between the two problems (7) and (8) expressed as the sum of the two objective function values [16]. The duality gap is always no less than 0 with any $\mathbf{X}$ and $\boldsymbol{\theta}$ which are feasible for the two problems, respectively. If the sum equals to 0, there is zero duality gap between the two problems, and the corresponding feasible solutions are actually the optimal solutions for both problems. With $\mathbf{X}^*$ and $\boldsymbol{\theta}^*$ as the feasible solutions, respectively, the duality gap is

$$\begin{aligned} & \mathrm{tr}[(\mathbf{X}^*)^H\mathbf{D}_0\mathbf{X}^*]+h(\boldsymbol{\theta}^*)\\ = & \mathrm{tr}[(\mathbf{X}^*)^H\mathbf{D}_0\mathbf{X}^*]+\sum_{r\in\mathcal{R}}\theta_r^*\eta_r-g(\boldsymbol{\theta}^*)\\ = & \mathrm{tr}[(\mathbf{X}^*)^H\mathbf{D}_0\mathbf{X}^*]+\sum_{r\in\mathcal{R}}\theta_r^*\{\mathrm{tr}[(\mathbf{X}^*)^H\mathbf{D}_r\mathbf{X}^*]+t_r^*\}\\ & -\mathrm{tr}[(\mathbf{X}^*)^H(\mathbf{D}_0+\sum_{r\in\mathcal{R}}\theta_r^*\mathbf{D}_r)\mathbf{X}^*]\\ = & \sum_{r\in\mathcal{R}}\theta_r^*t_r^*\\ = & 0. \end{aligned}$$

The second equality and the last equality hold from condition **KKT1** and **KKT2**, respectively. We deduce that the duality gap equals to 0, and consequently $\mathbf{X}^*$ is the optimal solution of (7). □

Shown by Theorem 1, the optimal solution of (7) is achieved by Algorithm 1.

3.4. Algorithm for the multiple stream TSTINR model

We conclude the algorithm to solve the multiple stream model problem (2) as follows.

input : initial value of $\mathbf{U}_k, k \in \mathcal{K}$ and $\mathbf{W}_r, r \in \mathcal{R}, C = 1$
output: $\mathbf{U}_k, \mathbf{V}_k, k \in \mathcal{K}$ and $\mathbf{W}_r, r \in \mathcal{R}$
repeat
  Update decoder $\mathbf{V}_k$ by solving (5), $k \in \mathcal{K}$;
  Update relay AF matrix $\mathbf{W}_r$ by solving (6), $r \in \mathcal{R}$;
  Update precoder $\mathbf{U}_k$ by solving (7), $k \in \mathcal{K}$;
  Update C as $C := \frac{P^S}{P^I + P^N}$;
until *Objective function value converges*;

Algorithm 2: Algorithm for multiple stream TSTINR model

Because each subproblem is solved optimally, the objective function value of (4) reduces monotonically, and consequently the objective function value of (2) "TSTINR" converges [11]. However as the variables have been separated into more than two blocks, there is no theoretical guarantee that the algorithm converges to the stationary point of (2).

4. SIMULATIONS

In this section, our proposed multiple stream TSTINR model is evaluated by simulations. Each element of $\mathbf{G}_{rk}$ and $\mathbf{H}_{kr}, k \in \mathcal{K}, r \in \mathcal{R}$ are generated as i.i.d complex Gaussian distribution with zero mean and unit variance. The noise variances are set as $\sigma_r^2 = \sigma^2 = 1$ and $\mu_k^2 = \mu^2 = 1$, for any $r \in \mathcal{R}$ and $k \in \mathcal{K}$. Initial values of $\{\mathbf{U}\}$ and $\{\mathbf{W}\}$ are randomly generated, and scaled to be feasible. Initially, the parameter C is set as 1. 100 random realizations of different channel coefficients are generated to evaluate the average performance. Here we define SNR as SNR$= \frac{p_k^U}{\mu^2} = \frac{p_r^R}{\sigma^2}$, thus for all $r \in \mathcal{R}$ and $k \in \mathcal{K}$, $p_k^U = p_r^R =$SNR. We use system sum rate R_{sum} as the measure of QoS.

We first compare our TSTINR model with the WMMSE model in [10], which is an effective model for the sum rate maximization problems. Consider the $(4 \times 4, 2)^3 + 4^3$ network, which means $K = R = 3$, $M_k = N_k = L_r = 4$ and $d_k = 2$ for all $k \in \mathcal{K}$ and $r \in \mathcal{R}$. Fig. 1 shows the average achieved sum rate with respect to different SNR values by both the multiple stream TSTINR model and the WMMSE model. The two curves representing the two models intersect at about SNR$=$ 20dB. With SNR above 20dB, our proposed TSTINR model achieves more sum rate than the WMMSE model[3]. In the simulation results we observe that the WMMSE model usually results in rank one precoding matrices. This might be the reason to restrain the WMMSE model to achieve high sum rate in medium to high SNR scenarios.

In Fig. 2 the multiple TSTINR models with individual and total relay power constraints [12] are compared. We consider the $2 \times 2 \times 2$ networks ($K = R = 2$) with 2 antennas for each user and 4 antennas for each relay. The achieved sum rate by transmitting different number of data streams with respect to different SNR values are depicted. "$(1, 1)$ data streams" means each user pair transmit single data stream. In multiple stream cases, we apply the multiple stream TSTINR model proposed here and in [12]. In single stream cases with both individual and total relay power constraints, the achieved sum rates are the maximum between the WMMSE model [10] and the TSTINR model. Indicated in Fig. 2, from one aspect, the multiple stream TSTINR model with individual relay power constraints has almost the same performance as that with total relay constraint[4]; from the other aspect, the achieved sum rate indeed benefits from the multiple stream transmission.

[3]The result of the WMMSE model is almost the same as the data in [10, Fig. 4], but scaled with $\frac{1}{2}$ here due to our different expression of sum rate from [10].

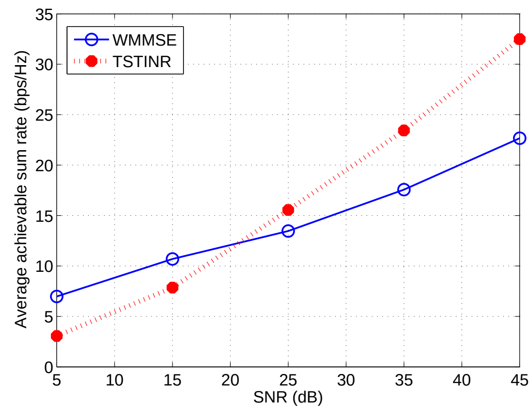

Fig. 1 Comparison between TSTINR and WMMSE

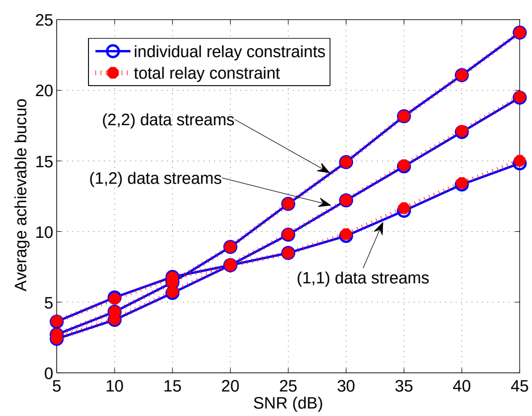

Fig. 2 Comparison of total and individual relay transmit power constraints

5. CONCLUSION

In this paper, we have built up the multiple stream TSTINR model with individual relay transmit power constraints. Applying the alternating iteration method, we have decomposed the optimization problem into several subproblems. The nonconvex precoding subproblem is solved by the dual method. And zero duality gap is proved between the primal and dual problems. Simulation results show our proposed new model outperforms the WMMSE model in [10] in medium to high SNR scenarios, and the new model with individual relay transmit power constraints achieved almost the same sum rate as that with total relay power constraint in [12].

[4]This performance is similar as that in [10, Fig. 4]

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