BLIND EQUALIZATION AND AUTOMATIC MODULATION CLASSIFICATION BASED ON PDF FITTING

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ABSTRACT

In this paper, a blind equalizer based on probability density function (pdf) fitting is proposed. It does not require any prior information about the transmission channel or the emitted constellation. We also investigate Automatic Modulation Classification (AMC) for Quadrature Amplitude Modulation (QAM) based on the pdf of the equalized signal. We propose three new approaches for AMC. The first employs maximum likelihood functions (ML) of the modulus of real and imaginary parts of the equalized signal. The second is based on the lowest quadratic or Bhattacharyya distance between the estimated pdf of the real and imaginary parts of the equalizer output and the theoretical pdfs of M-QAM modulations. The third approach is based on theoretical pdf dictionnary learning. The performance of the identification scheme is investigated through simulations.

Index Terms— Blind equalization, AMC, ML, Bhattacharyya distance, dictionary learning

1. INTRODUCTION

AMC is a high requirement of intelligent systems in both military and civil domains. It has been of significant importance for cognitive radios when the receiver has no knowledge about the channel and transmitted modulation. It is very useful in adaptive modulation contexts where the transmitter has to adapt the emitted modulation to the transmission conditions. [1] gives a detailed overview on the techniques developed in the field of AMC. There are two approaches for AMC [1]. One is based on likelihood functions where the detected modulation is the one that maximises the likelihood among all hypothesis [2] [3]. The second approach is based on statistical characteristics of the received signal and their comparison with the theoretical ones [4] [5].

Most of these techniques consider an additive white Gaussian noise (AWGN) channel. However, in real scenarios, signal propagation undergoes multipaths. In this case, Intersymbols Interference (ISI) has to be reduced before proceeding to AMC. In [6], Wu et *al.* proposed to estimate the multipah channel from the moments of the received signal before using a cumulant-based classifier. Instead of estimating the channel, an equalizer can be used to reduce the ISI. Among works in the literature that addressed joint blind equalization and AMC, we can mention [7], where S. Barbarossa et *al.* proposed to use multiple equalizing branches, each one adapted to a specific constellation. This leads to a complex architecture system where the filter that provides the smallest cost function indicates the correct constellation. In [8], the Constant Modulus Algorithm (CMA) was used as a generic equalizer with radius equal to 1 and the amplitude of the equalized signal Characteristic Function (CF) as a technique to recognize the transmitted modulation. In this paper, we propose to use a generic Multi-Modulus Stohastic Quadratic Distane (MSQD-ℓp) equalizer that is more efficient than the CMA [9]. Once the signal is equalized, we identify the transmitted constellation via an ML approach or pdf distance based methods. More specifically, the modulation we detect has the pdf that best fits, in some sense, that of the equalizer output. The key idea here is that we assume that after equalization, we roughly obtain a Gaussian mixture with modes centered on constellation points. The Gaussian nature of equalizer output conditional to transmitted symbol has been discussed in [10]. The rest of the paper is organized as follows. In section 2, the system model and the generic MSQD-ℓp equalizer are introduced. In section 3, the AMC approaches are detailed. In section 4, simulation results are presented. Conclusions of our work are given in section 5.

2. SYSTEM MODEL AND MSQD-ℓP$_{GEN}$ EQUALIZER

2.1. System model

The baseband model of a transmission system with an adaptive blind channel equalizer is shown in Fig.1, where $s(n), n \in \mathbb{Z}$, is the transmitted symbol at time n, that is assumed to be drawn from an M-QAM modulation, $\boldsymbol{h} = [h_0, h_1, ..., h_{L_h-1}]^T$ is the multipath channel finite impulse response with length L_h, while $(.)^T$ denotes the transpose operator, $b(n)$ is an additive white Gaussian noise, $x(n)$ is the equalizer input, $\boldsymbol{w} = [w_0, w_1, ..., w_{L_w-1}]^T$ is the equalizer impulse response, with length L_w and $y(n)$ is the equalized signal at time n. $x(n)$ and $y(n)$ can be modeled as $x(n) = \sum_{i=0}^{L_h-1} h_i s(n-i) + b(n)$ and $y(n) = \sum_{i=0}^{Lw-1} w_i x(n-i) = \boldsymbol{w}^T \boldsymbol{x}(n)$, where $\boldsymbol{x}(n) = [x(n), x(n-1), ..., x(n-Lw+1)]^T$.

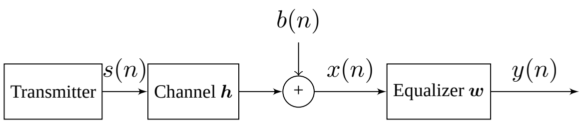


Fig. 1. Baseband model of a transmission system with an adaptive blind channel equalizer.

2.2. MSQD-ℓp$_{gen}$ equalizer

The MSQD-ℓp algorithm [9] aims at minimizing the distance error between observed and assumed pdfs for the real and imaginary parts of the equalizer output. The MSQD-ℓp cost function is given by

$$\begin{aligned} J(\boldsymbol{w}) &= \int_{-\infty}^{\infty} (\hat{p}_{|y_r|^p}(z) - \hat{p}_{|s_r|^p}(z))^2 dz \\ &+ \int_{-\infty}^{\infty} (\hat{p}_{|y_i|^p}(z) - \hat{p}_{|s_i|^p}(z))^2 dz \end{aligned} \tag{1}$$

where $y_r = \Re\{y(n)\}$, $y_i = \Im\{y(n)\}$. For instantaneous pdf estimation, we use the Parzen window method [11]:

$$\hat{p}_u(z) = \frac{1}{N_u} \sum_{k=1}^{N_u} K_\sigma(z - u_k) \tag{2}$$

where u stands for $|s_r|^p$, $|s_i|^p$, $|y_r|^p$ or $|y_i|^p$. We let $N_u = N_s$ for $u = |s_{r,i}|^p$ and $N_u = N_y$ (previous symbols) for $u = |y_{r,i}|^p$. K_σ is a Gaussian kernel with standard deviation σ. Since, in this paper we do not know the transmitted modulation, we propose to use the MSQD-ℓp criterion adapted for a 4-QAM modulation to equalize all emitted M-QAM constellations. Then, expending (1) with this choice where $|s_r|^p = |s_i|^p = 1$ and letting $N_s = 1$ and $N_y = 1$, we get the following generic cost function for the MSQD-ℓp$_{gen}$:

$$J_{p_{gen}}(\boldsymbol{w}) = -K_\sigma(|y_r(n)|^p - 1) - K_\sigma(|y_i(n)|^p - 1) + Cst. \tag{3}$$

Thus, the equalizer coefficient weights are adapted by

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) - \mu \nabla_{\boldsymbol{w}} J_{gen_p}(\boldsymbol{w}) \tag{4}$$

where μ is a fixed step-size. In the following we focus, as in [9], on the cases $p = 2$ and $p = 1$.

2.3. Output Constellations with MSQD-ℓp$_{gen}$ (p=1,2)

We now discuss the performance of the generic equalizers in terms of ISI. To check the reliability of both equalizers, we tested them with different multipath channels. The equalizer is initialized with a tap-centered strategy and its length is set to 21. The step size was blindly adapted to the transmitted modulation basing on the equalizer input power: $\mu_{1,2} = \frac{\mu_{2,1}^{cst}}{P_{\boldsymbol{x}}}$ where $P_{\boldsymbol{x}}$ is the power of $\boldsymbol{x}$. The values of μ_1^{cst} and μ_2^{cst} were set to $\mu_1^{cst} = 6 \times 10^{-3}$ and $\mu_2^{cst} = 4 \times 10^{-3}$ for MSQD-$\ell 1_{gen}$ and MSQD-$\ell 2_{gen}$ respectively after testing the equalizers with a 16-QAM modulation. The kernel bandwidth, σ, of $K_\sigma(x)$ was updated to control the convergence speed of the equalizer and its residual ISI [9]. Since we have no prior information about the emitted constellation, we propose to update the kernel size by

$$\sigma(n) = a\, G(n) + b \tag{5}$$

where,

$$G(n) = \alpha G(n-1) + (1-\alpha) \underbrace{\min}_{k=1,...,N_s} \left((|y(n)|^2 - |s_k|^2)^2\right). \tag{6}$$

$\alpha \in]0,1[$ is a forgetting factor and a and b are fixed empirically. N_s is the number of points in the largest constellation that transmitter may emit. In our case, we set N_s to 256. In figure 2, we show the ISI obtained with the generic equalizers and the input and output constellations with the MSQD-$\ell 2_{gen}$ equalizer for an emitted 16-QAM constellation. We used one of typical digital radio channel [12].

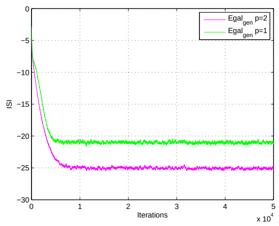
According to figure 2(a), we notice that MSQD-$\ell 2_{gen}$ is more efficient than MSQD-$\ell 1_{gen}$ in terms of ISI. We also notice that the equalizer tends to output a constellation inside the square $[-1,1] \times [-1,1]$. We notice the same thing when we use another channel like the Proakis A channel [13]. In the next section we detail on how to calculate the constellation scaling factor before proceeding to AMC.

3. NEW TECHNIQUES FOR AMC

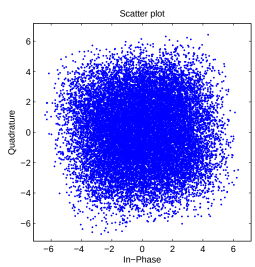
3.1. ML existing approach based on the pdf of the received signal [3] (ML)

Assuming we have K M-QAM modulations to classify. According to this approach, the detected modulation is the one that maximises $f(\boldsymbol{y}(n)|H_j)$ where H_j is the hypothesis to receive the j^{th} modulation $j = 1, 2, ..., K$. The pdf of $\boldsymbol{y}(n)$ is supposed to be a Gaussian mixture with means on the scaled constellation points and the same variance σ_j^2:

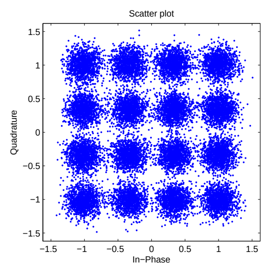
$$f(\boldsymbol{y}(n)|H_j))_{1 \le n \le N} = \frac{1}{M_j} \sum_{l=1}^{M_j} \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{|\boldsymbol{y}(n) - \alpha_j s_l|^2}{2\sigma_j^2}} \tag{7}$$



(a) ISI for MSQD-$\ell 2_{gen}$ and MSQD-$\ell 1_{gen}$ algorithms



(b) MSQD-$\ell 2_{gen}$ input



(c) MSQD-$\ell 2_{gen}$ output

Fig. 2. Convergence curves of MSQD-$\ell 2_{gen}$ and MSQD-$\ell 1_{gen}$ algorithms for SNR$=20$dB.

where M_j is the number of symbols in constellation j. Then the emitted modulation is decided according to (8) :

$$\hat{j} = \arg\max_{1\leq j\leq K} \sum_{n=1}^{N} \ln f(\boldsymbol{y}(n)|H_j) \tag{8}$$

The calculation of α_j and σ_j will be detailed in the following, where we introduce the three approaches that we propose for AMC.

3.2. ML approach based on the pdf of the modulus of real and imaginary parts of the equalized signal ($\mathrm{ML_{prop}}$)

With this approach, the detected modulation is the one that maximises (9) where
$\boldsymbol{\gamma} = [|\Re\{\boldsymbol{y(1)}\}|, .., |\Re\{\boldsymbol{y(N)}\}|, |\Im\{\boldsymbol{y(1)}\}|, .., |\Im\{\boldsymbol{y(N)}\}|]$.
Here, we take the absolute values of real and imaginary parts of the equalized signal to increase the number of data and decrease the number of modes in order to improve pdf calculation. If we suppose that the pdf of $\boldsymbol{y}$ is a 2D Gaussian mixture, then the pdfs of $\Re\{\boldsymbol{y}\}$ and $\Im\{\boldsymbol{y}\}$ are 1D Gaussian mixture. Thus, the pdf of $\boldsymbol{\gamma}$ is a mixture of folded normal distribution:

$$\begin{aligned} f(\boldsymbol{\gamma}(n)|H_j)_{1\leq n\leq 2N} &= \sum_{i=1}^{I_j} p_i \frac{1}{\sigma_j\sqrt{2\pi}} \Big(e^{-\frac{(\boldsymbol{\gamma}(n)+\alpha_j s_{Ri})^2}{2\sigma_j^2}} \\ &+ e^{-\frac{(\boldsymbol{\gamma}(n)-\alpha_j s_{Ri})^2}{2\sigma_j^2}}\Big), 1_{\boldsymbol{\gamma}(n)\geq 0} \end{aligned} \tag{9}$$

where I_j is the number of different positive real parts of symbols under hypothesis H_j, s_{Ri} are their values and p_i are their probabilities: $\sum_{i=1}^{I_j} p_i = 1$. Note that in modulations such as 32-QAM, weights p_i can be not uniform. The standard deviation σ_j measures the dispersion around the constellation points under each hypothesis and it is estimated by:

$$\sigma_j^2 = \frac{1}{2N}\sum_{n=1}^{2N} |\boldsymbol{\gamma}(n) - d_j(n)|^2 \tag{10}$$

where $d_j(n)$ is the absolute real value of the constellation symbols under hypothesis H_j that is the closest to $\boldsymbol{\gamma}(n)$. The signal scaling factor α_j is introduced to take into account the effect of the generic equalizer rescales the constellation as discussed at the end of section 2.1. α_j is calculated as a function of the mean of $\boldsymbol{\gamma}$, $m_{\text{est}} = \mathbb{E}\{\boldsymbol{\gamma}\}$, and the positive real parts of the constellation under the hypothesis H_j:

$$\alpha_j = \frac{m_{\text{est}}}{\sum_{k=1}^{M_j} p_k |\Re\{s_{jk}\}|} \tag{11}$$

where p_k is the weight of the mode s_{jk}. Then, we calculate the logarithm of the likelihood function of a sequence of $2N$ absolute real and imaginary parts of N consecutive symbols and define the decision variable D as

$$D_j = \arg\max_{H_j} \sum_{n=1}^{2N} \ln f(\boldsymbol{\gamma}(n)|H_j) \tag{12}$$

3.3. Bhattacharyya (DBch) or quadratic (DQ) pdf distance based approach for AMC

With this method, the real and imaginary parts of the equalized signal are considered:
$\boldsymbol{\Gamma} = [\Re\{\boldsymbol{y(1)}\}, .., \Re\{\boldsymbol{y(N)}\}, \Im\{\boldsymbol{y(1)}\}, .., \Im\{\boldsymbol{y(N)}\}]$. unlike to the previous approach, here we take the real and imaginary parts of the equalized symbols to make the Gaussian mixture assumption and the use of the Gaussian kernel estimator, for observed data pdf estimation, more meaningful. The scaling factors α_j and the standard deviations σ_j are estimated as above except that the decisions $d_j(n)$ are taken over the entire set of the real parts of the symbols s_{jk} under the hypothesis H_j. The pdf of $\boldsymbol{\Gamma}$ is then estimated by a Gaussian kernel estimator:

$$\hat{f}_{\boldsymbol{\Gamma}}(x|H_j) = \frac{1}{2Nh_j}\sum_{k=1}^{2N} K_{\sigma_j}\Big(\frac{x-\boldsymbol{\Gamma}(k)}{h_j}\Big) \tag{13}$$

where h_j is the bandwidth smoothing parameter such as $h_j = (\frac{4\sigma_j^2}{3\times 2N})^{\frac{1}{5}}$ [14]. The theoretical pdfs are calculated supposing that ideally, after eliminating the ISI by the generic equalizer, we get a Gaussian mixture pdf of the noisy emitted constellation with means the scaled constellation points. The theoretical pdfs are then given by:

$$f_{\boldsymbol{\Gamma}}(x|H_j) = \sum_{k=1}^{M_j} \frac{1}{M_j} K_{\sigma_j}(x - \alpha_j\,\Re\{s_{jk}\}) \tag{14}$$

Finally, we select the modulation basing on the quadratic or Bhattacharyya distances between $\hat{f}_\Gamma(x|H_j)$ and $f_\Gamma(x|H_j)$ such as

$$\hat{j} \;=\; \arg\min_{1\le j\le K} D_B\big(\hat{f}_{\boldsymbol{\Gamma}}(x|H_j), f_{\boldsymbol{\Gamma}}(x|H_j)\big) \qquad (15)$$

$$\hat{j} \;=\; \arg\min_{1\le j\le K} D_Q\big(\hat{f}_{\boldsymbol{\Gamma}}(x|H_j), f_{\boldsymbol{\Gamma}}(x|H_j)\big) \qquad (16)$$

where, $D_B(p,q) = -\ln \sum_{x\in X} \sqrt{p(x)q(x)}$ and $D_Q(p,q) = \sum_{x\in X} |p(x)-q(x)|^2$.

3.4. Dictionary learning based approach for AMC (DL)

With this method, we define a dictionary $\boldsymbol{A}$ of theoretical pdfs of all constellations that the transmitter may emit: $f_{\boldsymbol{\Gamma}}(x|H_j)$. $\boldsymbol{A}$ is a matrix such as each column j represents a samples version of $f_{\boldsymbol{\Gamma}}(x|H_j)$. The idea of this approach consists in minimising the following penalized criterion:

$$\hat{\boldsymbol{v}} = \arg\min_{\boldsymbol{v}}\big(||\hat{f}_{\boldsymbol{\Gamma}}(x) - \boldsymbol{A}\boldsymbol{v}||_2^2 + \lambda||\boldsymbol{v}||_1\big) \qquad (17)$$

where $\hat{f}_{\boldsymbol{\Gamma}}(x)$ is the sampled pdf of the equalized signal that is estimated by a Gaussian kernel estimator. The minimum $\hat{\boldsymbol{v}}$ in (17) should be sparse due to the $\ell 1$ penalty term. Ideally, we obtain a vector $\hat{\boldsymbol{v}}$ that has only one element equal to 1 and all the others equal to 0. The index j of 1 in $\hat{\boldsymbol{v}}$ indicates the right hypothesis H_j for the emitted modulation.

4. SIMULATION RESULTS

In this part, we show the results obtained with the complex channel of section 2.1 [12]. Modulation detection is performed in SNR intervals where modulations usually work. Figure 3 shows the probability of correct classification (Pcc) vs SNR for the four methods detailed in section 3 when a 16-QAM constellation is transmitted. It shows that the quadratic

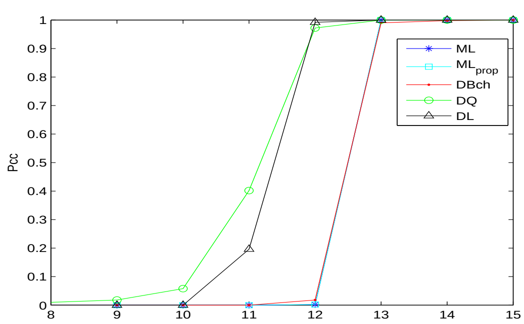


Fig. 3. Pcc for 16-QAM modulation.

and dictionnary learning based methods outperform both ML and Bhattacharyya distance approches. Figure 4 shows that for the 32-QAM modulation the existing ML approach outperforms other methods. In figure 5, a 64-QAM modulation is considered. We notice that for low values of SNR, the Bhattacharyya distance approach outperforms the others. Quadratic distance approach offers a good compromise between performance and detection curve slope.

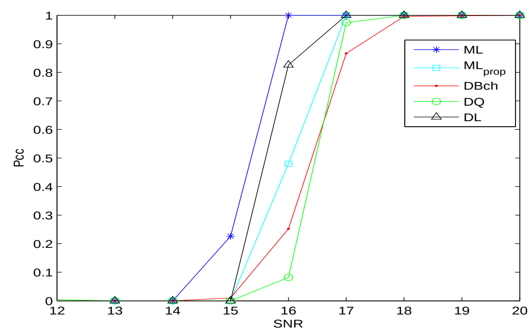


Fig. 4. Pcc for 32-QAM modulation.

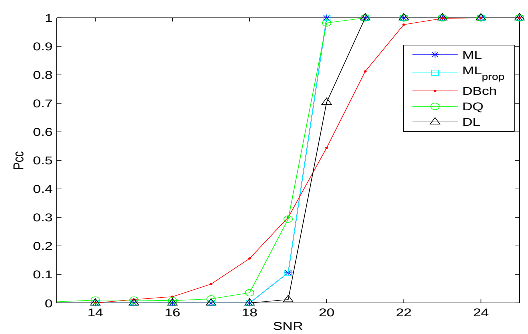


Fig. 5. Pcc for 64-QAM modulation.

We can also notice from these figures that the two ML approahes have quite same performance and that the increase of the number of symbols per mode does not provide a significant benefit in terms of Pcc. However the proposed method is less complex since it requires less computations of exponentials.

5. CONCLUSION

In this paper, a new joint blind equalization and AMC approach has been proposed. The key idea of the AMC process is based on the assumption that the equalized signal can be approximated as a Gaussian mixture with modes centered at constellation points. The results show that the quadratic distance approach is a good compromise.

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