COMPARATIVE PERFORMANCE EVALUATION OF ERROR REGULARIZED TURBO-MIMO MMSE-SIC DETECTORS IN GAUSSIAN CHANNELS

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ABSTRACT

We evaluate the performance of a set of low complexity successive interference cancellation (SIC) detection algorithms in comparison to optimal maximum a-posteriori probability (MAP) detection and low complexity linear filter detection in a Turbo multiple-input multiple-output (Turbo-MIMO) system. We show that both linear and SIC soft detection algorithms perform similarly poorly for iterative receivers, even if the channel decoder output is available at the detector. We propose a low complexity combined a-priori/a-posteriori information-based error regularization technique, that improves the performance of the Turbo-MIMO design considerably. With this regularization technique, we show that a decoding gain of 2.2 dB can be achieved in an LTE compliant Turbo-MIMO receiver.

***Index Terms*—** Turbo-MIMO, SIC, detection, error propagation, regularization

1. INTRODUCTION

The multiple-input multiple-output (MIMO) channel has found great attention in research and development of wireless transmitters and receivers, due to a potential increase of data rate that scales linearly with the minimum of the number of transmit and receive antennas [1], [2]. The achievable increase in data rate comes with the drawback of a high receiver complexity. Unfortunately, the optimal joint detection of the transmit vector mapping and decoding of the underlying channel code exhibits a prohibitive computational complexity which rises exponentially with the number of bits encoded in one code block and the number of transmit antennas. In a conventional MIMO receiver, the computational complexity can be reduced by performing the detection and decoding of the transmit signal separately. Furthermore, if the transmit information is transmitted using a bit interleaved coded modulation (BICM) scheme, the decoding of the transmit signal can be performed very efficiently using a bit-wise optimal channel decoder [3], [4]. To facilitate bitwise decoding, the MIMO detector must provide a-priori information on the coded bit sequence to the decoder, which can conveniently be expressed in terms of log likelihood ratios (LLRs). Often, the greater part of the receiver complexity is due to the computation of the a-priori LLRs in the MIMO soft detector, while the computational complexity of decoding is negligible due to the availability of highly efficient decoder implementations [5]–[7].

While in conventional MIMO receivers, without information given to the detector by the decoder, an unsuccessful attempt at decoding the transmit information in the channel decoder results in packet loss or the necessity for a request for retransmission of the code block, an approach called Turbo-MIMO [8]–[14] has recently found a lot of interest in receiver development. In contrast to conventional MIMO receivers, Turbo-MIMO receivers include a feedback loop from the channel decoder to the MIMO detector that is used to provide a-priori information on the coded bit sequence that aids to the detection. In case of an unsuccessful attempt at decoding the transmit information in the decoder, this feedback aids to a more accurate re-detection of the transmit vector. The probability of error-free decoding is increased in the next attempt, with no additional transmission from the transmitter to the receiver necessary. The feedback of a-posteriori information and the reattempts at detection and decoding can be performed iteratively until the information exchange between decoder and detector depletes, or a preset maximum number of Turbo-MIMO iterations can be performed. Turbo-MIMO can hence provide a significantly higher throughput than conventional MIMO systems.

While Turbo-MIMO systems reduce the performance degradation resulting from separate detection and decoding significantly, the bottleneck of the Turbo-MIMO system is the MIMO detection algorithm. While in conventional MIMO systems quasi-optimal detectors are more likely to be implemented in near future, there is a need for highly optimized low complexity MIMO detection algorithms for Turbo-MIMO systems. Unfortunately, promising suboptimal detection algorithms that have been delevoped for conventional MIMO systems often perform poorly w.r.t. optimal detection and even low complexity linear detectors. In this work, we focus on the class of successive interference cancellation (SIC) Turbo-MIMO receivers. Prominent realizations of SIC receivers are the Vertical-Bell Laboratories Layered Space-Time (VBLAST) [15] and the SoftSIC [16], [17] algorithms, which have been shown to provide a significant increase in bit error rate (BER) performance in conventional uncoded and coded MIMO systems, respectively. Despite their low computational complexity, SIC algorithms, in particular, have been shown to perform below expectations in iterative receivers. The reason for this shortcoming was found to be the error propagation effect that results from a symbol detection failure in the MIMO detector [18]. Hereby, *error propagation* describes a situation in that one or more incorrect symbol detections trigger a multitude of decoding failures of transmit information bits which relate to the incorrectly detected symbol via redundant code bits [19].

In the following, we will examine a set of methods that can be used to limit the impact of a false symbol detection through a regularization technique implemented in the linear filter. We will first introduce a system model for the Turbo-MIMO receiver. Secondly, we will provide an overview of the state-of-the-art optimal, linear, and standard SIC detection methods. We will then give a detailed overview of the regularization technique and provide alternative implementations of the same technique, which can be used to balance the affordable computational complexity with the required BER target of the receiver. We will conclude this paper with simulation results based on simulation parameters for a realistic, standardized LTE system.

2. SYSTEM MODEL

We model the desired information signal at the receiver as the binary bit vector $\boldsymbol{b} \in \mathbb{B}^{N_b}$, where $\mathbb{B} = \{-1, +1\}$ and the encoded binary transmit signal as $\boldsymbol{c} \in \mathbb{B}^{N_c}$, where N_c is the code block length, and $\eta = \frac{N_b}{N_c}$ is the rate of the outer channel code. We assume that both, $\boldsymbol{b}$ and $\boldsymbol{c}$ are uniformly distributed among the elements of $\mathbb{B}$. We describe the MIMO channel output as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}, \tag{1}$$

$$\boldsymbol{s} \in \mathcal{Q}_M^{N_\mathrm{T}},\ \boldsymbol{H} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{T}},\ \boldsymbol{n} \in \mathbb{C}^{N_\mathrm{R}}, \tag{2}$$

where N_R and N_T is the number of receive and transmit antennas, $\mathcal{Q}_M$ is a normalized, Gray coded quadrature amplitude modulation (QAM) alphabet of size M and symbol spacing $\sqrt{\frac{6}{M-1}}$. We assume that both the channel matrix $\boldsymbol{H}$ and the additive noise vector $\boldsymbol{n}$ are zero-mean complex Gaussian distributed with $\mathrm{vec}(\boldsymbol{H}) \sim \mathcal{N}_\mathbb{C}(\boldsymbol{0}, \mathbf{I})$ and $\boldsymbol{n} \sim \mathcal{N}_\mathbb{C}(\boldsymbol{0}, \rho^{-1}\mathbf{I})$, where we assume that the realization of $\boldsymbol{H}$ in (1) is perfectly known to the receiver, and ρ denotes the SNR. To constitute $\boldsymbol{s}$, we let $\boldsymbol{c}_\pi \in \mathbb{B}^{N_\mathrm{T} \log_2 M}$ denote a subset of the randomly permuted elements of $\boldsymbol{c}$. We assume that this "random" interleaving of the bits can be reversed at the receiver with the permutation function at hand. The transmit signal $\boldsymbol{s}$ is then given via the modulation $\boldsymbol{s} = \mathrm{m}(\boldsymbol{c}_\pi) \Leftrightarrow \boldsymbol{c}_\pi = \mathrm{m}^{-1}(\boldsymbol{s})$. In our notation, the superscript $\cdot^{(k)}$ references the occurrence of a value during the kth Turbo-MIMO iteration, and the superscript $\cdot^{(k,t)}$ refers to the detection of the tth symbol s_t of the transmit vector $\boldsymbol{s} = [s_1, \ldots, s_{N_\mathrm{T}}]^\mathrm{T}$ in the kth turbo iteration. A subscript $\cdot_i$ denotes the ith scalar element of a vector.

3. TURBO DETECTION ALGORITHMS

In order to facilitate the exchange of a-priori information on the ith coded bit of $\boldsymbol{c}_\pi$, the so called a-priori LLRs

$$\lambda_i^{(k)} = \ln \frac{\Pr\{c_i = +1 | \boldsymbol{y}, \boldsymbol{\ell}^{(k-1)}\}}{\Pr\{c_i = -1 | \boldsymbol{y}, \boldsymbol{\ell}^{(k-1)}\}}, \quad 1 \leq i \leq N_\mathrm{T} \log_2 M, \tag{3}$$

are used, where $\boldsymbol{\ell}^{(k-1)} \in \mathbb{R}^{N_\mathrm{T} \log_2 M}$ denotes the a-posteriori LLRs for $\boldsymbol{c}_\pi$ computed by the channel decoder in the $(k-1)$th Turbo-MIMO iteration. For $k = 1$, when no a-posteriori information on $\boldsymbol{c}_\pi$ is available, $\boldsymbol{\ell}^{(0)} = \boldsymbol{0}_{N_\mathrm{T} \log_2 M \times 1}$, i.e., all realizations of $\boldsymbol{c}_\pi$ are equally likely as in the conventional MIMO receiver. For $k \geq 1$, we assume that the extrinsic information $\boldsymbol{\lambda}_\mathrm{e}^{(k)} = \boldsymbol{\lambda}^{(k)} - \boldsymbol{\ell}^{(k-1)}$ gained in the kth detection cycle is forwarded to a bitwise optimal channel decoder [5], which then computes the a-posteriori LLRs $\boldsymbol{\ell}^{(k)}$ for $\boldsymbol{c}_\pi$. Given that a decoding error is detected, the extrinsic information $\boldsymbol{\ell}_\mathrm{e}^{(k)} = \boldsymbol{\ell}^{(k)} - \boldsymbol{\lambda}^{(k)}$ gained in the kth decoding cycle is fed back to the MIMO detector, and (3) is recomputed using a MIMO detection algorithm. This cyclic process repeats for $k > 1$, until all decoding errors have been corrected, or no more extrinsic information can be gained from further iterations.

3.1. Approximate LLR Computation

In order to compute or approximate (3), a variety of methods known from conventional MIMO systems can be extended to incorporate a-posteriori information from the channel decoder output. Similar to conventional MIMO systems, the choice of the detection algorithm is subject to the affordable computational power and the desired accuracy of the LLR values. Since the implementation of a-posteriori information is different for bitwise optimal maximum a-posteriori probability (MAP) detection, SIC detection, and linear detection, we will introduce the respective detection methods in the following.

3.1.1. Turbo (MaxLog) MAP Detection

In order to obtain the exact a-priori information, the two probabilities in (3) can be computed by two mutually disjoint sets of MAP hypotheses according to

$$\lambda_i^{(k)} = \ln \frac{\sum_{\boldsymbol{x} \in \mathcal{X}_i^{+1}} f_{\boldsymbol{y}|\boldsymbol{s},\boldsymbol{\ell}}(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{\ell}^{(k-1)})}{\sum_{\boldsymbol{x}' \in \mathcal{X}_i^{-1}} f_{\boldsymbol{y}|\boldsymbol{s},\boldsymbol{\ell}}(\boldsymbol{y}|\boldsymbol{x}', \boldsymbol{\ell}^{(k-1)})} \tag{4}$$
$$= \ln \frac{\sum\limits_{\boldsymbol{x} \in \mathcal{X}_i^{+1}} \exp(-\rho\|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}\|_2^2 + \frac{1}{2}\mathrm{m}^{-1}(\boldsymbol{x})^\mathrm{T}\boldsymbol{\ell}^{(k-1)})}{\sum\limits_{\boldsymbol{x}' \in \mathcal{X}_i^{-1}} \exp(-\rho\|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}'\|_2^2 + \frac{1}{2}\mathrm{m}^{-1}(\boldsymbol{x}')^\mathrm{T}\boldsymbol{\ell}^{(k-1)})}$$

where $\mathcal{X}_i^c = \{\boldsymbol{x} \in \mathcal{Q}_M^{N_\mathrm{T}} | [\mathrm{m}^{-1}(\boldsymbol{x})]_i = c\}$, i.e, $\mathcal{X}_i^c$ is the set of all possible transmit vectors for which the ith bit of $\boldsymbol{c}_\pi$ matches the binary value c. Since $|\mathcal{X}_i^c| = M^{N_\mathrm{T}}/2$, the number of computations required to compute the outcome of the hypothesis test (3) rises exponentially with the number of transmit antennas. However, $\lambda_i^{(k)}$ can be "max-log" approximated [20] for $\rho \to \infty$ based on the minimizer

$$h_{i,c}^\mathrm{MIN} = \min_{\boldsymbol{x} \in \mathcal{X}_i^c} \left\{ \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}\|_2^2 - \frac{1}{2}\mathrm{m}^{-1}(\boldsymbol{x})^\mathrm{T}\boldsymbol{\ell}^{(k-1)} \right\}, \tag{5}$$

with the approximate MaxLogMAP LLR given as

$$\lambda_i^{(k)} \approx \ln \frac{\exp(-h_{i,+1}^\mathrm{MIN})}{\exp(-h_{i,-1}^\mathrm{MIN})} = h_{i,-1}^\mathrm{MIN} - h_{i,+1}^\mathrm{MIN}. \tag{6}$$

For finding (5), a set of algorithms known as depth-first sphere detectors, K-best breadth-first, and tree search algorithms [21]–[25] are renowned for exploiting the structure of the problem (5). The quasi-optimal sphere detectors and fixed-complexity implementations thereof, in particular, are known to work efficiently only for $\rho \to \infty$, while otherwise, in low SNR situations, the numerical complexity of finding (5) is similar to that of (4).

3.1.2. Turbo MMSE Estimator

If a-posteriori information of the coded bits $\boldsymbol{c}_\pi$ is available, the well-known linear minimum mean square error (MMSE) estimator can be used to improve the estimate $\tilde{\boldsymbol{s}}^{(k)}$ of $\boldsymbol{s}$ iteratively. In contrast to the conventional linear filter approach, the optimization problem in the Turbo-MIMO system can be restated as the minimization of the estimation error w.r.t. the estimate of the previous Turbo-MIMO iteration [26], [27]. With the initial estimate $\tilde{\boldsymbol{s}}^{(0)} = \boldsymbol{0}_{N_\mathrm{T} \times 1}$, for $k \geq 1$, the optimization problem and its solution is given by

$$\boldsymbol{G}^{(k),\mathrm{H}} = \underset{\boldsymbol{G}^\mathrm{H}}{\arg\min}\, \mathrm{E}\{\| \boldsymbol{G}^\mathrm{H} \boldsymbol{y}_\triangle^{(k)} - \boldsymbol{s}_\triangle^{(k)} \|^2\} \tag{7}$$
$$= \mathrm{E}\{\boldsymbol{s}_\triangle^{(k)} \boldsymbol{y}_\triangle^{(k),\mathrm{H}}\}\, \mathrm{E}\{\boldsymbol{y}_\triangle^{(k)} \boldsymbol{y}_\triangle^{(k),\mathrm{H}}\}^{-1} \tag{8}$$
$$= \boldsymbol{C}_{\boldsymbol{s}_\triangle}^{(k)} \boldsymbol{H}^\mathrm{H} (\boldsymbol{H} \boldsymbol{C}_{\boldsymbol{s}_\triangle}^{(k)} \boldsymbol{H}^\mathrm{H} + \rho^{-1}\mathbf{I})^{-1}, \tag{9}$$

where we consider all expectations in (7)–(9) conditional to the a-posteriori decoder LLRs $\boldsymbol{\ell}^{(k-1)}$, e.g., $\boldsymbol{C}_{\boldsymbol{s}_\triangle}^{(k)} = \mathrm{E}\{\boldsymbol{s}_\triangle^{(k)} \boldsymbol{s}_\triangle^{(k),\mathrm{H}} | \boldsymbol{\ell}^{(k-1)}\}$,

$$\boldsymbol{y}_\triangle^{(k)} = \boldsymbol{y} - \boldsymbol{H}\mathrm{E}\{\boldsymbol{s}|\boldsymbol{\ell}^{(k-1)}\}, \tag{10}$$

$$\boldsymbol{s}_\triangle^{(k)} = \boldsymbol{s} - \mathrm{E}\{\boldsymbol{s}|\boldsymbol{\ell}^{(k-1)}\}. \tag{11}$$

Given the a-posteriori LLRs $\boldsymbol{\ell}^{(k-1)}$, the first and second-order moments in (9) can be computed as

$$\mathrm{E}\{s_t|\boldsymbol{\ell}^{(k-1)}\} = \sum_{x\in\mathcal{Q}_M} x \Pr\{s_t = x|\boldsymbol{\ell}^{(k-1)}\}, \quad (12)$$

$$\mathrm{E}\{|s_t|^2|\boldsymbol{\ell}^{(k-1)}\} = \sum_{x\in\mathcal{Q}_M} |x|^2 \Pr\{s_t = x|\boldsymbol{\ell}^{(k-1)}\}, \quad (13)$$

where

$$\Pr\{s_t = x|\boldsymbol{\ell}^{(k-1)}\} \approx \frac{1}{M}\times \prod_{i=1+(t-1)\log_2 M}^{t\log_2 M} \big(1-[\mathrm{m}^{-1}(x)]_i \tanh(\ell_i/2^{(k-1)})\big) \quad (14)$$

can be computed with the a-posteriori LLRs fed back from the decoder. The new estimate $\tilde{\boldsymbol{s}}^{(k)}$ of $\boldsymbol{s}$ in the kth Turbo-MIMO iteration is then given via the affine transformation

$$\tilde{\boldsymbol{s}}^{(k)} = \boldsymbol{G}^{(k),\mathrm{H}}\boldsymbol{y}_\Delta^{(k-1)} + \mathrm{E}\{\boldsymbol{s}|\boldsymbol{\ell}^{(k-1)}\}. \quad (15)$$

The extrinsic information gathered from the kth detection cycle is expressed in terms of LLRs. The computation of the LLRs is performed symbol-wise, similar to (5), based on the linear estimate

$$\tilde{s}_t^{(k)} = \boldsymbol{g}_{\mathrm{MMSE}}^{(k,t),\mathrm{H}}\boldsymbol{y}_\Delta^{(k-1)}, \quad (16)$$

where $\boldsymbol{g}_{\mathrm{MMSE}}^{(k,t)}$ is the tth column of $\boldsymbol{G}^{(k)}$. Each symbol estimate $\tilde{s}_t^{(k)}$ is systematically biased from $[\boldsymbol{s}_\Delta^{(k)}]_t$ by a scaling factor

$$\beta_t^{(k)} = |\boldsymbol{g}_{\mathrm{MMSE}}^{(k,t),\mathrm{H}}\boldsymbol{H}\mathbf{e}_t|\,[\boldsymbol{C}_{s_\Delta}^{(k)}]_{t,t} \quad (17)$$

and has the variance

$$\sigma^2_{\tilde{s}^{(k)}} = \boldsymbol{g}_{\mathrm{MMSE}}^{(k,t),\mathrm{H}}\big(\boldsymbol{H}\boldsymbol{C}_{s_\Delta}^{(k)}\boldsymbol{H}^{\mathrm{H}} + \sigma_n^2\mathbf{I}\big)\boldsymbol{g}_{\mathrm{MMSE}}^{(k,t)} - \beta_t^2. \quad (18)$$

Given (16)–(18), the detection output LLRs can be approximated by

$$\lambda_i^{(k)} \approx \frac{1}{\sigma^2_{\tilde{s}^{(k)}}}\Big(\min_{x\in\mathcal{Q}_{M,i}^{-1}}\big(|\tilde{s}_t^{(k)}-\beta_t^{(k)}x|^2 - \frac{1}{2}\mathrm{m}^{-1}(\mathbf{e}_t x)^{\mathrm{T}}\boldsymbol{\ell}^{(k-1)}\big) - \min_{x'\in\mathcal{Q}_{M,i}^{+1}}\big(|\tilde{s}_t^{(k)}-\beta_t^{(k)}x'|^2 - \frac{1}{2}\mathrm{m}^{-1}(\mathbf{e}_t x')^{\mathrm{T}}\boldsymbol{\ell}^{(k-1)}\big)\Big), \quad (19)$$

where $t = 1+\big\lfloor\frac{i}{\log_2 M}\big\rfloor$ and $\mathcal{Q}_{M,i}^{c} = \{x\in\mathcal{Q}_M|[\mathrm{m}^{-1}(\mathbf{e}_t x)]_i = c\}$ denotes all possible QAM symbols s_t with the ith bit of $\boldsymbol{c}_\pi$ matching the value c, and $\mathbf{e}_t$ is the tth unit vector of the N_{T}-dimensional Euclidean space. Note that in contrast to (4) and (6), for the above approximation, not more than $M/2$ hypotheses pairs need to be computed and the number of hypotheses per Turbo-MIMO iteration rises only linearly in M and N_{T}.

3.1.3. Turbo VBLAST and Soft Interference Cancellation (SoftSIC)

If a little more computational power is available at the receiver than necessary for Turbo MMSE detection, an alternative approach is to perform symbol-wise interference cancellation on the received signal vector $\boldsymbol{y}$ [12]. Noting that the order of the symbol-wise detection affects the performance of the interference cancellation technique [25], we assume, for the sake of simplicity, that the natural detection order $s_1,\dots,s_{N_{\mathrm{T}}}$ is optimal (w.l.o.g). Let $\boldsymbol{z}^{(k)} = [z_1^{(k)},\dots,z_{N_{\mathrm{T}}}^{(k)}]^{\mathrm{T}}$ denote an estimate of $\boldsymbol{s}$ in the kth Turbo-MIMO iteration. The interference cancellation is then performed symbol-wise on the receive vector $\boldsymbol{y}^{(k,1)} = \boldsymbol{y}$ by

$$\boldsymbol{y}^{(k,t)} = \boldsymbol{y}^{(k,t-1)} - \boldsymbol{H}\mathbf{e}_{t-1}z_{t-1}^{(k)}, \quad 2\le t\le N_{\mathrm{T}}. \quad (20)$$

Afterwards, the linear MMSE filter

$$\boldsymbol{g}^{(k,t)} = \arg\min_{\boldsymbol{g}} |\boldsymbol{g}^{\mathrm{H}}\boldsymbol{y}^{(k,t)} - s_t|^2, \quad (21)$$

cf. (7)–(9), is used to obtain the unbiased linear MMSE estimate

$$\zeta_t^{(k)} = \boldsymbol{g}^{(k,t),\mathrm{H}}\boldsymbol{y}^{(k,t)}/\gamma_t^{(k)}, \quad (22)$$

where $\gamma_t^{(k)} = |\boldsymbol{g}^{(k,t),\mathrm{H}}\boldsymbol{H}\mathbf{e}_t|$, and where $\zeta_t^{(k)}$ has the variance

$$\sigma_{\zeta_t}^{(k),2} = (\boldsymbol{g}^{(k,t),\mathrm{H}}(\boldsymbol{H}\boldsymbol{C}_s^{(t)}\boldsymbol{H}^{\mathrm{H}} + \boldsymbol{C}_n)\boldsymbol{g}^{(k,t)})/\gamma_t^{(k),2}, \quad (23)$$

and $\boldsymbol{C}_s^{(t)}$ is the $N_{\mathrm{T}}\times N_{\mathrm{T}}$ diagonal matrix $\mathrm{diag}([\mathbf{0}_{1\times t},1,\dots,1])$. Given $\zeta_t^{(k)}$, the detector output LLRs can be computed as

$$\lambda_i^{(k)} \approx \frac{1}{\sigma_{\zeta_t}^{(k),2}}\big(\min_{x\in\mathcal{Q}_{M,i}^{-1}}|\zeta_t^{(k)}-x|^2 - \min_{x'\in\mathcal{Q}_{M,i}^{+1}}|\zeta_t^{(k)}-x'|^2\big), \quad (24)$$

The symbol $z_t^{(k)}$ is then computed for the interference cancellation (20) according to

$$z_t^{(k)} = \begin{cases}\arg\max_{x\in\mathcal{Q}_M}\Pr\{s_t=x|\zeta_t^{(k)},\boldsymbol{\ell}^{(k-1)}\} & \text{(VBLAST)}\\ \mathrm{E}\{s_t|\zeta_t^{(k)},\boldsymbol{\ell}^{(k-1)}\} & \text{(SoftSIC)}\end{cases}, \quad (25)$$

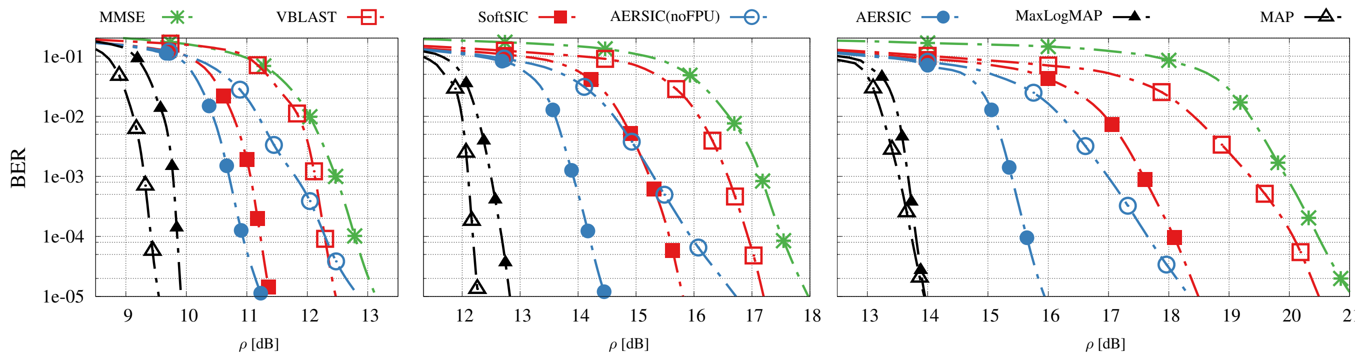
cf. [15], [16], which combines a-priori and a-posteriori information from the current linear detection estimate $\zeta_t^{(k)}$ and the previous decoding LLRs $\boldsymbol{\ell}^{(k-1)}$. Since $\boldsymbol{\lambda}_{\mathrm{e}}^{(k)} = \boldsymbol{\lambda}^{(k)} - \boldsymbol{\ell}^{(k-1)}$ and $\boldsymbol{\lambda}_{\mathrm{e}}^{(k)}$ is a consequence of $\boldsymbol{\zeta}^{(k)}$,

$$\Pr\{s_t = x|\zeta_t^{(k)},\boldsymbol{\ell}^{(k-1)}\} = \Pr\{s_t = x|\boldsymbol{\lambda}^{(k)}\}, \quad (26)$$

where $z_t^{(k)}$ can then be found directly by means of (25), (26), (12), and (14), whereby conditioning on $\boldsymbol{\lambda}^{(k)}$ instead of $\boldsymbol{\ell}^{(k)}$. $z_t^{(k)}$ then serves in (20) for the detection of the next symbol s_{t+1}.

3.1.4. A-Priori/Posteriori Error Regularizing SIC (AERSIC)

If $s_t \neq z_t^{(k)}$, due to an error during the detection and decoding cycle, the interference cancellation step (20) for VBLAST and SoftSIC fails. This error distorts $\lambda_i^{(k)}$ for all $i \ge 1+(t-1)\log_2 M$ in the current transmit vector detection. Unless this error can be corrected later by the channel decoder, the detection error is iteratively spread over the entire bit sequence, due to the iterative interleaver/de-interleaver operation. Particularly, for high efficiency LDPC [28] and turbo codes [29], incorrect detection LLRs are likely to cause system failures as a result of error cascades in conventional MIMO receivers [30]. While Turbo-MIMO systems are designed to correct detection errors by feeding back a-posteriori information of the transmit signal to the detector, no a-priori information on the transmit signal is available at the MIMO detector during the first Turbo-MIMO iteration. To reduce the error propagation in the first Turbo-MIMO iteration, we have proposed the AERSIC and the AERSIC(diag) regularization method in [30] and the reduced complexity AERSIC(noFPU) method in [31]. In what follows, we present an extension of AERSIC that combines the a-priori information that becomes successively



(a) $\eta = \frac{1}{2}$

(b) $\eta = \frac{2}{3}$

(c) $\eta = \frac{3}{4}$

Fig. 1: System Bit Error Rates

available during the SIC steps and the a-posteriori information that is available at the detector from the second Turbo-MIMO iteration on. To that end, we note that w.r.t. (20), the receive signal for the tth detection step $\boldsymbol{y}^{(k,t)}$ can explicitly be written as

$$\boldsymbol{y}^{(k,t)} = \boldsymbol{H}\boldsymbol{s}^{(t)} + \boldsymbol{H}\boldsymbol{d}^{(k,t)} + \boldsymbol{n}, \tag{27}$$

where $\boldsymbol{s}^{(t)} = \sum_{i=t}^{N_\mathrm{T}} \mathbf{e}_i s_i$ is the assumed residual transmit signal after $(t-1)$ perfect SIC steps, and $\boldsymbol{d}^{(k,t)} = \sum_{i=1}^{t-1} \mathbf{e}_i (s_i - z_i^{(k)})$ is the resulting error signal which models the imperfections of the SIC detection. Note that for the compound signal $\boldsymbol{q}^{(k,t)} = \boldsymbol{s}^{(t)} + \boldsymbol{d}^{(k,t)}$, the linear MMSE filter is directly given as

$$\boldsymbol{G}^{(k,t),\mathrm{H}} = (\boldsymbol{H}^\mathrm{H}\boldsymbol{H} + \rho\, \boldsymbol{Q}^{(k,t),-1})^{-1} \boldsymbol{H}^\mathrm{H}, \tag{28}$$

$$\boldsymbol{Q}^{(k,t)} = \mathrm{E}\{\boldsymbol{d}^{(k,t)}\boldsymbol{d}^{(k,t),\mathrm{H}}\} + \boldsymbol{C}_s^{(t)} \tag{29}$$

where $\mathrm{E}\{\boldsymbol{d}^{(k,t)}\boldsymbol{d}^{(k,t),\mathrm{H}}\}$ is the cancellation error covariance matrix. Because $\boldsymbol{d}^{(k,t)}$ is composed of $\boldsymbol{s}$ and $[z_1^{(k)}, \ldots, z_{t-1}^{(k)}]^\mathrm{T}$, $\boldsymbol{d}^{(k,t)}$ is unknown to the receiver, but the second order moments in (29) are determined by the detector a-priori information output during the SIC and for $k > 1$ by the a-posteriori information from the decoder output of the previous Turbo-MIMO iteration, additionally. Given $\boldsymbol{\ell}^{(k-1)}$, the ith row jth column element of the covariance matrix can be computed by

$$\left[\mathrm{E}\{\boldsymbol{d}^{(k,t)}\boldsymbol{d}^{(k,t),\mathrm{H}}|\boldsymbol{\ell}^{(k-1)}\}\right]_{i,j} = \sum_{x_i \in \mathcal{Q}_M} \sum_{x_j \in \mathcal{Q}_M} \cdots$$

$$(x_i - z_i^{(k)})(x_j - z_j^{(k)})^* \Pr\{s_i = x_i, s_j = x_j | \zeta_t^{(k)}, \boldsymbol{\ell}^{(k-1)}\}, \tag{30}$$

for which the posterior probability of $\boldsymbol{s}$ is given as

$$\Pr\{s_i, s_j | \zeta_t^{(k)}, \boldsymbol{\ell}^{(k-1)}\} = \begin{cases} \Pr\{s_i|\boldsymbol{\lambda}^{(k)}\} & i = j \\ \Pr\{s_i|\boldsymbol{\lambda}^{(k)}\} \Pr\{s_j|\boldsymbol{\lambda}^{(k)}\} & i \neq j, \end{cases} \tag{31}$$

assuming the estimates $\zeta_1^{(k)}, \ldots, \zeta_t^{(k)}$ are stochastically independent. (31) can now be used in combination with (14) to compute both the error regularized filter (28) and to evaluate a cancellation vector $z_t^{(k)}$ by means of (25). Beside the regularization via (29), the procedure of the detection is performed identically to Turbo VBLAST and Turbo SoftSIC after exchanging $\boldsymbol{C}_s^{(t)}$ and $\boldsymbol{Q}^{(k,t)}$. For the computational complexity of AERSIC, note that $\boldsymbol{Q}^{(k,t),-1}$ can be computed from $\boldsymbol{Q}^{(k,t-1),-1}$ via a low complexity rank-one matrix inversion update (cf. [31]). Therefore, AERSIC can be implemented under similar complexity constraints as VBLAST or SoftSIC in a Turbo MIMO receiver.

4. SYSTEM SIMULATION

In our simulations for a 4×4 antenna, 16-QAM Turbo-MIMO system, the LTE turbo code [32] with a code block length $N_c = 6144$ bits and the bitwise optimal BCJR decoder [5] have been used. The native code rate of $\frac{1}{3}$ has been punctured to obtain higher code rates, for which the results are depicted in the Figs. 1a, 1b, and 1c, respectively. A maximum number of 24 Turbo detection and decoding iterations was performed to obtain the BER results shown.

We see in Fig. 1a that for a half-rate code, the class of SIC algorithms performs considerably well w.r.t. the low complexity Turbo MMSE system and the optimal MAP detector. Among the SIC algorithms, the SoftSIC and the AERSIC algorithms perform superior to the AERSIC(noFPU) and the VBLAST algorithm. The total SNR gap in between optimal MAP and low-complexity Turbo MMSE detection at 10^{-4} BER is moderate (3.2 dB). By increasing the code rate to $\eta = \frac{2}{3}$, we observe different trends for the SIC algorithms in Fig. 1b. The performances of the SoftSIC, the VBLAST, and the MMSE Turbo MIMO system deteriorate similarly, while the relative SNR gaps among these methods remain almost unchanged. On the other hand, the performance of the error regularized AERSIC and AERSIC(noFPU) detection methods increases relatively to the SoftSIC method, clearly outperforming VBLAST and MMSE detection. Despite the increase of the total SNR gap to more than 5 dB, the performance of the AERSIC detection algorithm is clearly superior to all other SIC algorithms, leaving the SNR gap to optimal MAP detection almost unchanged from the lower code rate. This performance trend substantiates for the $\eta = \frac{3}{4}$ rate code, as it can be seen in Fig. 1c. Both error regularized detectors are now clearly superior to SoftSIC detection. It is noteworthy, that in order to achieve this performance, the AERSIC(noFPU) algorithm uses the VBLAST hard detection step (25) and computes the error regularization matrix by means of precomputed lookup tables only, while the yet inferior SoftSIC algorithm requires a floating point unit to compute (25).

5. CONCLUSION

Despite the availability of a-posteriori LLRs during detection, a drastic performance loss of Turbo SoftSIC and VBLAST systems is experienced w.r.t. optimal MAP detectors. AERSIC is a simple, yet effective, minimal extension of the VBLAST and SoftSIC algorithms, that improves the Turbo-MIMO performance significantly, with little increase of computational complexity. For a code rate of $\frac{3}{4}$, LTE-compliant Turbo-MIMO receiver, a detection and decoding gain of 2 dB can be achieved w.r.t. SoftSIC detection.

References

[1] G. Foschini and M. Gans, “On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas,” *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, 1998.

[2] I. Telatar, “Capacity of Multi-Antenna Gaussian Channels,” *Europ. Trans. Telecomm.*, vol. 10, no. 6, pp. 585–595, 1999, Also: Technical Memorandum, Bell Laboratories, Lucent Technologies, October 1995.

[3] G. Caire, G. Taricco, and E. Biglieri, “Bit-Interleaved Coded Modulation,” *IEEE Trans. Inform. Theory*, vol. 44, no. 3, pp. 927–946, 1998.

[4] E. Zehavi, “8-PSK Trellis Codes for a Rayleigh Channel,” *IEEE Trans. Commun.*, vol. 40, no. 5, pp. 873–884, 1992.

[5] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal Decoding of Linear Codes for Minimizing Symbol Error Rate (Corresp.),” *IEEE Trans. Inform. Theory*, vol. 20, no. 2, pp. 284–287, 1974.

[6] V. Franz and J. Anderson, “Concatenated Decoding with a Reduced-Search BCJR Algorithm,” *IEEE J. Select. Areas Commun.*, vol. 16, no. 2, pp. 186–195, 1998.

[7] M. Mansour and N. Shanbhag, “High-Throughput LDPC Decoders,” *IEEE Trans. VLSI Syst.*, vol. 11, no. 6, pp. 976–996, 2003.

[8] X. Wang and H. Poor, “Iterative (Turbo) Soft Interference Cancellation and Decoding for Coded CDMA,” *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1046–1061, 1999.

[9] D. Reynolds and X. Wang, “Low-Complexity Turbo-Equalization for Diversity Channels,” *Signal Processing*, vol. 81, no. 5, pp. 989–995, 2001.

[10] M. Tüchler, R. Kötter, and A. Singer, “Turbo Equalization: Principles and New Results,” *IEEE Trans. Commun.*, vol. 50, no. 5, pp. 754–767, 2002.

[11] T. Abe and T. Matsumoto, “Space-Time Turbo Equalization in Frequency-Selective MIMO Channels,” *IEEE Trans. Veh. Technol.*, vol. 52, no. 3, pp. 469–475, 2003.

[12] M. Sellathurai and S. Haykin, “Turbo-BLAST for Wireless Communications: Theory and Experiments,” *IEEE Trans. Signal Processing*, vol. 50, no. 10, pp. 2538–2546, 2002.

[13] H. Dai, A. Molisch, and H. Poor, “Downlink Capacity of Interference-Limited MIMO Systems with Joint Detection,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 2, pp. 442–453, 2004.

[14] A. Wolfgang, S. Chen, and L. Hanzo, “Parallel Interference Cancellation Based Turbo Space-Time Equalization in the SDMA Uplink,” *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 609–616, 2007.

[15] P. Wolniansky, G. Foschini, G. Golden, and R. Valenzuela, “V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel,” in *URSI Internat. Symp. on Signals, Systems, and Electronics (ISSSE)*, 1998, pp. 295–300.

[16] D. Seethaler, G. Matz, and F. Hlawatsch, “An Efficient MMSE-Based Demodulator for MIMO Bit-Interleaved Coded Modulation,” in *IEEE Glob. Telecomm. Conf. (GLOBECOM)*, vol. 4, 2004, 2455–2459 Vol.4.

[17] J. Choi, A. Singer, J. Lee, and N. Cho, “Improved Linear Soft-Input Soft-Output Detection Via Soft Feedback Successive Interference Cancellation,” *IEEE Trans. Commun.*, vol. 58, no. 3, pp. 986–996, 2010.

[18] G. Woodward, R. Ratasuk, M. Honig, and P. Rapajic, “Minimum Mean-Squared Error Multiuser Decision-Feedback Detectors for DS-CDMA,” *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 2104–2112, 2002.

[19] E. Zimmermann and G. Fettweis, “Adaptive vs. Hybrid Iterative MIMO Receivers Based on MMSE Linear and Soft-SIC Detection,” in *2006 IEEE 17th Internat. Symp. on Personal, Indoor and Mobile Radio Communications*, 2006, pp. 1–5.

[20] P. Robertson, E. Villebrun, and P. Höher, “A Comparison of Optimal and Sub-Optimal MAP Decoding Algorithms Operating in the Log Domain,” in *IEEE Internat. Conf. on Comm. (ICC)*, vol. 2, 1995, 1009–1013 vol.2.

[21] M.-O. Damen, H. El-Gamal, and G. Caire, “On Maximum-Likelihood Detection and the Search for the Closest Lattice Point,” *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2389–2402, 2003.

[22] H. Vikalo and B. Hassibi, “On the Sphere-Decoding Algorithm II. Generalizations, Second-Order Statistics, and Applications to Communications,” *IEEE Trans. Signal Processing*, vol. 53, no. 8, pp. 2819–2834, 2005.

[23] J. Jaldén and B. Ottersten, “On the Complexity of Sphere Decoding in Digital Communications,” *IEEE Trans. Signal Processing*, vol. 53, no. 4, pp. 1474–1484, 2005.

[24] R. Wang and G. Giannakis, “Approaching MIMO Channel Capacity with Soft Detection Based on Hard Sphere Decoding,” *IEEE Trans. Commun.*, vol. 54, no. 4, pp. 587–590, 2006.

[25] K. Kusume, M. Joham, W. Utschick, and G. Bauch, “Cholesky Factorization with Symmetric Permutation Applied to Detecting and Precoding Spatially Multiplexed Data Streams,” *IEEE Trans. Signal Processing*, vol. 55, no. 6, pp. 3089–3103, 2007.

[26] G. Dietl and W. Utschick, “Complexity Reduction of Iterative Receivers Using Low-Rank Equalization,” *IEEE Trans. Signal Processing*, vol. 55, no. 3, pp. 1035–1046, 2007.

[27] K. Fang, L. Rugini, and G. Leus, “Low-Complexity Block Turbo Equalization for OFDM Systems in Time-Varying Channels,” *IEEE Trans. Signal Processing*, vol. 56, no. 11, pp. 5555–5566, 2008.

[28] J. Hou, P. Siegel, and L. Milstein, “Design of Multi-Input Multi-Output Systems Based on Low-Density Parity-Check Codes,” *IEEE Trans. Commun.*, vol. 53, no. 4, pp. 601–611, 2005.

[29] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near Shannon Limit Error-Correcting Coding and Decoding: Turbo-Codes. 1,” in *IEEE Internat. Conf. Commun. (ICC)*, vol. 2, 1993, pp. 1064–1070.

[30] A. Krebs, M. Joham, and W. Utschick, “Soft Detection Constrained Achievable Rates for Nonlinear MIMO-BICM Receivers,” in *IEEE Internat. Conf. on Comm. (ICC)*, 2015.

[31] ——, “An Adaptive MMSE-SIC Soft Detector with Error Regularization for Iterative MIMO Receivers,” in *IEEE Internat. Conf. on Acoustics, Speech and Signal Processing (ICASSP)*, 2014, pp. 1961–1965.

[32] 3GPP TS 36.212, “Evolved Universal Terrestrial Radio Access (E-UTRA); Multiplexing and Channel Coding,” Specification, 3GPP, 2010.