A LOW COMPLEXITY ITERATIVE SOFT-DECISION FEEDBACK MMSE-PIC DETECTION ALGORITHM FOR MASSIVE MIMO

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ABSTRACT

In MIMO applications, the minimum mean square error parallel interference cancellation (MMSE-PIC) based Soft-Input Soft-Output (SISO) detector has been widely adopted because of its low complexity and good bit error rate (BER) performance. In this paper, we firstly propose to use a Gaussian model based MMSE detection algorithm to implement MMSE-PIC with low complexity. This algorithm, which can detect a length-N_r received data block by a single Hermitian matrix (sized $N_t \times N_t$) inversion, is especially preferable in Massive MIMO up-link applications where the number of transmit antennas N_t from each end terminal is much less than the number of receive antennas N_r in the Base Station. Then we derive a new method to calculate the matrix inversion by a linear combination of two matrices, which reduces the complexity from $\mathcal{O}(N_t^3)$ to $\mathcal{O}(N_t^2)$. At last, in order to improve the system performance for the first pass when there is no *a priori* information available, a self-iteration method is proposed and thus a system performance gain of 1dB to 2dB is achieved at the cost of modest complexity increase.

1. INTRODUCTION

Recent years, Massive MIMO (also known as "Large-Scale Antenna Systems", "Very Large MIMO") has attracted great interest from wireless communication research community [1]. Research shows that with Massive MIMO, the throughput and spectrum efficiency of wireless systems can be greatly improved [2]. Together with the iterative detection and decoding (IDD) technology, linear detection algorithm like the minimum mean square error parallel interference cancellation (MMSE-PIC) algorithm [3] [4] is attractive for detection of Massive MIMO signals because of its low complexity and good bit error rate (BER) performance.

To reduce the burden of performing matrix inversion for detecting every symbol in MMSE-PIC algorithm, an iterative method to implement the MMSE filter was proposed in [5]. Then [6] presented a method which needs pre-computing one matrix inversion only and then detects every symbol with low complexity incremental calculations. In 2011, [7] proposed a well optimized version of MMSE-PIC and implemented it in ASIC which has been widely cited as the state-of-the-art MIMO detection implementation benchmark. Based on the result of [7], MMSE-PIC has been employed in Massive MIMO detection applications in [8]. Both [7] and [8] selected the non-Hermitian matrix to perform MMSE filter calculation, as the Hermitian matrix inversion had numerical stability issue. Recently, [9] proposed a more efficient ASIC implementation compared with [7]. The major computing saving comes from the matrix inversion part where LDL decomposition based matrix inversion algorithm can be employed because the matrix to be inverted in [9] is Hermitian positive definite (HPD) [10] [11]. But unfortunately, because the matrix to be inverted has the size of $N_r \times N_r$ where N_r is the number of receive antennas, that algorithm is not suitable for Massive MIMO up-link applications, where the number of transmit antennas N_t from each end terminal is much less than the number of receive antennas N_r in the Base Station.

In [12], we proposed a generic method to implement a Soft-Input Soft-Output (SISO) detector, where the *a posteriori* distribution of a multivariate Gaussian vector was calculated first, followed by the calculation of the *extrinsic* information of each individual variable. The calculation of multiple variables together naturally enables sharing of computational units, thereby reducing system complexity. So in this paper, we firstly employ [12] to implement the MMSE-PIC in MIMO applications, which can reduce system complexity as the matrix to be inverted is a HPD matrix with size $N_t \times N_t$. A HPD matrix enables us to use the more computational efficient matrix inversion method.

In order to reduce the complexity of the second and subsequent passes, we derive a new method to calculate the matrix inversion by a linear combination of two matrices which have been computed in the first pass (from detector to decoder). With this method, we can reduce the complexity of matrix inversion from $\mathcal{O}(N_t^3)$ to $\mathcal{O}(N_t^2)$ along with small performance penalty. In comparison to other matrix inversion approximation methods, the proposed method does not rely on any special requirement for the size of the random channel matrix.

The power of turbo processing comes from the more and more reliable *a priori* information from the decoder, but for the first pass, there is no such information available. At the same time, as the employed iterations between the decoder and the detector will inevitably reduce the throughput and increase the system latency, for high speed applications it is difficult to perform IDD when they run at the highest throughput [7] [9]. Considering this, it is desirable to improve the first pass performance. So, we propose a self-iteration method, which feeds back the detector's soft decision output directly to its *a priori* input, to improve the performance of the detector. By employing a low cost approximation of matrix inversion, the method of self-iteration is attractive due to the fact that with only a slight

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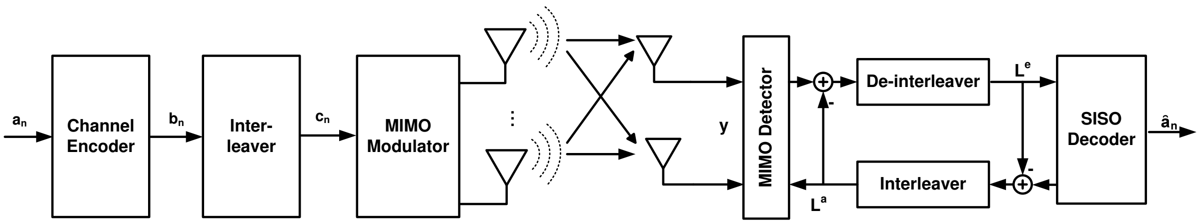

Fig. 1. Iterative Detection and Decoding of a MIMO Communication System

increase of complexity, a performance gain of 1dB to 2dB can be achieved. It is worth noting that this self-iteration method also applies to non-turbo systems to improve system performance.

The remainder of this paper is organized as follows. Section II describes the turbo-MIMO system model. Then the Gaussian model based MMSE detection algorithm is detailed in Section III. In Section IV, we introduce a proposal of how to reduce the complexity of matrix inversion and the self-iteration method to improve the first pass BER performance. Simulation results are shown in Section V.

The notations used in this paper are as follows. Lower and upper case letters denote scalars. Bold lower and upper case letters represent column vectors and matrices, respectively. We use $\propto$ to denote equality of functions up to a scale factor. The superscriptions "^{T}" and "^{H}" denote the transpose and conjugate transpose, respectively.

2. SYSTEM MODEL

As shown in Fig. 1, we consider a coded MIMO system with N_r receive antennas and N_t transmit antennas. The received signal at the receiver is as follows

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{y}$ denotes a length-N_r observation vector, $\mathbf{H}$ denotes an $N_r \times N_t$ MIMO system transfer matrix, $\mathbf{w}$ denotes a length-N_r circularly symmetric additive white Gaussian noise (AWGN) vector with PDF $\mathcal{CN}(\mathbf{w}; 0, 2\sigma^2\mathbf{I})$, and $\mathbf{x} = [x_1, x_2, \cdots, x_{Nt}]^T$ is mapped from an interleaved code sequence $\mathbf{c}$, i.e., each $x_n \in \mathcal{A} = \{\alpha_1, \alpha_2, \cdots, \alpha_{2Q}\}(|\mathcal{A}| = 2^Q)$ corresponds to a length-Q subsequence of $\mathbf{c}$ denoted by $\mathbf{c}_n = [c_{n,1}, c_{n,2}, \cdots, c_{n,Q}]^T$.

The task of the detector is to compute the log-likelihood ratio (LLR) for each code bit $c_{n,q}$, which can be expressed as [13]

$$L(c_{n,q}) = \ln \frac{P(c_{n,q}=0|\mathbf{y})}{P(c_{n,q}=1|\mathbf{y})} = \ln \frac{\sum\limits_{x_n \in \mathcal{A}_q^0} P(x_n|\mathbf{y})}{\sum\limits_{x_n \in \mathcal{A}_q^1} P(x_n|\mathbf{y})} \tag{2}$$

where $\mathcal{A}_q^0$ ($\mathcal{A}_q^1$) denotes the subset of all $\alpha_i \in \mathcal{A}$ corresponding to a binary subsequence with the qth bit given by 0 (1). The extrinsic LLR [12]

$$\begin{aligned} L^e(c_{n,q}) &= L(c_{n,q}) - L^a(c_{n,q}) \\ &= \ln \frac{\sum\limits_{x_n \in \mathcal{A}_q^0} P(\mathbf{y}|x_n)P(x_n)}{\sum\limits_{x_n \in \mathcal{A}_q^1} P(\mathbf{y}|x_n)P(x_n)} - L^a(c_{n,q}) \end{aligned} \tag{3}$$

will be the input to the decoder, where $L^a(c_{n,q})$ is the output extrinsic LLR of the decoder in the last iteration and $P(x_n)$ can be calculated from $L^a(c_{n,q})$.

3. GAUSSIAN MODEL BASED MMSE DETECTION ALGORITHM

Let $\mathbf{G} = \mathbf{H}^H\mathbf{H}$ and $\hat{\mathbf{y}} = \mathbf{H}^H\mathbf{y}$, the linear MMSE detection algorithm in [12] is shown in **Algorithm 1**. Due to the use of the interleaver, different bits of a symbol can be assumed to be independent, and thus $P(x_n = \alpha_i) = \prod_{j=1}^{Q} p(c_{n,j} = s_{i,j})$ where $p(c_{n,j} = s_{i,j})$ is calculated from the *a priori* LLR of $\mathbf{L}_j^a$ with the LLR definition of $L_j^a = ln\frac{p(c_{n,j}=0)}{p(c_{n,j}=1)}$.

Algorithm 1 Gaussian model based MMSE detection

Input: $\hat{\mathbf{y}}$,$\mathbf{G}$, $\mathbf{L}^a$
Output: $\mathbf{L}^e$ $\triangleright$ *extrinsic* LLR value for every bit
1: Calculate *a priori* mean $\mathbf{m}$ and variance $\mathbf{V}$
2: $m_n = \sum\limits_{\alpha_i \in \mathcal{A}} \alpha_i P(x_n = \alpha_i)$ $\triangleright$ $\mathbf{m} = [m_1, m_2, \cdots, m_{N_t}]^T$
3: $v_n = \sum\limits_{\alpha_i \in \mathcal{A}} |\alpha_i - m_n|^2 P(x_n = \alpha_i)$ $\triangleright$ $\mathbf{V} = diag[v_1, v_2, \cdots, v_{N_t}]$
4: Calculate *a posteriori* mean $\mathbf{m}^p$ and variance $\mathbf{V}^p$
5: $\hat{\mathbf{m}} = \mathbf{G}\mathbf{m}$
6: $\mathbf{V}^p = (\mathbf{V}^{-1} + \frac{1}{2\sigma^2}\mathbf{G})^{-1}$
7: $\mathbf{m}^p = \mathbf{m} + \frac{1}{2\sigma^2}\mathbf{V}^p(\hat{\mathbf{y}} - \hat{\mathbf{m}})$
8: Calculate *extrinsic* mean m_n^e and variance v_n^e
9: $v_n^e = (\frac{1}{v_n^p} - \frac{1}{v_n})^{-1}$ $\triangleright$ v_n^p is the nth diagonal element of $\mathbf{V}^p$
10: $m_n^e = v_n^e(\frac{m_n^p}{v_n^p} - \frac{m_n}{v_n})$ $\triangleright$ m_n^p is the nth element of $\mathbf{m}^p$
11: Calculate *extrinsic* LLR $\mathbf{L}^e$
12: $L^e(c_{n,q}) = \ln \frac{\sum\limits_{\alpha_i \in \mathcal{A}_q^0} \exp\left(-\frac{|\alpha_i - m_n^e|^2}{v_n^e}\right) \prod\limits_{q' \neq q} P(c_{n,q'} = s_{i,q'})}{\sum\limits_{\alpha_i \in \mathcal{A}_q^1} \exp\left(-\frac{|\alpha_i - m_n^e|^2}{v_n^e}\right) \prod\limits_{q' \neq q} P(c_{n,q'} = s_{i,q'})}$

It is worth noting that the LLR calculation in Line 12 can be further simplified by employing the constellation regularity after applying the log_max approximation and ignoring the *a priori* terms like [14].

4. COMPLEXITY REDUCTION

4.1. Low Complexity Matrix Inversion

It can be seen that the matrix inversion in Line 6 of **Algorithm 1** contributes the major complexity of $N_t^3/2$. If N_r and N_t are large enough (e.g. greater than 200 [15]), matrix $\mathbf{G}$ tends to be an identity matrix from random matrix theory, which makes the computational complexity of this matrix inversion trivial. On the other hand, if N_r is much bigger than N_t (like $N_r/N_t > 8$ [16]), matrix $\mathbf{G}$ becomes diagonal dominant, then the 2-term Neumann series can be employed to approximate this matrix inversion with complexity of $\mathcal{O}(N_t^2)$. We aim to find a more generic method which does not depend on any special requirement for the size of random matrix $\mathbf{H}$. As [17], by averaging the diagonal elements of $\mathbf{V}$, we have $\mathbf{V} = k\mathbf{I}$

where $k = \frac{\sum_n v_n}{N_t}$. So, Line 6 of **Algorithm 1** can be rewritten as

$$\mathbf{V}^p = (\bar{k}\mathbf{I} + \frac{1}{2\sigma^2}\mathbf{G})^{-1} \tag{4}$$

where $\bar{k} = 1/k = 1/(\sum_n v_n/N_t)$. For the first pass, there is no *a priori* information available, thus we assume $\mathbf{m}$ to be a zero vector and $\mathbf{V}$ to be the identity matrix $\mathbf{I}$. So, we change (4) to

$$\begin{aligned}\mathbf{V}^p &= \big((\mathbf{I} + \frac{1}{2\sigma^2}\mathbf{G}) + (\bar{k}-1)\mathbf{I}\big)^{-1} \\ &= (\mathbf{A} + (\bar{k}-1)\mathbf{I})^{-1}\end{aligned} \tag{5}$$

where $\mathbf{A} = \mathbf{I} + \frac{1}{2\sigma^2}\mathbf{G}$. Thus, we can represent (5) as a function of $\bar{k}$ as $\mathbf{V}^p = f(\bar{k})$. By using the approximation of $f(\bar{k}) = f(1) + f^{'}(1)(\bar{k}-1)$ and the derivative of a matrix inverse $\frac{\mathrm{d}\mathbf{M}^{-1}}{\mathrm{d}k^{'}} = -\mathbf{M}^{-1}\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}k^{'}}\mathbf{M}^{-1}$, we have a direct formula to compute this matrix inversion as

$$\mathbf{V}^p = \mathbf{A}^{-1} - (\bar{k}-1)\mathbf{A}^{-1}\mathbf{A}^{-1}. \tag{6}$$

We can pre-compute $\mathbf{E}_1 = \mathbf{A}^{-1}$ and $\mathbf{E}_2 = \mathbf{A}^{-1}\mathbf{A}^{-1}$ and save them in memory. Then the matrix inversion can be calculated by linear combination of these two fixed matrices as

$$\mathbf{V}^p = \mathbf{E}_1 - (\bar{k}-1)\mathbf{E}_2. \tag{7}$$

Using this method, we reduce the complexity of matrix inversion from $\mathcal{O}(N_t^3)$ to $\mathcal{O}(N_t^2)$. It is worth noting that [6] also proposed an approximation method which incrementally calculates the second and subsequent pass matrix inversion based on a pre-computed exact matrix inversion result, but the method is only applicable to constant envelope constellations. And in [18], a singular value decomposition (SVD) based matrix inversion method was proposed, but this method needs linear combination of N_t pre-computed matrices and thus has higher computational complexity than the proposed method.

4.2. A Heuristic Approach to Solve the Stability Problem

As the approximation of $f(x) = f(1) + f^{'}(1)(\bar{k}-1)$ has an error term of $\mathcal{O}((\bar{k}-1)^2)$, to achieve a high accuracy $(\bar{k}-1)$ must be small enough ($|\bar{k}-1| < 1$). But unfortunately this constraint cannot always be met because when the *a priori* information becomes more and more reliable, v_n will be less than 0.5, leading to a unstable BER performance. Heuristically, we propose to revise $\bar{k}$ as $\bar{k} = 1/(\sum_n v_n/N_t + 0.5)$, thus Line 6 of **Algorithm 1** is replaced with the following:

1: $\bar{k} = 1/(\sum_n v_n/N_t + 0.5)$
2: $\mathbf{V}^p = \mathbf{E}_1 - (\bar{k}-1)\mathbf{E}_2$

Hereafter, we refer this updated algorithm as **Algorithm $\hat{\mathbf{1}}$**.

4.3. Computational Complexity Comparison

In [7], a well optimized MMSE-PIC algorithm, which employs only one matrix inversion to detect a length-N_r received data block for every iteration, has been proposed and implemented in ASIC and now it has been widely cited as a MMSE-PIC implementation benchmark. The core part of this algorithm is listed in **Algorithm 2** which is equivalent to Line 4 to Line 10 of **Algorithm 1**[1]. From **Algorithm 2**, it is easy to see that the computational complexity of Line 1 is $N_t^2 + N_t^3$ as the matrix to be inverted is not Hermitian. By contrast, the complexity of the matrix inversion in **Algorithm 1** is $N_t^3/2$ by using LDL decomposition and modified backwards substitution [11]. As $\mathbf{H}^H\mathbf{H}$ is a Hermitian matrix, we assume that this matrix multiplication has a complexity of $N_r N_t^2/2$.

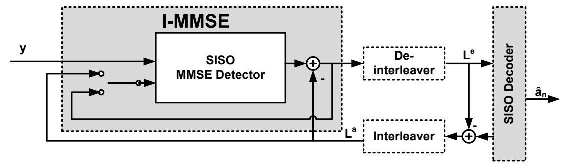

Fig. 2. Iterative Soft-in Soft-Out MMSE Detector

Algorithm 2 Core Part of MMSE-PIC Algorithm in [7]

1: $\mathbf{A}^{-1} = (\mathbf{G}\mathbf{V} + 2\sigma^2\mathbf{I})^{-1}$ ▷ One matrix inversion per iteration
2: **for** $n = 1$ to N_t **do**
3: $\quad\bar{\mathbf{y}}_n = \hat{\mathbf{y}} - \sum_{j,j\neq n}\mathbf{g}_j m_j$ ▷ $\mathbf{g}_j$ is *j*th column of $\mathbf{G}$
4: $\quad\mu_n = \mathbf{a}_n^H\mathbf{g}_n$ ▷ $\mathbf{a}_n$ is the *n*th row of $\mathbf{A}^{-1}$
5: $\quad\hat{x}_n = \mathbf{a}_n^H\bar{\mathbf{y}}_n$
6: $\quad m_n^e = \hat{x}_n/\mu_n$ ▷ *extrinsic* mean
7: $\quad v_n^e = 1/\mu_n - v_n$ ▷ *extrinsic* variance
8: **end for**

We summarize the complexity of above mentioned algorithms in **Table 1**. From this table, **Algorithm 1** and **Algorithm 2** have the same pre-computing complexity. But for every pass **Algorithm 1** has only half of the complexity of **Algorithm 2**. At the same time, compared to **Algorithm 2**, the proposed **Algorithm $\hat{\mathbf{1}}$** has great computation saving for the second and subsequent pass processing while maintaining the same level of pre-computing complexity.

Table 1. Computational Complexity Comparison

	Pre-computing	Every Pass
Algorithm 2	$\frac{1}{2}N_rN_t^2 + N_rN_t$	$4N_t^2 + N_t^3$
Algorithm 1	$\frac{1}{2}N_rN_t^2 + N_rN_t$	$2N_t^2 + \frac{1}{2}N_t^3$
Algorithm $\hat{\mathbf{1}}$	$\frac{1}{2}N_rN_t^2 + N_rN_t + N_t^3$	$4N_t^2$

4.4. Iterative Method to Improve First-pass Performance

By employing SISO MMSE's soft decision output as its *a priori* input (see Fig. 2), the SISO MIMO detector itself can run in an iterative manner and we call it iterative MMSE detection algorithm (I-MMSE). Compared to conventional MMSE turbo receiver, a 1dB to 2dB performance gain can be obtained. Actually, the fact that self-iteration can improve system performance had been observed in other literatures [8] [19] where only one self-iteration has been reported. By contrast, the simulations show that there is performance gain up to four iterations. More importantly, after employing the proposed low complexity matrix inversion, I-MMSE seems more attractive because of its much lower complexity cost of $4N_t^2$ for the second and subsequent pass calculation.

[1] Algorithm in [17] and **Algorithm 2** in [7] are both equivalent or closely approximate to the famous Wang-Poor algorithm [3], and in [12] we have proved that **Algorithm 1** is equivalent to [17]. Thus, **Algorithm 1** and **Algorithm 2** are approximately equivalent.

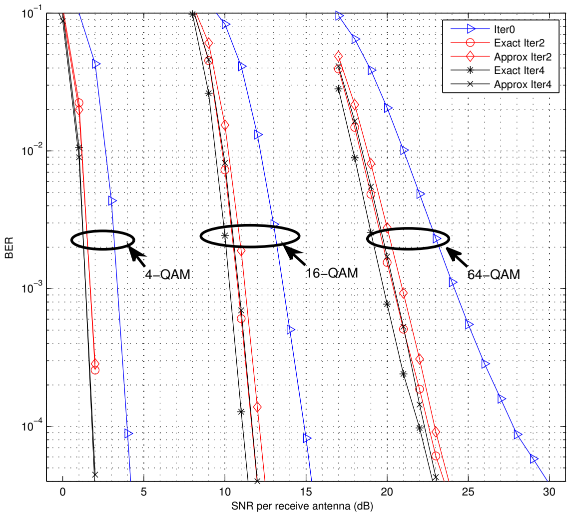

Fig. 3. BER Performance Comparison Between Exact Implementation and Proposed Approximation for a 16×16 MIMO System.

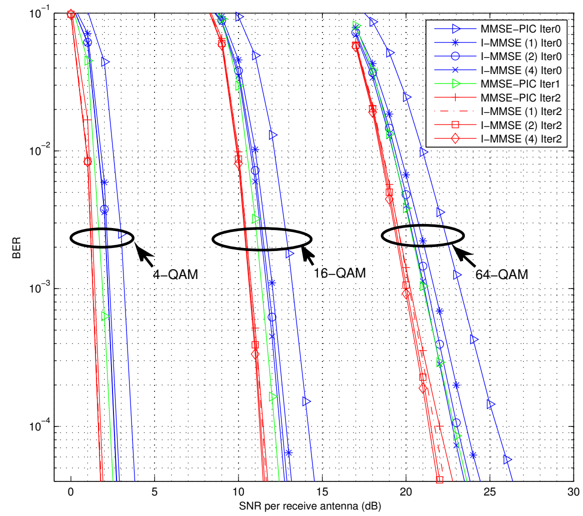

Fig. 4. BER Performance Comparison Between Different Number of Self-iterations for 32×32 MIMO.

5. SIMULATION RESULTS

5.1. Simulation Setup

We consider a Rayleigh slow fading random channel so **H** does not change over a codeword. The elements of **H** is independent and identically Gaussian distributed with zero mean and variance 1. During simulation, we assume perfect channel information is available in the detection module. A rate-1/2, regular (3,6) low-density parity-check (LDPC) code with codeword length of 2000 bits is employed as the channel code and the maximum number of iterations of the decoder is 25. The square quadrature amplitude modulations (2^Q-QAM) with Gray mapping are used. For each signal-to-noise (SNR) value, we run at least 100000 codewords in the Monte Carlo simulations. We set the scaling factor of output LLR to 0.7 [20]. In the simulations, there are clipping both in soft-output part and soft-input part of the detector. The soft-in clipping threshold[2] for the *a priori* LLR is ± 2, and soft-output module constrains the output LLR range to $[-50, 50]$.

5.2. BER Performance

Fig. 3 shows the performance comparison between exact implementation (**Algorithm 1**) and the proposed approximation (**Algorithm $\hat{1}$**) of a 16×16 MIMO system with 4-QAM, 16-QAM and 64-QAM signaling. The legend of *Iter=0* stands for the the first pass without the *a priori* information. The legend of *Iter=2* stands for performance after running two outer loops (between decoder to detector). It is clear that this proximation has nearly no performance loss for 4-QAM signaling, but has small performance loss for 16-QAM and 64-QAM compared to the exact one. For IDD systems employing I-MMSE algorithm, there exist two iterative loops. The self-iteration of detector is the inner loop. The outer loop from the decoder to the detector is the same as that in a typical turbo system. Through extensive simulation we have found that the self-iteration method has only marginal performance gain beyond the first outer pass, thus we only perform the inner loop for the first outer pass. Then we compare performances between I-MMSE (using **Algorithm $\hat{1}$** together with low complexity inner loop) and the conventional MMSE-PIC (using **Algorithm 1**) for MIMO systems with different sizes and various modulation signaling. In Fig. 4, the number in the bracket denotes the number of self-iterations and *IterX* denotes *X* outer iterations. It is clear that the proposed method can significantly improve the first pass system performance (1dB to 2dB at BER of 10^{-4}). It can also be seen that with more than two self-iterations there is still performance gain although after four self-iterations the performance gain is marginal. The simulations show that similar performance gain can also be obtained in other sized MIMO systems like 4×4 and 16×16 (not shown due to the page limitation).

[2] This clipping threshold can also help resolve the numerical stability issue of Line 10 of **Algorithm 1** when the *a priori* variance v_n is close to zero.

6. CONCLUSION

In this paper, we firstly employed a low complexity Gaussian model based MMSE algorithm to perform the MMSE-PIC detection. This algorithm can detect a length-N_r received data block with only one Hermitian matrix inversion, and the matrix to be inverted has the size of $N_t \times N_t$ which is especially preferable for Massive MIMO uplink applications where $N_t << N_r$. Then we proposed a generic method to reduce the computational complexity of the matrix inversion from $\mathcal{O}(N_t^3/2)$ to $\mathcal{O}(2N_t^2)$ without the dependance on the size of the random channel matrix. At last, a self-iteration method was proposed to improve a turbo receiver's first pass performance by 1dB to 2dB with only a small complexity increase.

7. REFERENCES

[1] E. G. Larsson, F. Tufvesson, O. Edfors, and T. L. Marzetta, "Massive MIMO for Next Generation Wireless Systems,"

Communications Magazine, IEEE , vol.52, no.2, pp.186,195, February 2014.

[2] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: Opportunities and Challenges with Very Large Arrays," *IEEE Signal Proces. Mag.*, vol. 30, no. 1, pp. 40 - 46, Jan. 2013.

[3] X. Wang and H. V. Poor, "Iterative (turbo) soft interference cancellation and decoding for coded CDMA, *IEEE Trans. Comm.*, vol. 47, no. 7, pp. 10461061, Jul. 1999.

[4] M. Witzke, S. Bro, F. Schreckenbach, and J. Hagenauer, "Iterative detection of MIMO signals with linear detectors," in *Proc. Asilomar Conf. on Signals, Systems and Computers (ACSSC)*, Monterey, CA, USA, Nov. 2002, pp. 289-293.

[5] D. N. Liu and M. P. Fitz, "Low Complexity Affine MMSE detector for Iterative Detection-Decoding MIMO OFDM system," *Proc. IEEE Int. Conf. Commun. (ICC)*, vol. 10, pp.4654 -4659 2006

[6] Seunghwan Choi, Jongkyung Kim and Jong-Soo Seo, "A simplified MMSE detection for iterative receivers in multiple antenna systems," *Broadband Multimedia Systems and Broadcasting (BMSB), 2011 IEEE International Symposium on*, vol., no., pp.1,5, 2011.

[7] C. Studer, S. Fateh, and D. Seethaler, "ASIC implementation of soft-input soft-output MIMO detection using MMSE parallel interference cancellation," *IEEE J. Solid-State Circuits*, vol. 46, no. 7, pp. 1754-1765, Jul. 2011.

[8] M. Wu, C. Dick, J. R. Cavallaro and C. Studer, "Iterative Detection and Decoding in 3GPP LTE-based Massive MIMO Systems,"*Signal Processing Conference (EUSIPCO), 2014 Proceedings of the 22nd European* , vol., no., pp.96,100, 1-5 Sept. 2014

[9] D. Auras, R. Leupers, and G. Ascheid, "A novel reduced-complexity soft-input soft-output MMSE MIMO detector: Algorithm and ecient VLSI architecture" *Communications (ICC), 2014 IEEE International Conference on*, vol., no., pp.4722,4728, 10-14 June 2014.

[10] D. Auras, R. Leupers, and G. Ascheid, "Efficient VLSI architectures for matrix inversion in soft-input soft-output MMSE MIMO detectors," *Circuits and Systems (ISCAS), 2014 IEEE International Symposium on*, vol., no., pp.1018,1021, 1-5 June 2014.

[11] A. Krishnamoorthy and D.Menon, "Matrix inversion using Cholesky decomposition," *Signal Processing: Algorithms, Architectures, Arrangements, and Applications (SPA)*, vol., no., pp.70,72, 26-28 Sept. 2013.

[12] Q. Guo and D. Huang, "A concise representation for the soft-in soft-out LMMSE detector," *IEEE Communications Letters*, 15, 5, pp. 566-568. 2011.

[13] M. Tüchler, A. C. Singer, and R. Koetter, "Minimum mean squared error equalization using a priori information," *IEEE Trans. Signal Processing*, vol. 50, pp. 673-683, Mar. 2002.

[14] A. Tomasoni, M. Ferrari, D. Gatti, F. Osnato, and S. Bellini, "A Low Complexity Turbo MMSE Receiver for W-LAN MIMO Systems," in *Proc. IEEE Int. Conf. Communications*, June 2006, pp. 4119-4124.

[15] P. Suthisopapan, K. Kasai, A. Meesomboon and V. Imtawil, "Achieving Near Capacity of Non-Binary LDPC Coded Large MIMO Systems with a Novel Ultra Low-Complexity Soft-Output Detector," *IEEE Trans. Wireless Commun.*, vol.12, no.10, pp.5185,5199, October 2013.

[16] Yin Bei, M. Wu, C.Studer, J. R. Cavallaro and C. Dick, "Implementation trade-offs for linear detection in large-scale MIMO systems," *Proc. IEEE ICASSP*, pp.2679 -2683 2013.

[17] M. Tüchler, R. Koetter, and A. C. Singer, "Turbo equalization: prinicples and new results," *IEEE Trans. Commun.*, vol. 50, pp. 754-767, May. 2002.

[18] Yuan Yang and Hai-lin Zhang, "A simplified MMSE-based iterative receiver for MIMO systems," *Journal of Zhejiang University SCIENCE A*, 2009 10(10):1389-1394.

[19] M. Senst and G. AscheidHow, "The Framework of Expectation Propagation Yields an Iterative IC-LMMSE MIMO Receiver, *Global Telecommunications Conference (GLOBECOM 2011), 2011 IEEE*, vol., no., pp.1,6, 5-9 Dec. 2011.

[20] J. Vogt and A. Finger, "Improving the max-log-MAP turbo decoder," *Electronics Letters*, vol.36, no.23, pp.1937-1939, 9 Nov 2000.