JOINT GROUP POWER ALLOCATION AND PREBEAMFORMING FOR JOINT SPATIAL-DIVISION MULTIPLEXING IN MULTIUSER MASSIVE MIMO SYSTEMS

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ABSTRACT

We investigate the joint optimization of the group power allocation and prebeamformer for joint spatial division and multiplexing (JSDM) in massive MIMO downlink systems. In contrast with the approximated block diagonalization (ABD) prebeamformer which is derived by heuristic method in the original JSDM scheme and is restricted to semi-unitary matrix, general prebeamforming matrix together with group power is optimized in the framework of per-user ergodic rate balancing. Thus, both flexibility and optimality are integrated in our work. To ease the difficulty in optimizing the exact ergodic rate, the deterministic approximation is employed. Based on the uplink-downlink duality approach, an iterative algorithm alternatively updating group power and prebeamformers is designed. It is shown that the optimization subproblems of the algorithm can be solved efficiently. Compared with the classic signal-to-noise-and-interference ratio (SINR) balancing algorithm for MISO downlink, the proposed algorithm has similar properties of both convergence and optimality. Numerical experiments are conducted to validate the effectiveness of the proposed algorithm.

***Index Terms*—** Ergodic rate balancing, joint spatial division and multiplexing, power allocation, prebeamforming, massive MIMO

1. INTRODUCTION

Through deploying basestation (BS) with large-scale antenna array, massive MIMO [1–5], the newly emerged technique, aims at boosting the downlink performance such as spectral efficiency, reliability, energy efficiency, etc [4] to meet the challenge of the next generation wireless communications. The frequency-division multiplexing (FDD) system is a candidate for massive MIMO implementation, but downlink beamforming for FDD massive MIMO is generally thought of as improper since the cost of training and channel state information (CSI) feedback brought out by large-scale array are too high to be affordable. To alleviate the problem, many advanced techniques have been developed [6–10].

The joint spatial division and multiplexing (JSDM) scheme proposed by Nam *et al.* in [7, 8] is among the recent efforts to make FDD massive MIMO effective. Especially the per-group processing (PGP) approach for JSDM can greatly reduce the training and CSI feedback load. In JSDM based on PGP, users are categorized into different groups according to the similarity of their channel covariance matrices (CCM). Prebeamforming is performed upon the channel matrix of each group to yield an effective channel matrix of lower dimension, and the conventional zero-forcing (ZF) or regularized ZF (RZF) beamforming is then applied to user signals within each group. Because for each group the conventional beamforming is performed in a low-dimensional space, the amount of training and CSI feedback can thus be greatly saved.

In [7,8], the approximated block diagonalization (ABD) method is used to derive the prebeamforming matrices. However, ABD method is heuristic and imposes semi-unitary constraints upon the prebeamforming matrices. So it is possible that prebeamforming based on ABD method is not able to fully cancel the inter-group interference (IGI). To overcome such shortcomings, we consider the design of general prebeamforming matrix under the per-user ergodic rate balancing criterion, which guarantees fair rate allocation when users are experiencing uneven channel conditions. In addition, we also introduce the group power allocation into the optimization to further improve the performance. But for simplicity, ZF beamforming is used within each group. As the exact ergodic rate is difficult to optimize, deterministic approximation in [11] is used to yield an approximated rate balancing problem. Based on the uplink-downlink duality [12], we treat the problem in the dual uplink channel. Similar to the signal-to-interference-and-noise ratio (SINR) balancing algorithm for MISO downlink [12], we propose an iterative algorithm which alternatively updates the uplink group power and the prebeamformers. In the algorithm, the power allocation is solved by an eigenvalue problem while the prebeamformer is solved by a subproblem, the optimal solution of which can be obtained in semi-closed form in the sense that eigenvalue decomposition is the only step involving iterative computation. Due to the resemblance in structures, the results on convergence and global optimality of the classic SINR balancing algorithm in [12] can be directly applied to the proposed algorithm.

Notations: $\mathbf{I}_N$ is the identity matrix of $N \times N$, and $\mathbb{R}^{M\times N}$ and $\mathbb{C}^{M\times N}$ stand for the sets of $M \times N$ real and complex matrix, respectively. $\succ$ are $\succeq$ are the positive definite and semidefinite signs respectively. $\lambda_i(\mathbf{A})$ is the ith largest eigenvalue of matrix $\mathbf{A}$. $\mathrm{diag}\{x_1, \ldots, x_N\}$ is a diagonal matrix with diagonal entries $x_1, \ldots, x_n$. $\mathbf{1} = [1, \ldots, 1]^T$. $\mathcal{R}(\mathbf{A})$ is the range space and $\mathcal{N}(\mathbf{A})$ is the null space of matrix $\mathbf{A}$. $\|\cdot\|$ is the spectral norm.

2. SYSTEM MODEL

Suppose the BS has M antennas. In JSDM scheme based on PGP, the total K users are divided into N groups, where group n

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$(n = 1, \ldots, N)$ has K_n users with $\sum_{n=1}^{N} K_n = K$. The prebeamformer for group n is denoted as $\mathbf{B}_n \in \mathbb{C}^{M\times M_n}$. The maximum BS transmit power is P_T, and the group power vector is $\mathbf{p} \in \mathbb{R}^{N\times 1}$ with its nth entry $p_n \geq 0$ as the power allocated to group n. Then, $\mathbf{p}$ should satisfy the total power constraint $\mathbf{1}^T\mathbf{p} \leq P_T$.

Let $\mathbf{H}_n = [\mathbf{h}_{k_1}, \ldots, \mathbf{h}_{K_n}]^H \in \mathbb{C}^{K_n\times M}$ be the channel matrix for group n with $\mathbf{h}_{k_n} \in \mathbb{C}^{M\times 1}$ as the channel vector for the k_nth user $(k_n = 1, \ldots, K_n)$. Suppose the channels of users in group n undergo Rayleigh fading with CCM $\mathbf{R}_n$, then we have $\mathbf{h}_{k_n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_n)$. For rank$(\mathbf{R}_n) = r_n$, the dimension of $\mathcal{R}(\mathbf{B}_n)$, M_n should satisfy $M_n \leq r_n$. With prebeamforming matrices $\{\mathbf{B}_n\}$, the effective channel matrix of group n is $\hat{\mathbf{H}}_n = \mathbf{H}_n\mathbf{B}_n$ whose CCM is $\mathbf{\Sigma}_n = \mathbf{B}_n^H\mathbf{R}_n\mathbf{B}_n$. In this paper, we assume that the BS has only statistical CSI of $\mathbf{H}_n$ (the CCM) but perfect knowledge of instantaneous $\hat{\mathbf{H}}_n$.

By employing ZF beamforming and allocating equal power to users within each group, the received signal vector for group n is given by

$$\mathbf{y}_n = \mathbf{H}_n\mathbf{B}_n\mathbf{F}_n\mathbf{s}_n + \mathbf{H}_n\sum_{l\neq n}\mathbf{B}_l\mathbf{F}_l\mathbf{s}_l + \mathbf{v}_n, \tag{1}$$

where the signal vector of group n is $\mathbf{s}_n$, the ZF beamformer $\mathbf{F}_n = \frac{\xi_n}{K_n}\hat{\mathbf{H}}_n^H(\hat{\mathbf{H}}_n\hat{\mathbf{H}}_n^H)^{-1}$ with $\xi_n = p_n/\mathrm{tr}((\hat{\mathbf{H}}_n\hat{\mathbf{H}}_n)^{-1})$, and the noise vector $\mathbf{v}_n \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{K_n})$.

With notations above, the achievable ergodic rate of user k_n in group n is $R_{k_n} = \mathbb{E}[\log_2(1 + \gamma_{k_n})]$ where

$$\gamma_{k_n} = \frac{\xi_n/K_n}{\mathbf{h}_{k_n}^H\left(\sum_{l\neq n}\mathbf{B}_l\mathbf{F}_l\mathbf{F}_l^H\mathbf{B}_l^H\right)\mathbf{h}_{k_n} + 1} \tag{2}$$

is the instantaneous SINR at user k_n.

3. JOINT GROUP POWER ALLOCATION AND PREBEAMFORMER OPTIMIZATION

With group power $\mathbf{p}$ and prebeamformers $\{\mathbf{B}_n\}$ as the optimization variables, the per-user ergodic rate balancing problem, or the minimum ergodic user rate maximization problem, can be formulated as

$$\text{(P1)}\ \max_{\mathbf{p},\{\mathbf{B}_n\}}\ \min_{k_n,n} R_{k_n}, \text{s.t. } \mathbf{1}^T\mathbf{p} \leq P_T. \tag{3}$$

Unfortunately, the exact expression of the ergodic rate R_{k_n} is difficult to attain and therefore the problem (P1) is not amenable to optimization. We bypass such difficult by appealing to the deterministic approximation of the ergodic rate, which was first proposed in [11] and was later applied to anaylze the performance of JSDM scheme in [7, 8]. Based on the results in [7, 8], we have the approximated per-user ergodic rate balancing problem which is equivalent to the following $\bar{\gamma}_n^{\text{DL}}$ balancing problem.

$$\text{(P2)}\ \max_{\mathbf{p},\mathbf{e},\{\mathbf{B}_n\}}\ \min_{n} \bar{\gamma}_n^{\text{DL}}, \text{s.t. } e_n = \frac{1}{M_n}\mathrm{tr}(\mathbf{\Sigma}_n\mathbf{T}_n), \forall n, \mathbf{1}^T\mathbf{p} \leq P_T, \tag{4}$$

where

$$\bar{\gamma}_n^{\text{DL}} = \frac{p_n/(c_nK_n)}{\sum_{l=1,l\neq n}^{N} a_{nl}p_l/c_l + 1}, \tag{5}$$

is the deterministic counterpart of γ_{k_n} with

$$a_{nl} = \frac{\frac{1}{M_n^2}\mathrm{tr}(\mathbf{\Sigma}_n\mathbf{T}_n\mathbf{\Sigma}_{nl}\mathbf{T}_n)}{e_n^2 - \frac{K_n}{M_n^2}\mathrm{tr}((\mathbf{\Sigma}_n\mathbf{T}_n)^2)}, c_n = \frac{\frac{1}{M_n^2}\mathrm{tr}(\mathbf{\Sigma}_n\mathbf{T}_n\mathbf{B}_n^H\mathbf{B}\mathbf{T}_n)}{e_n^2 - \frac{K_n}{M_n^2}\mathrm{tr}((\mathbf{\Sigma}_n\mathbf{T}_n)^2)}, \tag{6}$$

and $\mathbf{T}_n = (\frac{K_n}{M_n}\frac{\mathbf{\Sigma}_n}{e_n} + \mathbf{I}_{M_n})^{-1}$, $\mathbf{e} = [e_1, \ldots, e_n]^T$..

Notice that for fixed $\{\mathbf{B}_n\}$, optimization over $\mathbf{p}$ is identical to the power allocation problem for SINR balancing in MISO downlink [12]. So the result in [12] for $\mathbf{p}^{\text{opt}}$ is immediately applicable, and we simply state the result. Denote $\mathbf{C} = \mathrm{diag}\{c_1, \ldots, c_N\}$ and $\mathbf{K} = \mathrm{diag}\{K_1, \ldots, K_N\}$, and nonnegative matrix $\mathbf{A} \in \mathbb{R}^{N\times N}$ whose elements $[\mathbf{A}]_{nl} = a_{nl}$ for $n \neq l$ and 0 for $n = l$. The optimal $\mathbf{p}^{\text{opt}}$ is solved by the eigensystem below.

$$\begin{bmatrix} \mathbf{CKAC}^{-1} & \mathbf{CK1} \\ \frac{\mathbf{1}^T\mathbf{CKAC}^{-1}}{P_T} & \frac{\mathbf{1}^T\mathbf{CK1}}{P_T} \end{bmatrix}\begin{bmatrix}\mathbf{p}^{\text{opt}} \\ 1\end{bmatrix} = \mathbf{\Upsilon}\begin{bmatrix}\mathbf{p}^{\text{opt}} \\ 1\end{bmatrix} = \lambda_1(\mathbf{\Upsilon})\begin{bmatrix}\mathbf{p}^{\text{opt}} \\ 1\end{bmatrix}, \tag{7}$$

and the optimal balanced $\bar{\gamma}_n^{\text{DL}}$ is $\bar{\gamma}^{\text{opt}} = 1/\lambda_1(\mathbf{\Upsilon})$.

To solve the optimal $\{\mathbf{B}_n\}$ and $\mathbf{e}$, we resort to the uplink-downlink duality approach in [12] and have the dual uplink power allocation problem:

$$\max_{\mathbf{q}}\ \min_{l} \bar{\gamma}_l^{\text{UL}}, \text{s.t. } \mathbf{1}^T\mathbf{q} \leq P_T \tag{8}$$

where $\mathbf{q} \in \mathbb{R}^{N\times 1}$ is the uplink power allocation vector, and

$$\bar{\gamma}_l^{\text{UL}} = \frac{q_l/K_l}{\sum_{n=1,n\neq l}^{N} a_{ln}q_n + c_l}, l = 1, \ldots, N, \tag{9}$$

is the uplink counterpart of $\bar{\gamma}_n^{\text{DL}}$. Likewise, the optimal uplink power allocation is solved by the eigensystem:

$$\begin{bmatrix} \mathbf{KA}^T & \mathbf{CK1} \\ \frac{\mathbf{1}^T\mathbf{KC}^T\mathbf{1}}{P_T} & \frac{\mathbf{1}^T\mathbf{CK1}}{P_T} \end{bmatrix}\begin{bmatrix}\mathbf{q}^{\text{opt}} \\ 1\end{bmatrix} = \mathbf{\Lambda}\begin{bmatrix}\mathbf{q}^{\text{opt}} \\ 1\end{bmatrix} = \lambda_1(\mathbf{\Lambda})\begin{bmatrix}\mathbf{q}^{\text{opt}} \\ 1\end{bmatrix}, \tag{10}$$

with $\lambda_1(\mathbf{\Lambda}) = \lambda_1(\mathbf{\Upsilon}) = 1/\bar{\gamma}^{\text{opt}}$.

For fixed $\mathbf{q}$, as in [12], the prebeamformer optimization for $\{\mathbf{B}_n\}$ and $\mathbf{e}$ is treated in the uplink channel and is a $\bar{\gamma}_l^{\text{UL}}$ maximization problem which by simple derivation can be cast as

$$\text{(Q1)}\ \min_{\mathbf{B}_l,e_l} \frac{\frac{1}{M_l^2}\mathrm{tr}(\mathbf{\Sigma}_l\mathbf{T}_l\mathbf{\Theta}_l\mathbf{T}_l)}{e_l^2 - \frac{K_l}{M_l^2}\mathrm{tr}(\mathbf{\Sigma}_l\mathbf{T}_l\mathbf{\Sigma}_l\mathbf{T}_l)}, \text{s.t. } e_l = \frac{1}{M_l}\mathrm{tr}(\mathbf{\Sigma}_l\mathbf{T}_l), \tag{11}$$

where $\mathbf{\Theta}_l = \mathbf{B}_l^H\mathbf{S}_l\mathbf{B}_l$ with $\mathbf{S}_l = \mathbf{I}_M + \sum_{n=1,n\neq l} q_n\mathbf{R}_n$. In such formulation, the problems of prebeamforming in different groups are decoupled. But without simple structure to utilize, solving the optimal $\mathbf{B}_l^{\text{opt}}$ and e_l^{opt} of (Q1) could be quite involved. Next we give the key theorem in our work which reveals the structure of the solution of (Q1) and greatly facilitates the optimization.

Theorem 1. *The problem (Q1) is equivalent to*

$$\begin{aligned} \textit{(Q2)}\ \min_{0\prec\mathbf{X}_l\prec\frac{1}{K_l}\mathbf{I}_{M_l},\mathbf{W}_l}\ & \frac{tr(\mathbf{X}_l\mathbf{W}_l^H\mathbf{S}_l\mathbf{W}_l\mathbf{X}_l)}{1 - K_l tr(\mathbf{X}_l^2)}, \\ \textit{s.t. } tr(\mathbf{X}_l) = 1,\ & \mathbf{W}_l^H\mathbf{R}_l\mathbf{W}_l = \mathbf{I}_{M_l}, \end{aligned} \tag{12}$$

where $\mathbf{X}_l = diag\{x_1, \ldots, x_{M_l}\}$ with $x_1 \geq \cdots \geq x_{M_l} > 0$, and $\mathbf{W}_l \in \mathbb{C}^{M\times M_l}$. If $\mathbf{X}_l^{opt}$ and $\mathbf{W}_l^{opt}$ solve (Q2), the solution for (Q1) is $\mathbf{B}_l^{opt} = \mathbf{W}_l^{opt}(\frac{1}{M_l}(\mathbf{X}_l^{opt})^{-1} - \frac{K_l}{M_l}\mathbf{I}_{M_l})^{-1/2}$ and $e_l^{opt} = 1$.

Proof. First take variable change $\mathbf{W}_l = \mathbf{B}_l/\sqrt{e_l}$ to get the problem

$$\min_{\mathbf{W}_l} \frac{\frac{1}{M_l^2}\mathrm{tr}\left(\tilde{\mathbf{\Sigma}}_l\tilde{\mathbf{T}}_l\tilde{\mathbf{\Theta}}_l\tilde{\mathbf{T}}_l\right)}{1 - \frac{K_l}{M_l^2}\mathrm{tr}\left((\tilde{\mathbf{\Sigma}}_l\tilde{\mathbf{T}}_l)^2\right)}, \text{s.t. } \frac{1}{M_l}\mathrm{tr}(\tilde{\mathbf{\Sigma}}_l\tilde{\mathbf{T}}_l) = 1. \tag{13}$$

where $\tilde{\boldsymbol{\Sigma}}_l = \mathbf{W}_l^H \mathbf{R}_l \mathbf{W}_l = \boldsymbol{\Sigma}_l / e_l^2$, $\tilde{\boldsymbol{\Theta}}_l = \boldsymbol{\Theta}/e_l^2 = \mathbf{W}_l^H \mathbf{S}_l \mathbf{W}_l$, and $\mathbf{T}_l = (\frac{K_l}{M_l}\boldsymbol{\Sigma}_l + \mathbf{I}_{M_l})^{-1}$.

The problem in (13) is equivalent to (Q1) in that for any $\mathbf{W}_l$ we can always let $\mathbf{B}_l = \mathbf{W}_l$ and $e_l = 1$ to get feasible $\mathbf{B}_l$ and e_l pair for (Q1) which also yields the same objective function value of (Q1) as that of (13). We further introduce auxiliary variable $\mathbf{Y} \succ 0$ and consider the problem below.

$$\min_{\mathbf{W}_l, \mathbf{Y} \succ 0} \frac{\frac{1}{M_l^2} \mathrm{tr}\left(\tilde{\boldsymbol{\Sigma}}_l \left(\frac{K_l}{M_l}\tilde{\boldsymbol{\Sigma}}_l + \mathbf{Y}\right)^{-1} \tilde{\boldsymbol{\Theta}}_l \left(\frac{K_l}{M_l}\tilde{\boldsymbol{\Sigma}}_l + \mathbf{Y}\right)^{-1}\right)}{1 - \frac{K_l}{M_l^2}\mathrm{tr}\left(\left(\tilde{\boldsymbol{\Sigma}}_l \left(\frac{K_l}{M_l}\tilde{\boldsymbol{\Sigma}}_l + \mathbf{Y}\right)^{-1}\right)^2\right)},$$
$$\text{s.t. } \frac{1}{M_l}\mathrm{tr}\left(\tilde{\boldsymbol{\Sigma}}_l \left(\frac{K_l}{M_l}\tilde{\boldsymbol{\Sigma}}_l + \mathbf{Y}\right)^{-1}\right) = 1. \tag{14}$$

The problem in (14) is also equivalent to (13) and hence to (Q1). To see this, note that with Cholesky decomposition $\mathbf{Y} = \mathbf{G}^H \mathbf{G}$, the invertible transform $\mathbf{W}_l \to \mathbf{G}^{-H}\mathbf{W}_l \mathbf{G}^{-1}$ will give a feasible solution to (13) and make the objective function in (14) equal to that in (13).

Now let $\mathbf{X}_l = \frac{1}{M_l}(\frac{K_l}{M_l}\tilde{\boldsymbol{\Sigma}}_l + \mathbf{Y})^{-1} \prec \frac{1}{K_l}\tilde{\boldsymbol{\Sigma}}_l$. With fixed $\mathbf{W}_l$, the transform between $\mathbf{X}_l$ and $\mathbf{Y}$ is always invertible, so the problem in (14) can be reformulated as an optimization over $\mathbf{X}_l$ and $\mathbf{W}_l$:

$$\min_{\mathbf{W}_l, 0 \prec \mathbf{X} \prec \frac{1}{K_l}\tilde{\boldsymbol{\Sigma}}_l} \frac{\mathrm{tr}\left(\tilde{\boldsymbol{\Sigma}}_l \mathbf{X}_l \tilde{\boldsymbol{\Theta}}_l \mathbf{X}_l\right)}{1 - K_l \mathrm{tr}\left(\left(\tilde{\boldsymbol{\Sigma}}_l \mathbf{X}_l\right)^2\right)}, \text{s.t. } \mathrm{tr}\left(\tilde{\boldsymbol{\Sigma}}_l \mathbf{X}_l\right) = 1. \tag{15}$$

It can be verified that the objective function and the constraints in (15) are invariant to transform $\mathbf{W}_l \to \mathbf{W}_l \mathbf{P}$ with any invertible matrix $\mathbf{P}$, so we can add an extra constraint $\tilde{\boldsymbol{\Sigma}}_l = \mathbf{W}_l^H \mathbf{R}_l \mathbf{W}_l = \mathbf{I}_{M_l}$ to the problem in (15). Further notice that as for the problem in (15), both the objective function and the constraints with the additional one just mentioned are also invariant to transforms $\mathbf{X}_l \to \mathbf{Q}\mathbf{X}_l\mathbf{Q}^H$ and $\mathbf{W}_l \to \mathbf{W}_l \mathbf{Q}^H$ with any unitary matrix $\mathbf{Q}$, we may therefore as well assume that $\mathbf{X}_l$ is diagonal and without loss of generality the diagonal elements of $\mathbf{X}_l$ are arranged in descending order.

In sum, after a series of transforms above, we arrive at the problem (Q2) which is equivalent to (Q1). Once we solve (Q2), we can get the optimal solution of (Q1) as is described in the proposition by performing these transforms backwards. □

Theorem 1 reveals that the solution for (Q1) is composed of two parts: the matrix $\mathbf{W}_l$ which reduces the dimension and diagonalizes $\mathbf{R}_l$, and the diagonal matrix $\mathbf{X}_l$ which puts weights along the directions of $\mathbf{W}_l$'s column vectors. Propositions 1 and 3 give a full characterization of the optimal $\mathbf{W}_l^{\text{opt}}$ and $\mathbf{X}_l^{\text{opt}}$. Proposition 1 solves $\mathbf{W}_l^{\text{opt}}$ for fixed $\mathbf{X}_l$, which turns out to be independent from specific $\mathbf{X}_l$ and thus is optimal for (Q2). In Proposition 3, the optimal $\mathbf{X}_l^{\text{opt}}$ is solved with $\mathbf{W}_l^{\text{opt}}$. The computation of $\mathbf{W}_l^{\text{opt}}$ and $\mathbf{X}_l^{\text{opt}}$ only involves eigenvalue decomposition and therefore can be carried out efficiently.

Proposition 1. *Let $\mathbf{S}_l = \mathbf{G}_l^H \mathbf{G}_l$ with invertible upper-triangular matrix $\mathbf{G}_l \in \mathbb{C}^{M \times M}$, and the eigenvalue decomposition $\mathbf{R}_l = \mathbf{U}_l \boldsymbol{\Lambda}_l \mathbf{U}_l^H$ with diagonal matrix $\boldsymbol{\Lambda}_l \in \mathbb{R}^{r_l \times r_l}$ and semi-unitary matrix $\mathbf{U}_l \in \mathbb{C}^{M \times r_l}$. Denote semi-unitary matrix $\mathbf{U}_l^{\perp} \in \mathbb{C}^{M \times (M - r_l)}$ with $\mathbf{U}_l^{\perp H}\mathbf{U}_l = \mathbf{O}$, $\mathbf{Z}_1 = \mathbf{G}_l \mathbf{U}_l^{\perp}$, $\mathbf{Z}_2 = \mathbf{G}_l \mathbf{U}_l \boldsymbol{\Lambda}_l^{-1/2}$, $\mathbf{Z}_3 = \mathbf{Z}_2^H(\mathbf{I}_M - \mathbf{Z}_1\mathbf{Z}_1^+)\mathbf{Z}_2 \succeq 0$ with $\mathbf{Z}_1^+$ as the Moore-Penrose inverse of $\mathbf{Z}_1$, and the diagonal matrix $\mathbf{D} = diag\{\lambda_1, \ldots, \lambda_{M_l}\}$ where $\lambda_i = \lambda_{r_l - i + 1}(\mathbf{Z}_3)$ $(i = 1, \ldots, M_l)$. Then, the optimal $\mathbf{W}_l^{opt}$ of (Q2) is given by*

$$\mathbf{W}_l^{opt} = \mathbf{U}_l \boldsymbol{\Lambda}_l^{-1/2}\mathbf{Q}^{opt} + \mathbf{U}_l^{\perp}\mathbf{Z}^{opt}, \tag{16}$$

where $\mathbf{Q}^{opt} \in \mathbb{C}^{r_l \times M_l}$ satisfies $\mathbf{Q}^{optH}\mathbf{Q}^{opt} = \mathbf{I}_{M_l}$ and $\mathbf{Q}^{optH}\mathbf{Z}_3\mathbf{Q}^{opt} = \mathbf{D}$, and $\mathbf{Z}^{opt} = -\mathbf{Z}_1^+\mathbf{Z}_2\mathbf{Q}^{opt}$.

Proof. Any $\mathbf{W}_l$ satisfying $\mathbf{W}_l^H \mathbf{R}_l \mathbf{W}_l = \mathbf{I}_{M_l}$ can be written as $\mathbf{W}_l = \mathbf{U}_l \boldsymbol{\Lambda}_l^{-1/2}\mathbf{Q} + \mathbf{U}_l^{\perp}\mathbf{Z}$ where $\mathbf{Q} \in \mathbb{C}^{r_l \times M_l}$ with $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}_{M_l}$, and $\mathbf{Z} \in \mathbb{C}^{(M - r_l) \times M_l}$. So $\mathbf{W}_l^{\text{opt}}$ is solved by optimizing $\mathbf{Z}$ and $\mathbf{Q}$. For fixed $\mathbf{Q}$, the optimization over $\mathbf{Z}$ can be identified as an matrix-weighted minimum norm problem, which gives solution $\mathbf{Z}' = -\mathbf{Z}_1^+\mathbf{Z}_2\mathbf{Q}$. Substituting $\mathbf{Z}'$ back into the objective function of (Q2) gives the optimization problem for $\mathbf{Q}^{\text{opt}}$:

$$\min_{\mathbf{Q}^H\mathbf{Q} = \mathbf{I}_{M_l}} \mathrm{tr}\left(\mathbf{X}_l^2 \mathbf{Q}^H \mathbf{Z}_3 \mathbf{Q}\right). \tag{17}$$

$\mathbf{Q}^{\text{opt}}$ is then obtained by the results in [13] and [14]. □

The property of the diagonal elements of $\mathbf{D}$ is presented in the proposition below.

Proposition 2. *Suppose that $\max_l \|\mathbf{R}_l\| < C < \infty$ and $\lambda_{r_l}(\mathbf{R}_l) > \epsilon > 0$ for $\forall l$. The diagonal elements of $\mathbf{D}$ satisfy $1/(C(1 + P_T NC)) < \lambda_1 \leq \cdots \leq \lambda_{M_l} < 1/\epsilon$.*

Proof. By using the bounds on $\{\mathbf{R}_l\}$ and the expression of $\mathbf{S}_l$, we have $\mathbf{I}_M \prec \mathbf{S}_l \preceq (1 + P_T N \max_l \|\mathbf{R}_l\|)\mathbf{I}_M \prec (1 + P_T NC)\mathbf{I}_M$. Define $\mathbf{Z}_1^{\perp} = \mathbf{G}_l^{-H}\mathbf{U}_l$ which satisfies $\mathrm{rank}(\mathbf{Z}_1^{\perp}) + \mathrm{rank}(\mathbf{Z}_1) = M$ and $(\mathbf{Z}_1^{\perp})^H\mathbf{Z}_1 = \mathbf{O}$. Therefore, $\mathcal{R}(\mathbf{Z}_1^{\perp}) = \mathcal{N}(\mathbf{Z}_1)$ and $\mathbf{I}_M - \mathbf{Z}_1\mathbf{Z}_1^+ = \mathbf{Z}_1^{\perp}(\mathbf{Z}_1^{\perp})^+ = \mathbf{G}_1^{-H}\mathbf{U}_l(\mathbf{U}_l^H\mathbf{S}_l\mathbf{U}_l)^{-1}\mathbf{U}_l^H\mathbf{G}_l^{-1}$, which gives $\mathbf{Z}_3 = \boldsymbol{\Lambda}_l^{-1/2}(\mathbf{U}_l^H\mathbf{S}_l\mathbf{U}_l)^{-1}\boldsymbol{\Lambda}_l^{-1/2}$. Consequently, with the bounds on $\mathbf{S}_l$ and $\{\mathbf{R}_l\}$, we get $\frac{1}{C(1+P_T NC)}\mathbf{I}_{r_l} \prec \mathbf{Z}_3 \prec \frac{1}{\epsilon}\mathbf{I}_{r_l}$ by which the proposition is proved. □

Proposition 3. *Denote the largest generalized eigenvalue of matrix pencil $\{\mathbf{1}\mathbf{1}^T - K_l\mathbf{I}_{M_l}, \mathbf{D}\}$ as σ and the corresponding generalized eigenvector as $\mathbf{x}_{ext}$. Then $\sigma > 0$ and if the assumptions on $\{\mathbf{R}_l\}$ in Proposition 2 hold, the optimization problem for $\mathbf{X}_l = diag\{\mathbf{x}_l\}$ with $\mathbf{W}_l^{opt}$*

$$\min_{\mathbf{x}_l} \frac{\mathbf{x}_l^T\mathbf{D}\mathbf{x}_l}{1 - K_l\|\mathbf{x}_l\|^2}, \text{s.t. } \mathbf{1}^T\mathbf{x}_l = 1, K_l^{-1} > x_1 \geq \cdots \geq x_{M_l} > 0 \tag{18}$$

has optimal solution $\mathbf{x}_l^{opt} = \mathbf{x}_{ext}/\mathbf{1}^T\mathbf{x}_{ext}$. Further, the entries of $\mathbf{x}_l^{opt}$ satisfy $1/K_l > 1/(K_l + \delta(M_l - K_l)) > x_1^{opt} \geq \cdots \geq x_{M_l}^{opt} > 1/(K_l + \delta^{-1}(M_l - K_l)) > 0$ where $\delta = \epsilon/(C(1 + P_T NC)) > 0$.

Proof. $1/\sigma$ is in fact a lower bound of the optimal objective function value of (18), which is from

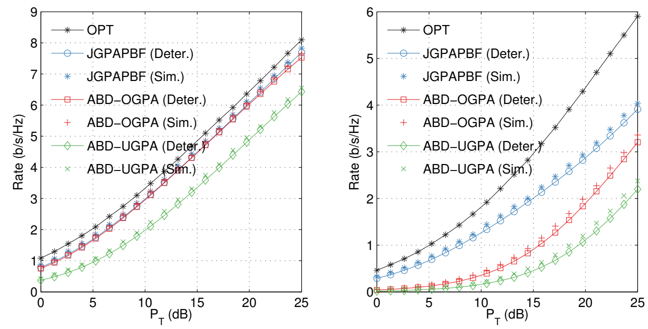
$$\sigma = \max_{\mathbf{x}} \frac{\mathbf{x}^T\left(\mathbf{1}\mathbf{1}^T - K_l\mathbf{I}_{M_l}\right)\mathbf{x}}{\mathbf{x}^T\mathbf{D}\mathbf{x}} \geq \max_{\mathbf{1}^T\mathbf{x}=1} \frac{1 - K_l\mathbf{x}^T\mathbf{x}}{\mathbf{x}^T\mathbf{D}\mathbf{x}}. \tag{19}$$

In addition, $\lambda_1(\mathbf{1}\mathbf{1}^T - K_l\mathbf{I}_{M_l}) = M_l - K_l$ and $\frac{1}{C(1+P_T NC)}\mathbf{I}_{M_l} \prec \mathbf{D} \prec \frac{1}{\epsilon}\mathbf{I}_{M_l}$, which is from Proposition 2. So $\epsilon(M_l - K_l) < \sigma < (M_l - K_l)(C(1 + P_T NC))$. The rest is to prove $\mathbf{x}^{\text{opt}} = \mathbf{x}_{\text{ext}}/\mathbf{1}^T\mathbf{x}_{\text{ext}}$ is feasible for (18) and satisfies the inequality in the proposition.

As $(\mathbf{1}\mathbf{1}^T - K_l\mathbf{I}_{M_l})\mathbf{x}_{\text{ext}} = \sigma\mathbf{D}\mathbf{x}_{\text{ext}} \Rightarrow \mathbf{x}_l^{\text{opt}} = \mathbf{x}_{\text{ext}}/\mathbf{1}^T\mathbf{x}_{\text{ext}} = (K_l\mathbf{I}_{M_l} + \sigma\mathbf{D})^{-1}\mathbf{1}$, we get $x_i^{\text{opt}} = 1/(K_l + \sigma\lambda_i)$ where x_i^{opt} is the ith entry of $\mathbf{x}_l^{\text{opt}}$. Proposition 3 is then readily proved through the bounds on σ above and the inequalities of λ_i in Proposition 2. □

initialization: $m = 0$, $\mathbf{q}^{(0)} = \frac{P_T}{N}\mathbf{1}$, $R^{(0)} = 0$, tol $= 10^{-3}$;
repeat
  Solve $\mathbf{B}_l^{(m+1)}$ and $e_l^{(m+1)}$ through Theorem 1, and Propositions 1 and 3, $l = 1, \dots, N$;
  Solve (8) for $\mathbf{q}^{(m+1)}$ and $\bar{\gamma}^{\text{opt}(m+1)}$;
  $R^{(m+1)} = \log_2(1 + \bar{\gamma}^{\text{opt}(m+1)})$;
  $m = m + 1$;
until $R^{(m)} - R^{(m-1)} < tol$;
Solve (7) for $\mathbf{p}^{\text{opt}}$;

Table 1: Joint group power allocation and prebeamforming (JGPAPBF) algorithm.



(a) $N = 3$ (b) $N = 9$

Fig. 1: Minimum per-user ergodic rate versus P_T.

Now we have finished the derivation for $\mathbf{W}_l^{\text{opt}}$ and $\mathbf{X}_l^{\text{opt}}$ which can give $\mathbf{B}_l^{\text{opt}}$ and e_l^{opt} for (Q1). Combining the result of $\mathbf{q}^{\text{opt}}$, we can design an iterative JGPAPBF algorithm alternatively updating $\mathbf{q}$ and $\{\mathbf{B}_l, \mathbf{e}\}$ which is listed in Table 1.

The convergence of the JGPAPBF algorithm and its optimality for (P2) are guaranteed by Theorem 2 below. But the optimality of the JGPAPBF algorithm for the exact ergodic rate balancing problem (P1) is still unresolved. Note that we can expect asymptotic optimaltiy for large M which could be proved following the proof of Proposition 3 in [15]. As more technical details are required, we settle with Theorem 2 for now and leave the proof for future work.

Theorem 2. *JGPAPBF algorithm in Table 1 improves $R^{(m)}$ monotonically and the limit value of sequence $R^{(m)}$ is the global optimum of (P2).*

Proof. The proof mimics that for Theorem 3 in [12] and is omitted due to space limitation. □

To get accurate deterministic approximation of the ergodic rate for ZF beamforming as $M \to \infty$, the solution given by JGPAPBF algorithm need to fulfill Assumptions 1-4 in [11]. This is guaranteed in the following proposition.

Proposition 4. *Suppose that the group number N is fixed, and M_n, K_n, and r_n grow linearly with M. Under the assumptions on $\{\mathbf{R}_n\}$ in Proposition 2, we have $R_{k_n} \to \log_2(1+\bar{\gamma}_n^{DL})$ as $M \to \infty$ for the optimal solutions $\{\mathbf{B}_n^{opt}\}$ and $\mathbf{p}^{opt}$ produced by JGPAPBF algorithm.*

Proof. From Theorem 1, $\mathbf{\Sigma}_n^{\text{opt}} = (\mathbf{B}_n^{\text{opt}})^H \mathbf{R}_n \mathbf{B}_n^{\text{opt}} = (\frac{1}{M_n}\mathbf{X}_n^{-1} - \frac{K_n}{M_n}\mathbf{I}_{M_n})^{-1}$ which according to Proposition 3 yields $0 < \delta < \frac{M_n \delta}{M_n - K_n} < \|\mathbf{\Sigma}_n^{\text{opt}}\| < \frac{M_n \delta^{-1}}{M_n - K_n} < \frac{\delta^{-1}}{1-\beta_n} < \infty$ with $\beta_n = K_n/M_n$. Base on the bounds, it is easy to verify that $\{\mathbf{B}_n^{\text{opt}}\}$ fulfills Assumptions 1-4 in [11] and the proposition is proved. □

4. SIMULATIONS

The system configuration is as follow. Uniform circular array with $M = 64$ elements is used. $M_n = 6$ and $K_n = 3$ for $\forall n$, and two cases of $N = 3$ and 9 are considered. CCMs are given by $\{w_n \mathbf{R}_n\}$ where the weight w_n is taken from $[0.1, 0.5, 1]$ for $N = 3$ and $[0.1, 0.1, 0.1, 0.5, 0.5, 0.5, 1, 1, 1]$ for $N = 9$ to emulate the uneven channel qualities of different groups, and the entries of $\mathbf{R}_n$ are defined by [8, 16]

$$[\mathbf{R}_n]_{mp} = \frac{1}{2\Delta} \int_{-\Delta+\theta_n}^{\Delta+\theta_n} e^{-2\pi j D(\cos(\alpha - \frac{2\pi}{m}) - \cos(\alpha - \frac{2\pi}{p}))} d\alpha \quad (20)$$

with the angular spread $\Delta = 15°$, the angle of arrival (AoA) $\theta_n = -\pi + \Delta + 2\pi(n-1)/N$, and $D = 1/(4\sin \pi/M)$.

In the experiment, JGPAFBF algorithm is compared with ABD prebeamforming with uniform group power allocation ("ABD-UGPA"), ABD prebeamforming with optimal group power allocation calculated by (7) ("ABD-OGPA"), and the optimal SINR balancing algorithm in [12] ("OPT"). The ergodic rates from 1000 Monte Carlo simulations (Sim.) and the deterministic approximation (Deter.) are both presented. For ABD-based algorithms, the effective rank r_n^*, which is the number of dominant eigenvalues of CCM, should be selected [8]. We set $r_n^* = 7$ which through our experiments gives the best performance for ABD method. To make fair comparison, we adopt the same r_n^* for JGPAPBF instead of the true matrix rank which is required in Theorem 1.

Figs. 1a and 1b show the curves of minimum ergodic rate versus P_T. As seen from the figures, the rates of all algorithms grow with P_T. For $N = 3$, the advantage of JGPAPBF algorithm over ABD-OGPA algorithm is not evident and both algorithms have small degradation compared with the optimal one as the AoA spacing between groups is larger and IGI can be fully canceled by both JGPAPBF and ABD-OGPA algorithms. But for $N = 9$, JGPAPBF algorithm is more superior to ABD-OPGA algorithm, which indicates that JGPAPBF is more capable in IGI cancellation in the case of dense group AoAs. Nevertheless, the gap between the optimal rate and that of JGPAPBF algorithm is also very obvious and widens as P_T increases. Such performance loss is the result of the use of ZF beamformer and the much reduced CSI requirement ($6 \times 3 \times 9 = 162$ channel coefficients for JGPAPBF compared with $64 \times (3 \times 9) = 1728$ for the optimal method). Unsurprisingly, in both cases of $N = 3$ and 9, ABD-UGPA is the worst. Furthermore, as shown by the figures, the approximated ergodic rate of JGPAPBF algorithm fits very well to the simulated one, which verifies Proposition 4.

5. CONCLUSION

We have investigated the design of joint group power allocation and prebeamforming problem for JSDM. The general prebeamforming matrices and group power allocation were optimized under the criterion of approximated ergodic rate balancing. Based on uplink-downlink duality, iterative JGPAPBF algorithm was proposed. It was shown that the subproblems in each step of the proposed algorithm could be solved efficiently. Simulations have shown that the proposed algorithm can offer a performance gain over the heuristic ABD method especially in the case of dense groups.

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