BLIND ESTIMATION OF EFFECTIVE DOWNLINK CHANNEL GAINS IN MASSIVE MIMO

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ABSTRACT

We consider the massive MIMO downlink with time-division duplex (TDD) operation and conjugate beamforming transmission. To reliably decode the desired signals, the users need to know the effective channel gain. In this paper, we propose a blind channel estimation method which can be applied at the users and which does not require any downlink pilots. We show that our proposed scheme can substantially outperform the case where each user has only statistical channel knowledge, and that the difference in performance is particularly large in certain types of channel, most notably keyhole channels. Compared to schemes that rely on downlink pilots (e.g., [1]), our proposed scheme yields more accurate channel estimates for a wide range of signal-to-noise ratios and avoid spending time-frequency resources on pilots.

Index Terms— Blind channel estimation, downlink, massive MIMO, time-division duplex.

1. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is one of the most promising technologies to meet the demands for high throughput and communication reliability of next generation cellular networks [2–5]. In massive MIMO, time-division duplex (TDD) operation is preferable since then the pilot overhead does not depend on the number of base station antennas. With TDD, the channels are estimated at the base station through the uplink training. For the downlink, under the assumption of channel reciprocity, the channels estimated at the base station are used to precode the data, and the precoded data are sent to the users. To coherently decode the transmitted signals, each user should have channel state information (CSI), that is, know its effective channel from the base station.

In most previous works, the users are assumed to have statistical knowledge of the effective downlink channels, that is, they know the mean of the effective channel gain and use this for the signal detection [6,7]. In these papers, Rayleigh fading channels were assumed. Under the Rayleigh fading, the effective channel gains become nearly deterministic (the channel "hardens") when the number of base station antennas grows large, and hence, using the mean of the effective channel gain for signal detection works very well. However, in practice, propagation scenarios may be encountered where the channel does not harden. In that case, using the mean effective channel gain may not be accurate enough, and a better estimate of the effective channel should be used. In [1], we proposed a scheme where the base station (in addition to the beamformed data) also sent a beamformed downlink pilot sequence to the users. With this scheme, a performance improvement (compared to the case when the mean of the effective channel gain is used) was obtained. However, this scheme requires time-frequency resources in order to send

the downlink pilots. The associated overhead is proportional to the number of users which can be in the order of several tens, and hence, in a high-mobility environment (where the channel coherence interval is short) the spectral efficiency is significantly reduced.

Contribution: In this paper, we consider the massive MIMO downlink with conjugate beamforming.¹ We propose a scheme with which the users *blindly* estimate the effective channel gain from the received data. The scheme exploits the asymptotic properties of the mean of the received signal power when the number of base station antennas is large. The accuracy of our proposed scheme is investigated for two specific, very different, types of channels: (i) independent Rayleigh fading and (ii) keyhole channels. We show that when the number of base station antennas goes to infinity, the channel estimate provided by our scheme becomes exact. Also, numerical results quantitatively show the benefits of our proposed scheme, especially in keyhole channels, compared to the case where the mean of the effective channel gain is used as if it were the true channel gain, and compared to the case where the beamforming training scheme of [1] is used.

Notation: We use boldface upper- and lower-case letters to denote matrices and column vectors, respectively. The superscripts ()^T and ()^H stand for the transpose and conjugate transpose, respectively. The Euclidean norm, the trace, and the expectation operators are denoted by $\|\cdot\|$, Tr (·), and E {·}, respectively. The notation $\stackrel{P}{\rightarrow}$ means convergence in probability, and $\stackrel{a.s.}{\rightarrow}$ means almost sure convergence. Finally, we use $z \sim CN(0, \sigma^2)$ to denote a circularly symmetric complex Gaussian random variable (RV) z with zero mean and variance σ^2 .

2. SYSTEM MODEL

Consider the downlink of a massive MIMO system. An *M*-antenna base station serves *K* single-antenna users, where $M \gg K \gg 1$. The base station uses conjugate beamforming to simultaneously transmit data to all *K* users in the same time-frequency resource. Since we focus on the downlink channel estimation here, we assume that the base station perfectly estimates the channels in the uplink training phase. (In future work, this assumption may be relaxed.) Denote by \mathbf{g}_k the $M \times 1$ channel vector between the base station and the *k*th user. The channel \mathbf{g}_k results from a combination of small-scale fading and large-scale fading, and is modeled as:

$$\mathbf{g}_k = \sqrt{\beta_k} \mathbf{h}_k,\tag{1}$$

where β_k represents large-scale fading which is constant over many coherence intervals, and \mathbf{h}_k is an $M \times 1$ small-scale channel vector. We assume that the elements of \mathbf{h}_k are i.i.d. with zero mean and unit variance.

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¹We consider conjugate beamforming since it is simple and nearly optimal in many massive MIMO scenarios. More importantly, conjugate beamforming can be implemented in a distributed manner.

Let s_k , $\mathsf{E} \{ |s_k|^2 \} = 1$, k = 1, ..., K, be the symbol intended for the *k*th user. With conjugate beamforming, the $M \times 1$ precoded signal vector is given by

$$\mathbf{x} = \sqrt{\alpha} \mathbf{G} \mathbf{s},\tag{2}$$

where $\mathbf{s} \triangleq [s_1, s_2, \dots, s_K]^T$, $\mathbf{G} \triangleq [\mathbf{g}_1 \dots \mathbf{g}_K]$ is an $M \times K$ channel matrix between the K users and the base station, and α is a normalization constant chosen to satisfy the average power constraint at the base station: $\mathsf{E} \{ \|\mathbf{x}\|^2 \} = \rho.$

Hence,

$$\alpha = \frac{\rho}{\mathsf{E}\left\{\mathrm{Tr}\left(\mathbf{G}\mathbf{G}^{H}\right)\right\}}.$$
(3)

The signal received at the kth user is

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$$\begin{aligned} \mu_{k} &= \mathbf{g}_{k}^{H} \mathbf{x} + n_{k} = \sqrt{\alpha} \mathbf{g}_{k}^{H} \mathbf{G} \mathbf{s} + n_{k} \\ &= \sqrt{\alpha} \|\mathbf{g}_{k}\|^{2} s_{k} + \sqrt{\alpha} \sum_{k' \neq k}^{K} \mathbf{g}_{k}^{H} \mathbf{g}_{k'} s_{k'} + n_{k}, \end{aligned}$$
(4)

where $n_k \sim C\mathcal{N}(0, 1)$ is the additive Gaussian noise at the kth user. Then, the desired signal s_k is decoded.

3. PROPOSED DOWNLINK BLIND CHANNEL ESTIMATION TECHNIQUE

The *k*th user wants to detect s_k from y_k in (4). For this purpose, it needs to know the effective channel gain $||\mathbf{g}_k||^2$. If the channel is Rayleigh fading, then by the law of large numbers, we have

$$\frac{1}{M} \|\mathbf{g}_k\|^2 \xrightarrow{P} \beta_k,$$

as $M \to \infty$. This implies that when M is large, $||\mathbf{g}_k||^2 \approx M\beta_k$ (we say that the channel *hardens*). So we can use the statistical properties of the channel, i.e., use $\mathsf{E} \{ ||\mathbf{g}_k||^2 \} = M\beta_k$ as a good estimate of $||\mathbf{g}_k||^2$ when detecting s_k . This assumption is widely made in the massive MIMO literature. However, in practice, the channel is not always Rayleigh fading, and does not always harden when $M \to \infty$. For example, consider a keyhole channel, where the small-scale fading component \mathbf{h}_k is modeled as follows [8,9]:

$$\mathbf{h}_k = \nu_k \bar{\mathbf{h}}_k,\tag{5}$$

where ν_k and the *M* elements of $\bar{\mathbf{h}}_k$ are i.i.d. $\mathcal{CN}(0,1)$ RVs. For the keyhole channel (5), by the law of large numbers, we have

$$\frac{1}{M} \|\mathbf{g}_k\|^2 - \beta_k |\nu_k|^2 \stackrel{P}{\to} 0,$$

which is not deterministic, and hence the channel does not harden. In this case, using $\mathsf{E} \{ \| \mathbf{g}_k \|^2 \} = M \beta_k$ as an estimate of the true effective channel $\| \mathbf{g}_k \|^2$ to detect s_k may result in poor performance.

For the reasons explained, it is desirable that the users estimate their effective channels. One way to do this is to have the base station transmit beamformed downlink pilots as proposed in [1]. With this scheme, at least K downlink pilot symbols are required. This can significantly reduce the spectral efficiency. For example, suppose M = 300 antennas serve K = 50 terminals, in a coherence interval of length 200 symbols. If half of the coherence interval is used for the downlink, then with the downlink beamforming training

of [1], we need to spend at least 50 symbols for sending pilots. As a result, less than 50 of the 100 downlink symbols are used for payload in each coherence interval, and the insertion of the downlink pilots reduces the overall (uplink+downlink) spectral efficiency by a factor of 1/4.

In what follows, we propose a blind channel estimation method which does not require any downlink pilots.

3.1. Mathematical Preliminaries

Consider the average power of the received signal at the kth user (averaged over s and n_k). From (4), we have

$$\mathsf{E}\left\{\left|y_{k}\right|^{2}\right\} = \alpha \left\|\mathbf{g}_{k}\right\|^{4} + \alpha \sum_{k' \neq k}^{K} \left|\mathbf{g}_{k}^{H}\mathbf{g}_{k'}\right|^{2} + 1.$$
(6)

The second term of (6) can be rewritten as

$$\alpha \sum_{k'\neq k}^{K} \left| \mathbf{g}_{k}^{H} \mathbf{g}_{k'} \right|^{2} = \alpha \sum_{k'\neq k}^{K} \mathbf{g}_{k'}^{H} \mathbf{g}_{k} \mathbf{g}_{k}^{H} \mathbf{g}_{k'} = \alpha \tilde{\mathbf{g}}_{k}^{H} \mathbf{A} \tilde{\mathbf{g}}_{k}, \quad (7)$$

where $\tilde{\mathbf{g}}_k \triangleq [\mathbf{g}_1^T \dots \mathbf{g}_{k-1}^T \mathbf{g}_{k+1}^T \dots \mathbf{g}_K^T]^T$, and \mathbf{A} is an $M(K-1) \times M(K-1)$ block-diagonal matrix whose (i, i)-block is the $M \times M$ matrix $\mathbf{g}_k \mathbf{g}_k^H$. Since \mathbf{A} and $\tilde{\mathbf{g}}_k$ are independent, as $M(K-1) \to \infty$, the Trace lemma gives [10]

$$\frac{1}{M(K-1)} \sum_{k' \neq k}^{K} \left| \mathbf{g}_{k}^{H} \mathbf{g}_{k'} \right|^{2} - \frac{1}{M(K-1)} \sum_{k' \neq k}^{K} \beta_{k'} \| \mathbf{g}_{k} \|^{2} \stackrel{a.s.}{\to} 0.$$
(8)

Substituting (8) into (6), as $M(K-1) \rightarrow \infty$, we have

$$\frac{\mathsf{E}\left\{|y_{k}|^{2}\right\}}{M(K-1)} - \frac{1}{M(K-1)} \left(\alpha \|\mathbf{g}_{k}\|^{4} + \alpha \sum_{k' \neq k}^{K} \beta_{k'} \|\mathbf{g}_{k}\|^{2} + 1\right)$$

$$\stackrel{a.s.}{\to} 0. \quad (9)$$

The above result implies that when M and K are large,

$$\mathsf{E}\left\{\left|y_{k}\right|^{2}\right\} \approx \alpha \|\mathbf{g}_{k}\|^{4} + \alpha \sum_{k' \neq k}^{K} \beta_{k'} \|\mathbf{g}_{k}\|^{2} + 1.$$
(10)

Therefore, the effective channel gain $\|\mathbf{g}_k\|^2$ can be estimated from $\mathsf{E}\{|y_k|^2\}$ by solving the quadratic equation (10).

3.2. Downlink Blind Channel Estimation Algorithm

As discussed in Section 3.1, we can estimate the effective channel gain $||\mathbf{g}_k||^2$ by solving the quadratic equation (10). It is then required that the *k*th user knows α , $\sum_{k'\neq k}^{K} \beta_{k'}$, and $\mathsf{E}\{|y_k|^2\}$. We assume that the *k*th user knows α and $\sum_{k'\neq k}^{K} \beta_{k'}$. This assumption is reasonable since the terms α and $\sum_{k'\neq k}^{K} \beta_{k'}$ depend on the large-scale fading coefficients, which stay constant over many coherence intervals. Note that the expectation in (3) is performed over small-scale fading. The *k*th user can estimate these terms, or the base station may inform the *k*th user about them. Regarding $\mathsf{E}\{|y_k|^2\}$, in practice, it is unavailable. However, we can use the received samples during a whole coherence interval to form a sample estimate of $\mathsf{E}\{|y_k|^2\}$ as follows:

$$\mathsf{E}\left\{|y_k|^2\right\} \approx \xi_k \triangleq \frac{|y_k(1)|^2 + |y_k(2)|^2 + \ldots + |y_k(T)|^2}{T}, \quad (11)$$



Fig. 1. Normalized MSE versus SNR for different channel estimation schemes, for Rayleigh fading channels.

where $y_k(n)$ is the *n*th receive sample, and *T* is the length (in symbols) of the coherence interval used for the downlink transmission.

The algorithm for estimating $\|\mathbf{g}_k\|^2$ is summarized as follows:

Algorithm 1 (Proposed blind downlink channel estimation method)

- **1.** Using a data block of T samples, compute ξ_k as (11).
- **2.** The channel estimate of $||\mathbf{g}_k||^2$, denoted by a_k , is determined as

$$a_k = \frac{-\alpha \sum_{k' \neq k}^{K} \beta_{k'} + \sqrt{\alpha^2 \left(\sum_{k' \neq k}^{K} \beta_{k'}\right)^2 + 4\alpha(\xi_k - 1)}}{2\alpha}.$$
 (12)

Note that a_k in (12) is the positive root of the quadratic equation: $\xi_k = \alpha a_k^2 + \alpha \sum_{k' \neq k}^K \beta_{k'} a_k + 1$ which comes from (10) and (11).

3.3. Asymptotic Performance Analysis

In this section, we analyze the accuracy of our proposed scheme for two specific propagation environments: Rayleigh fading and keyhole channels. For keyhole channels, we use the model (5). We assume that the *k*th user perfectly estimates $E\{|y_k|^2\}$. This is true when the number of symbols of the coherence interval allocated to the downlink, *T*, is large. In the numerical results, we shall show that the estimate of $E\{|y_k|^2\}$ in (11) is very close to $E\{|y_k|^2\}$ even for modest values of *T* (e.g. $T \approx 100$ symbols). With the assumption $\xi_k = E\{|y_k|^2\}$, from (6) and (12), the estimate of $||\mathbf{g}_k||^2$ can be written as:

$$a_{k} = -\frac{\sum_{k'\neq k}^{K} \beta_{k'}}{2} + \sqrt{\left(\frac{\sum_{k'\neq k}^{K} \beta_{k'}}{2} + \|\mathbf{g}_{k}\|^{2}\right)^{2} + \epsilon_{k}}, \quad (13)$$

where

$$\epsilon_k \triangleq \sum_{k' \neq k}^{K} \left| \mathbf{g}_k^H \mathbf{g}_{k'} \right|^2 - \left(\sum_{k' \neq k}^{K} \beta_{k'} \right) \| \mathbf{g}_k \|^2.$$
(14)



Fig. 2. Normalized MSE versus SNR for different channel estimation schemes, for keyhole channels.

We can see from (13) that if $|\epsilon_k| \ll \left(\frac{\sum_{k'\neq k}^{K} \beta_{k'}}{2} + \|\mathbf{g}_k\|^2\right)^2$, then $a_k \approx \|\mathbf{g}_k\|^2$. In order to see under what conditions $|\epsilon_k| \ll \left(\frac{\sum_{k'\neq k}^{K} \beta_{k'}}{2} + \|\mathbf{g}_k\|^2\right)^2$, we consider ϱ_k which is defined as:

$$\varrho_{k} \triangleq \mathsf{E}\left\{\left|\epsilon_{k}/\mathsf{E}\left\{\left(\frac{1}{2}\sum_{k'\neq k}^{K}\beta_{k'}+\|\mathbf{g}_{k}\|^{2}\right)^{2}\right\}\right|^{2}\right\}.$$
 (15)

Hence,

$$\varrho_{k} = \begin{cases}
\frac{M(M+1)\beta_{k}^{2} \sum_{k'\neq k}^{K} \beta_{k'}^{2}}{\left(\frac{1}{4}\bar{\beta}_{k}^{2} + M\beta_{k} \sum_{k'=1}^{K} \beta_{k'} + \beta_{k}^{2}M^{2}\right)^{2}}, \text{ for Rayleigh fading channels,} \\
\frac{6M(M+1)\beta_{k}^{2} \sum_{k'\neq k}^{K} \beta_{k'}^{2}}{\left(\frac{1}{4}\bar{\beta}_{k}^{2} + M\beta_{k} \sum_{k'=1}^{K} \beta_{k'} + \beta_{k}^{2}M(2M+1)\right)^{2}}, \text{ for keyhole channels,} \end{cases}$$
(16)

where $\bar{\beta}_k \triangleq \sum_{k'\neq k}^K \beta_{k'}$. The detailed derivations of (16) are presented in the Appendix. We can see that $\varrho_k = O(1/M^2)$. Thus, when $M \gg 1$, $|\epsilon_k|$ is much smaller than $\left(\frac{\sum_{k'\neq k}^K \beta_{k'}}{2} + ||\mathbf{g}_k||^2\right)^2$. As a result, our proposed channel estimation scheme is expected to work well.

4. NUMERICAL RESULTS

In this section, we provide numerical results to evaluate our proposed channel estimation scheme for finite M. As performance metric we consider the normalized mean-square error (MSE) at the kth user:

$$\mathsf{MSE}_{k} \triangleq \mathsf{E}\left\{ \left| \frac{a_{k} - \|\mathbf{g}_{k}\|^{2}}{\mathsf{E}\left\{ \|\mathbf{g}_{k}\|^{2} \right\}} \right|^{2} \right\}.$$
(17)

For the simulation, we choose M = 100, K = 20, and $\beta_k = 1, \forall k = 1, \ldots, K$. We define SNR $\triangleq \rho$. Figures 1 and 2 show the normalized MSE versus SNR for Rayleigh fading and keyhole channels, respectively. The curves labeled "without channel estimation, use $\mathbb{E} \{ \|\mathbf{g}_k\|^2 \}$ " represent the case when the *k*th user uses the statistical properties of the channels, i.e., it uses $\mathbb{E} \{ \|\mathbf{g}_k\|^2 \}$ as estimate of $\|\mathbf{g}_k\|^2$. The curves "DL pilots [1]" represent the case when the beamforming training scheme of [1] with MMSE channel estimation is applied. The curves "proposed scheme (Algorithm 1)" represent our proposed scheme for different T ($T = \infty$ implies that the *k*th user perfectly knows $\mathbb{E} \{ |y_k|^2 \}$). For the beamforming training scheme, the duration of the downlink training is *K*. For our proposed blind channel estimation scheme, $s_k, k = 1, \ldots, K$, are random 4-QAM symbols.

We can see that in Rayleigh fading channels, the MSEs of the three schemes are comparable. Using $E \{ \|\mathbf{g}_k\|^2 \}$ in lieu of the true $\|\mathbf{g}_k\|^2$ for signal detection works rather well. However, in keyhole channels, since the channels do not harden, the MSE when using $E \{ \|\mathbf{g}_k\|^2 \}$ as estimate of $\|\mathbf{g}_k\|^2$ is very large. In both propagation environments, our proposed scheme works very well. For a wide range of SNRs, our scheme outperforms the beamforming training scheme, even for short coherence intervals (e.g., T = 100 symbols). Note again that, with the beamforming training scheme of [1], we additionally have to spend at least *K* symbols on training pilots (this is not accounted for here, since we only evaluated MSE). By contrast, our proposed scheme does not requires any resources for downlink training.

5. CONCLUDING REMARKS

Massive MIMO systems may encounter propagation conditions when the channels do not harden. Then, to facilitate detection of the data in the downlink, the users need to estimate their effective channel gain rather than relying on knowledge of the *average* effective channel gain. We proposed a channel estimation approach by which the users can blindly estimate the effective channel gain from the data received during a coherence interval. The approach is computationally easy, it does not requires any resource for downlink pilots, it can be applied regardless of the type of propagation channel, and it performs very well.

Future work may include studying rate expressions rather than channel estimation MSE, and taking into account the channel estimation errors in the uplink. (We hypothesize, that the latter will not qualitatively affect our results or conclusions.) Blind estimation of β_k by the users may also be addressed.

6. APPENDIX

Here, we provide the proof of (16). From (15), we have

$$\varrho_{k} = \mathsf{E}\left\{|\epsilon_{k}|^{2}\right\} / \mathsf{E}\left\{\left(\frac{1}{2}\sum_{k'\neq k}^{K}\beta_{k'} + \|\mathbf{g}_{k}\|^{2}\right)^{2}\right\}^{2}.$$
 (18)

• Rayleigh Fading Channels:

For Rayleigh fading channels, we have

$$\mathsf{E}\left\{\left(\frac{1}{2}\sum_{k'\neq k}^{K}\beta_{k'} + \|\mathbf{g}_{k}\|^{2}\right)^{2}\right\} = \frac{1}{4}\left(\sum_{k'\neq k}^{K}\beta_{k'}\right)^{2} + \left(\sum_{k'\neq k}^{K}\beta_{k'}\right)\mathsf{E}\left\{\|\mathbf{g}_{k}\|^{2}\right\} + \mathsf{E}\left\{\|\mathbf{g}_{k}\|^{4}\right\} = \frac{1}{4}\left(\sum_{k'\neq k}^{K}\beta_{k'}\right)^{2} + M\beta_{k}\sum_{k'=1}^{K}\beta_{k'} + \beta_{k}^{2}M^{2},$$
(19)

where the last equality follows [11, Lemma 2.9]. We next compute $E \{ |\epsilon_k|^2 \}$. From (14), we have

$$\mathsf{E}\left\{\left|\epsilon_{k}\right|^{2}\right\} = \mathsf{E}\left\{\left(\sum_{k'\neq k}^{K}\left|\mathbf{g}_{k}^{H}\mathbf{g}_{k'}\right|^{2}\right)^{2}\right\} + \left(\sum_{k'\neq k}^{K}\beta_{k'}\right)^{2}\mathsf{E}\left\{\left\|\mathbf{g}_{k}\right\|^{4}\right\} - 2\left(\sum_{k'\neq k}^{K}\beta_{k'}\right)\mathsf{E}\left\{\sum_{k'\neq k}^{K}\left|\mathbf{g}_{k}^{H}\mathbf{g}_{k'}\right|^{2}\left\|\mathbf{g}_{k}\right\|^{2}\right\}.$$
 (20)

We have,

$$\mathsf{E}\left\{\left(\sum_{k'\neq k}^{K} \left|\mathbf{g}_{k}^{H}\mathbf{g}_{k'}\right|^{2}\right)^{2}\right\} = \mathsf{E}\left\{\left\|\mathbf{g}_{k}\right\|^{4} \left(\sum_{k'\neq k}^{K} \left|z_{k'}\right|^{2}\right)^{2}\right\}, \quad (21)$$

where $z_{k'} \triangleq \frac{\mathbf{g}_{k}^{H} \mathbf{g}_{k'}}{\|\mathbf{g}_{k}\|}$. Conditioned on \mathbf{g}_{k} , $z_{k'}$ is complex Gaussian distributed with zero mean and variance $\beta_{k'}$ which is independent of \mathbf{g}_{k} . Thus, $z_{k'} \sim \mathcal{CN}(0, \beta_{k'})$ and is independent of \mathbf{g}_{k} . This yields

$$\mathsf{E}\left\{\left(\sum_{k'\neq k}^{K} \left|\mathbf{g}_{k}^{H}\mathbf{g}_{k'}\right|^{2}\right)^{2}\right\} = \mathsf{E}\left\{\left\|\mathbf{g}_{k}\right\|^{4}\right\} \mathsf{E}\left\{\left(\sum_{k'\neq k}^{K} \left|z_{k'}\right|^{2}\right)^{2}\right\}$$
$$= \beta_{k}^{2}M\left(M+1\right)\left(\sum_{i\neq k}^{K} \beta_{i}^{2} + \sum_{i\neq k}^{K} \sum_{j\neq k}^{K} \beta_{i}\beta_{j}\right).$$
(22)

Similarly,

$$\mathsf{E}\left\{\sum_{k'\neq k}^{K} \left|\mathbf{g}_{k}^{H}\mathbf{g}_{k'}\right|^{2} \|\mathbf{g}_{k}\|^{2}\right\} = \mathsf{E}\left\{\|\mathbf{g}_{k}\|^{4}\right\} \mathsf{E}\left\{\sum_{k'\neq k}^{K} |z_{k'}|^{2}\right\}$$
$$= \beta_{k}^{2}M\left(M+1\right)\sum_{k'\neq k}^{K} \beta_{k'}^{2}.$$
 (23)

Substituting (22), (23), and $\mathsf{E}\left\{\|\mathbf{g}_k\|^4\right\} = \beta_k^2 M(M+1)$ into (20), we obtain

$$\mathsf{E}\left\{|\epsilon_{k}|^{2}\right\} = M(M+1)\beta_{k}^{2}\sum_{k'\neq k}^{K}\beta_{k'}^{2}.$$
(24)

Inserting (19) and (24) into (18), we obtain (16) for the Rayleigh fading case.

Keyhole Channels:

By using the fact that

$$z_{k'} = \frac{\mathbf{g}_k^H \mathbf{g}_{k'}}{\|\mathbf{g}_k\|} = \sqrt{\beta_{k'}} \nu_{k'} \frac{\mathbf{g}_k^H \bar{\mathbf{h}}_{k'}}{\|\mathbf{g}_k\|},$$
(25)

is the product of two independent Gaussian RVs, and following a similar methodology used in the Rayleigh fading case, we obtain (16) for keyhole channels.

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