

# BASE STATION CLUSTERING IN HETEROGENEOUS NETWORK WITH FINITE BACKHAUL CAPACITY

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## ABSTRACT

This paper considers the Base Station (BS) clustering in a downlink heterogeneous network with finite backhaul capacity. We consider a tree structure network where each BS has only one incoming link and several outgoing links. The objective is to maximize the minimum rate among all users while satisfying the backhaul capacity constraint and the per-BS power constraint. We propose an algorithm that combines the bisection and the Alternating Direction Method of Multipliers (ADMM). The bisection search is conducted for the minimum rate. When it is given, we use ADMM to check the feasibility of the network while obeying the backhaul and power constraints. There are two steps involved in ADMM: i) a second order conic programming is used to calculate the beamformer; ii) a closed-form rule is used to determine the BS clustering. Due to the non-convexity and the non-smoothness, ADMM is not guaranteed to solve the problem. Therefore, we propose a revise step to further improve the performance. The simulation results show that the proposed algorithm outperforms the heuristic method and the revise step does improve the performance in all considered scenarios.

**Index Terms**— heterogeneous network, finite backhaul capacity, base station clustering

## 1. INTRODUCTION

With the increasing demanding of high-speed data rate, the Heterogeneous Network (HetNet), in which multiple low-power Base Stations (BS) are overlaid with a conventional macro BS, has been paid much attention as a promising paradigm for improving the system performance. To further improve the signal strength, the cell size is also shrinking. Therefore, the intercell interference gradually becomes the main performance-limiting factor. To deal with it, the coordinated transmission, which allows multiple BSs to design the beamformer jointly, is proposed. There are two common cooperation types. One is Coordinated Multi-Point (CoMP) transmission and the other one is Joint Processing (JP) [1]. For CoMP, each user is served by only one BS while for JP, there is no constraint on the number of serving BSs.

Intuitively, the performance of joint processing should be better. In an ideal network where the backhaul capacity is infinite, we can just assign all the BSs to form a cluster serving all the users. In practice, however, the backhaul links are rate-limited, which makes the one-cluster scheme impractical. Therefore, how many and which BSs are assigned to each user, which is called *BS clustering* in this paper, is a very important problem for a finite backhaul

network. The finite backhaul capacity is dealt by either a compression scheme [2] [3] or a data sharing scheme [4] [5]. In this work, we adopt the latter one. Since the total rate on a backhaul link is a discrete value based on the number of served users and their rates, the problem has mixed integer variables, which is very difficult to solve globally. Therefore suboptimal algorithms have been proposed. A simple way to control the backhaul overhead is to use fixed clustering, in which several neighboring BSs are formed in a cluster and each user selects one cluster to get service [6]. Although it may have performance improvement, this strategy has no flexibility for the varying channel. Another heuristic scheme for the BS clustering is to choose a fixed number of BSs with the largest channel gain for each user. When there are only a few users, this method provides good performance. But when the number of users increases, many users may select the same BS which causes its backhaul link to be crowded. As a result, some literatures focus on a more flexible scheme, where a cardinality-unconstrained set of BSs is assigned for each user. The number of serving BSs for a user can be viewed as a  $l_0$  norm of the beamformer [7–12]. In [9] [10], the authors formulate the BS clustering as a sparse beamforming problem and utilize the reweighted  $l_1$  technique [13]. In [11], the cluster is designed via the weighted sum rate maximization with a mixed  $l_2/l_1$  regularizer. Unlike the traditional convex  $l_1$  approximation,  $l_0$  norm is approximated as a nonconvex function in [12] in order to improve the approximation accuracy. However, these methods focus on the total backhaul capacity and ignores the fact that each link has its own capacity constraint.

Motivated by this, in this paper we consider the per-link backhaul capacity constraint and aim at maximizing the minimum rate among users, which is a commonly used objective that guarantees the user fairness. In addition, we assume the considered HetNet is in a tree structure and only macro BSs are connected to a central processor (CP). Unlike the literatures above, we put the individual backhaul capacity in constraints instead of objectives. Due to the nature of beamforming design and BS clustering, the formulated problem is nonsmooth and nonconvex. In this paper, we adopt the bisection method for the minimum rate  $R$  and when  $R$  is given, we use the Alternating Direction Method of Multipliers (ADMM) algorithm [14]. The two subproblems in ADMM are an Second Order Conic Programming (SOCP) [15] and a problem with closed-form solution. Since the convergence of the nonconvex ADMM is not guaranteed, we propose a revise step for the clustering. The simulation results show that our proposed algorithm outperforms the heuristic method and the revise step does improve the performance.

## 2. SYSTEM MODEL

In this paper, we consider a downlink HetNet with  $K$  single-antenna users and  $N$  BSs including both macro and pico ones, as shown in Fig. 1. Each BS has  $A_t$  antennas. All the BSs are connected via wired backhaul links. Here, we assume the network is in a tree

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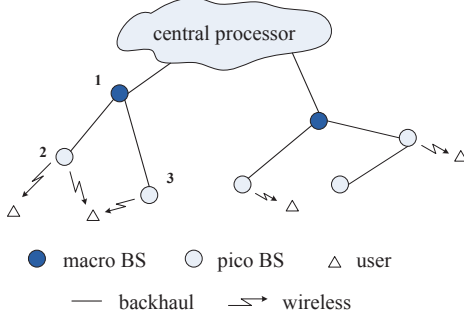


Fig. 1. System Model

structure. The macro BSs are directly connected to a CP while a pico BS connects to either a macro BS or another pico BS. In our model, we denote the only one incoming link of BS  $n$  as link  $n$  with a capacity limit  $C_n$ . In the HetNet, each user can be served by multiple BSs via joint processing. The user data is distributed to each serving BS from CP along the tree structure backhaul network. For example, if BS 1  $\sim$  3 jointly serve user  $k$ , the data is first send to BS 1 and then BS 1 distributes the data to both BS 2 and 3.

We denote the network-wide beamformer from all BSs to user  $k$  as  $\mathbf{v}_k = [\mathbf{v}_{1k}^H \dots \mathbf{v}_{Nk}^H]^H \in \mathbb{C}^{N A_t \times 1}$ , where  $\mathbf{v}_{nk}$  is associated with BS  $n$ . Here,  $\mathbf{v}_{nk} = \mathbf{0}$  indicates that BS  $n$  does not serve user  $k$ . Therefore, the received signal of user  $k$  can be written as

$$y_k = \mathbf{h}_k \mathbf{v}_k s_k + \sum_{j \neq k} \mathbf{h}_k \mathbf{v}_j s_j + z_k \quad (1)$$

where  $\mathbf{h}_k = [\mathbf{h}_{1k}, \dots, \mathbf{h}_{Nk}] \in \mathbb{C}^{1 \times N A_t}$  is the concatenated channel to user  $k$  and  $z_k \sim \mathcal{CN}(0, \sigma_k^2)$  is the complex additive white Gaussian noise. The data of each user is assumed to be independent and have the same power, i.e.  $\mathbb{E}[|s_k|^2] = 1, \mathbb{E}[s_k^* s_j] = 0, \forall j \neq k$ . The signal-to-interference-plus-noise-ratio (SINR) of user  $k$  is

$$\gamma_k = \frac{|\mathbf{h}_k \mathbf{v}_k|^2}{\sum_{j \neq k} |\mathbf{h}_k \mathbf{v}_j|^2 + \sigma_k^2} \quad (2)$$

### 3. PROBLEM FORMULATION

In this paper, we solve the BS clustering with finite backhaul capacity by maximizing the minimum rate among users.

If BS  $n$  is scheduled to serve user  $k$ , the data of user  $k$  need to be propagated to BS  $n$  from CP using the backhaul links. In other words, the backhaul link  $n$  contains the data of user  $k$  if BS  $n$  or its descendants are assigned to serve user  $k$ . We denote  $\mathcal{D}_n$  as the set containing BS  $n$  and its descendants, e.g.,  $\mathcal{D}_1 = \{1, 2, 3\}$ . A matrix  $\mathbf{T}$  is defined with  $(n, m)$ -th entry

$$t_{nm} = \begin{cases} 1, & m \in \mathcal{D}_n \\ 0, & m \notin \mathcal{D}_n \end{cases} \quad (3)$$

If  $\sum_m t_{nm} \|\mathbf{v}_{mk}\| > 0$ , it means the data of user  $k$  is on link  $n$ . The total flow rate on link  $n$  can be written as

$$\sum_k \mathbf{1} \left( \sum_m t_{nm} \|\mathbf{v}_{mk}\| > 0 \right) r_k$$

where  $r_k$  is the rate of user  $k$  and  $\mathbf{1}(\cdot)$  is an indicator which equals to one if the clause inside the parenthesis holds true, otherwise it is

zero. The BS clustering with finite backhaul capacity can be determined via the following problem:

$$(\mathbf{P}) : \max_{\mathbf{V}, \mathbf{P}, \mathbf{r}} \min_k r_k \quad (4)$$

$$\text{s.t.} \quad r_k \leq \log_2(1 + \gamma_k), \forall k$$

$$\sum_k \|\mathbf{v}_{nk}\|^2 \leq P_n, \forall n \quad (5)$$

$$\|\mathbf{v}_{nk}\| \leq p_{nk} \sqrt{P_n}, p_{nk} \leq 1, \forall n, k \quad (6)$$

$$\sum_k \mathbf{1}(\mathbf{t}_n^T \mathbf{p}_k > 0) r_k \leq C_n, \forall n \quad (7)$$

where  $\mathbf{V} = [\mathbf{v}_1 \dots \mathbf{v}_K]$ ,  $\mathbf{r} = [r_1 \dots r_K]^T$  and  $\gamma_k$  is presented in (2). The objective is the minimum rate among users and (4) indicates that the achievable rate should be smaller than the capacity of the wireless link.  $p_{n,k}$  is a newly introduced slack variable that denotes the portion of the total transmit power that BS  $n$  used to serve user  $k$ . For notational convenience, we denote  $\mathbf{p}_k = [p_{1k} \dots p_{Nk}]^T$  and  $\mathbf{P} = [\mathbf{p}_1 \dots \mathbf{p}_K]$ . The finite backhaul capacity constraint is modeled in (7) where  $\mathbf{t}_n^T$  is the  $n$ th row of  $\mathbf{T}$ . When  $\mathbf{t}_n^T \mathbf{p}_k = \sum_m t_{nm} p_{mk} > 0$ , at least one BS in  $\mathcal{D}_n$  serves user  $k$ , which also means link  $n$  transmits  $r_k$  data of user  $k$ . We can see that if there is no constraints in (6) and (7), problem (P) turns out to be the max-min rate optimization problem which is well studied in [16–19]. Compared to the existing works [9–12] which formulate the finite backhaul capacity by introducing a regularization in the objective, we, however, directly formulate it in the constraints (6) and (7). Unfortunately, due to the non-convexity of rate constraint (4) and the non-smoothness of capacity constraint (7), solving (P) to global optimality is very difficult. Therefore, in this paper we focus on designing algorithms with high-quality suboptimal solution.

### 4. PROPOSED ALGORITHMS

In this section, we describe the proposed algorithm that uses bisection and ADMM as well as a revise step for the clustering result.

#### 4.1. Algorithm using Bisection and ADMM

By introducing a variable  $R$  which represents the minimum rate among users, (P) is equivalent to

$$(\mathbf{P1}) : \max_{\mathbf{V}, \mathbf{P}, \mathbf{r}, R} R \quad (8)$$

$$\text{s.t.} \quad R \leq r_k, \forall k$$

$$\text{constraints in (4)-(7)}$$

A common way is to conduct bisection search for  $R$ . Given  $R$ , we check whether there exists a  $\{\mathbf{V}, \mathbf{P}\}$  such that all users can achieve a rate that is at least  $R$ , i.e.,

$$(\mathbf{F}) : \min_{\mathbf{V}, \mathbf{P}} 0 \quad (9)$$

$$\text{s.t.} \quad \gamma_k \geq 2^R - 1, \forall k$$

$$\sum_k \mathbf{1}(\mathbf{t}_n^T \mathbf{p}_k > 0) R \leq C_n, \forall n \quad (10)$$

$$\text{constraints in (5) and (6)}$$

For the backhaul capacity constraint (9), we introduce a non-negative variable  $z_{nk}$  to replace  $\mathbf{t}_n^T \mathbf{p}_k$ . Then ADMM is adopted to deal with the equality constraint  $z_{nk} = \mathbf{t}_n^T \mathbf{p}_k$ . In iteration  $i$ , ADMM includes the following three steps:

$$\mathbf{Z}^{(i)} = \arg \min_{\mathbf{Z}} L(\mathbf{V}^{(i-1)}, \mathbf{P}^{(i-1)}, \mathbf{Z}, \boldsymbol{\lambda}^{(i-1)}), \text{s.t. (9), } \mathbf{Z} \geq \mathbf{0} \quad (11)$$

$$(\mathbf{V}^{(i)}, \mathbf{P}^{(i)}) = \arg \min_{\mathbf{V}, \mathbf{P}} L(\mathbf{V}, \mathbf{P}, \mathbf{Z}^{(i)}, \boldsymbol{\lambda}^{(i-1)}), \text{s.t. (5), (6), (8)} \quad (12)$$

$$\boldsymbol{\lambda}_{nk}^{(i)} = \boldsymbol{\lambda}_{nk}^{(i-1)} + \rho(z_{nk}^{(i)} - \mathbf{t}_n^T \mathbf{p}_k^{(i)}), \forall n, k$$

with  $L(\mathbf{V}, \mathbf{P}, \mathbf{Z}, \boldsymbol{\lambda}) = \sum_{n,k} (z_{nk} - \mathbf{t}_n^T \mathbf{p}_k + \lambda_{nk}/\rho)^2$ .  $\{\lambda_{nk}\}_{\forall n,k}$  denote the Lagrangian multipliers and  $\rho > 0$  is the penalty parameter.  $\mathbf{z}_k = [z_{1k} \dots z_{Nk}]^T$ ,  $\mathbf{Z} = [\mathbf{z}_1 \dots \mathbf{z}_K]$  and  $\boldsymbol{\lambda} = [\lambda_{11} \dots \lambda_{NK}]$ .

When  $(\mathbf{V}, \mathbf{P})$  is fixed, the  $\mathbf{Z}$  updated (10) can be written as

$$\begin{aligned} \min_{\mathbf{Z}} \quad & \sum_{n,k} (z_{nk} - \mathbf{t}_n^T \mathbf{p}_k + \lambda_{nk}/\rho)^2 \\ \text{s.t.} \quad & \sum_k \mathbf{1}(z_{nk} > 0) R \leq C_n, \forall n \\ & z_{nk} \geq 0, \forall n, k \end{aligned} \quad (12)$$

It can be seen that (12) is separable in terms of  $n$ . For each  $n$ , we define  $a_k = \mathbf{t}_n^T \mathbf{p}_k - \frac{\lambda_{nk}}{\rho}$  and sort them in descending order as  $a_{i_1} \geq a_{i_2} \geq \dots \geq a_{i_K}$  where  $i_j$  denotes the original user label of  $a_{i_j}$ . Due to the fact that  $z_{nk} \geq 0$  and the number of  $z_{nk} > 0$  is bounded by  $\lfloor C_n/R \rfloor$ , the optimal solution for  $n$  can be obtained in closed form:

$$z_{ni_j} = \begin{cases} \max(a_{i_j}, 0), & j \leq \lfloor \frac{C_n}{R} \rfloor \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

When  $\mathbf{Z}$  is fixed, the  $(\mathbf{V}, \mathbf{P})$  update (11) can be easily reformulated as an SOCP and solved by some efficient solvers such as SeDuMi [20] and Mosek [21].

Even though the ADMM approach cannot globally solve (F) due to the non-convexity and non-smoothness, the clustering result,  $z_{nk}$ , computed by ADMM should still be a good estimate of the optimal clustering. Therefore, in the following we fix the clustering and check the feasibility of the network. We define a variable  $x_{nk} = \mathbf{1}(z_{nk} > 0)$  which indicates whether BS  $n$  can obtain the data of user  $k$  from link  $n$ . If  $x_{nk} = 1$ , BS  $n$  can serve user  $k$ , otherwise,  $\mathbf{v}_{nk}$  should be  $\mathbf{0}$ . When  $\{x_{nk}\}_{\forall n,k}$  are fixed, the feasibility of  $R$  is checked via

$$\begin{aligned} \min_{\mathbf{V}} \quad & 0 \\ \text{s.t.} \quad & \gamma_k \geq 2^R - 1, \forall k \\ & \sum_k \|\mathbf{v}_{nk}\|^2 \leq P_n, \forall n \\ & \mathbf{v}_{nk} = x_{nk} \mathbf{v}_{nk}, \forall n, k \end{aligned} \quad (14)$$

where the last constraint guarantees  $\mathbf{v}_{nk} = \mathbf{0}$  when  $x_{nk} = 0$  and becomes redundant when  $x_{nk} = 1$ . Again, (14) can be efficiently solved since it is an SOCP. If (14) is feasible, (F) is also feasible. All the user can get rate  $R$  with the serving cluster of user  $k$  being  $\{n \mid x_{nk} = 1\}$ . When (14) is infeasible, we just view (F) as infeasible, although it may be feasible in theory. As a result, we can obtain a lower bound of the optimal  $R$  of (P1) via bisection search since we may treat feasible  $R$  as infeasible but every feasible  $R$  we find is definitely feasible for (F).

The whole algorithm is summarized in Algorithm 1. The bisection method is terminated when the gap between  $R_{\max}$  and  $R_{\min}$  is smaller than  $\eta$ . We set a maximum number of iterations  $i_{\max}$  as well as an accuracy  $\epsilon$  for ADMM, so it stops when the relative error  $\|\mathbf{Z}^{(i)} - \mathbf{TP}^{(i)}\|/\|\mathbf{Z}^{(i)}\| \leq \epsilon$  or  $i \geq i_{\max}$ .

#### 4.2. Revising the Base Station Clustering

In Algorithm 1, we determine the BS clustering just by judging whether  $\mathbf{Z}$  is above zero. It is possible that  $\mathbf{Z}$  has a all-zero column, i.e.  $z_{nk} = 0, \forall n$ . If it happens, (14) is definitely infeasible because no BSs will serve user  $k$ . Therefore, we provide an optional revise step for dealing with this situation.

If there exists all-zero columns, the idea is to set some links with high contributions to be active for the users associated with all-zero

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#### Algorithm 1 Algorithm based on Bisection and ADMM for (P1)

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**Require:**  $R_{\min}, R_{\max}, \eta, \epsilon, \rho$  and  $i_{\max}$

- 1: **while**  $R_{\max} - R_{\min} > \eta$  **do**
- 2:   Set  $R = (R_{\min} + R_{\max})/2$ , flag = 1;
- 3:   Set  $i = 0$ , initialize  $(\mathbf{V}^{(0)}, \mathbf{P}^{(0)})$  and  $\lambda_{nk}^{(0)}, \forall n, k$ ;
- 4:   **while** flag  $\wedge (i < i_{\max})$  **do**
- 5:     Set  $i = i + 1$ , update  $\mathbf{Z}^{(i)}$  in closed form (13);
- 6:     Calculate  $\mathbf{V}^{(i)}$  and  $\mathbf{P}^{(i)}$  by solving an SOCP (11);
- 7:     Update  $\lambda_{nk}^{(i)} = \lambda_{nk}^{(i-1)} + \rho(z_{nk}^{(i)} - \mathbf{t}_n^T \mathbf{p}_k^{(i)}), \forall n, k$ ;
- 8:     Set flag =  $\mathbf{1}(\|\mathbf{Z}^{(i)} - \mathbf{TP}^{(i)}\|/\|\mathbf{Z}^{(i)}\| > \epsilon)$ ;
- 9:   **end while**
- 10:   Set  $\mathbf{X} \in \{0, 1\}^{N \times K}$  with  $x_{nk} = \mathbf{1}(z_{nk}^{(i)} > 0), \forall n, k$ ;
- 11:   (optional) revise the clustering (Algorithm 2, sec. 4.2);
- 12:   Solve (14) with fixed  $\mathbf{X}$ ;
- 13:   **if** (14) is feasible **then**
- 14:     Set  $R_{\min} = R$ ;
- 15:   **else**
- 16:     Set  $R_{\max} = R$ ;
- 17:   **end if**
- 18: **end while**

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#### Algorithm 2 Revising the clustering result (Step 11 of Alg. 1)

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**Require:**  $\mathbf{V}, \mathbf{X}, \mathbf{T}$  and  $R$

- 1: Set  $\mathcal{K} = \{k \mid \mathbf{x}_k = \mathbf{0}\}$ ;
- 2: **if**  $\mathcal{K}$  is not empty **then**
- 3:   Define  $\mathbf{Q} \in \mathbb{R}^{N \times K}$  with  $q_{nk} = \frac{\|\mathbf{v}_{nk}\|^2}{\sum_n \|\mathbf{v}_{nk}\|^2}$ ;
- 4:   Define  $\mathbf{G} \in \mathbb{R}^{N \times |\mathcal{K}|}$  with  $g_{ni} = (1 - x_{n\mathcal{K}(i)})q_{n\mathcal{K}(i)}$ ;
- 5:   Define  $\mathbf{F} \in \mathbb{R}^{N \times K}$  with  $f_{nk} = x_{nk}q_{nk}$ ;
- 6:   **while**  $\max_{n,i} g_{ni} > 0$  **do**
- 7:     Set  $(\bar{n}, \bar{i}) = \arg \max_{n,i} g_{ni}$ ;
- 8:     **if**  $\sum_k x_{\bar{n}k} < \lfloor C_{\bar{n}}/R \rfloor$  **then**
- 9:       Set  $x_{\bar{n}\mathcal{K}(\bar{i})} = 1$ ;
- 10:    **else**
- 11:     Calculate  $\bar{k} = \arg \min_{x_{\bar{n}k} > 0} \mathbf{t}_{\bar{n}}^T \mathbf{q}_k$ ;
- 12:     **if**  $\mathbf{t}_{\bar{n}}^T \mathbf{q}_{\bar{k}} \leq g_{\bar{n}\bar{i}}$  **then**
- 13:       Set  $x_{\bar{n}\mathcal{K}(\bar{i})} = 1, x_{m\bar{k}} = 0, \forall m \in \mathcal{D}_{\bar{n}}$ ;
- 14:     **else**
- 15:       Set  $q_{\bar{n}j} = 0, \forall j \in \mathcal{K}$ ;
- 16:     **end if**
- 17:    **end if**
- 18:    Update  $g_{ni}, \forall n, i$  and  $f_{nk}, \forall n, k$ ;
- 19:   **end while**
- 20: **end if**

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columns and deactivate some comparatively low contributing links corresponded to other users. We first define a set  $\mathcal{K}$  containing the indices of the all-zero columns. If  $\mathcal{K}$  is not empty, we need to revise the clustering  $\mathbf{X}$ . We define  $q_{nk} = \frac{\|\mathbf{v}_{nk}\|^2}{\sum_n \|\mathbf{v}_{nk}\|^2}, \forall n, k$  as the measure of the contribution of BS  $n$  to user  $k$ . For each user  $k$ ,  $\sum_n q_{nk} = 1$ . We also define  $g_{ni} = (1 - x_{n\mathcal{K}(i)})q_{n\mathcal{K}(i)}, i = 1, \dots, |\mathcal{K}|$  and  $f_{nk} = x_{nk}q_{nk}$  to represent the contribution factors associated with the users belong to  $\mathcal{K}$  and the active links. Then we find the largest  $g_{\bar{n}\bar{i}}$  and compare it with the minimum non-zero  $\sum_m t_{\bar{n}m}q_{m\bar{k}}$ . If  $g_{\bar{n}\bar{i}}$  is larger, we activate this link  $(\bar{n}, \bar{i})$  and set the links  $(m, \bar{k})$  inactive, where  $m \in \mathcal{D}_{\bar{n}}$ . Otherwise, the inactive links with respect to  $\bar{n}$  will not be activated so we set  $q_{\bar{n}j} = 0, \forall j \in \mathcal{K}$ . After that, both  $g_{ni}, \forall n, i$  and  $f_{nk}, \forall n, k$  are updated. These steps are repeated until all  $g_{ni} = 0$ . The details is described in Algorithm 2.

## 5. SIMULATION

In this section, we evaluate the performance of the proposed algorithms and that of a heuristic method which determines the BS clustering based on the channel gain. The network structure is shown in Fig. 2, in which there are  $N = 20$  BSs (4 macro BSs and 16 pico BSs) with fixed locations and the users are randomly located within the 2km square area. We assume each BS has  $A_t = 4$  antennas. The variance of the noise is -174dBm/Hz and the bandwidth is 10kHz. The backhaul capacity is set as 100bps/Hz for the incoming link of macro BSs. For the pico BS connected to a macro BS, the backhaul capacity is 50bps/Hz and it is 30bps/Hz for the other pico BSs. The channel between BS  $n$  and user  $k$  is modeled as  $\mathbf{h}_{nk} = \sqrt{\beta_{nk}} \tilde{\mathbf{h}}_{nk}$  where the entries of  $\tilde{\mathbf{h}}_{nk}$  are drawn from i.i.d random variables with zero mean and unit variance.  $\beta_{nk}$  denotes the path loss.  $\beta_{nk} = 128.1 + 37.6 \log_{10}(d_{nk})$  (dB) for macro BS and  $\beta_{nk} = 38 + 30 \log_{10}(d_{nk} \times 10^3)$  (dB) for pico BS, where  $d_{nk}$  is the distance between BS  $n$  and user  $k$ . In our simulation, we set  $\eta = 0.5$ ,  $\epsilon = 0.01$ ,  $\rho = 1$ ,  $i_{\max} = 5$  and Mosek is used to solve SOCPs.

The max-min rate performance is illustrated in Fig. 3. Here, the heuristic approach is set to choose three<sup>1</sup> BSs with the largest channel gain for each user. The maximum transmission power for a macro BS is 30dBm while it is 24dBm for a pico BS. The simulation result shows that the proposed algorithm significantly outperforms the heuristic method, especially when the number of users is large. When  $K$  is small, the backhaul link is not crowded. Hence the heuristic that assigns base stations by simply looking at the channel gain suffices to give reasonable performance. When more and more users compete for the limited backhaul, the heuristic method may cause a BS serving too many users and result in a small minimum rate. This is because the heuristic method does not consider the overall objective when determine the BS clustering. However, our approach considers this by combining bisection and ADMM. Therefore, the proposed algorithm performs better when the network is crowded. Moreover, when  $K$  is larger, using the optional revise step does improve the performance. The reason is that the larger the  $k$ , the higher the probability that all-zero columns happen in  $\mathbf{Z}$ .

In Fig. 4, we evaluate the performance of the algorithms with various signal-to-noise ratio (SNR) when  $K = 20$ . The transmission power of a pico BS and a macro BS is set to have fixed ratio, i.e.  $P_{\text{macro}}(\text{dBm}) = 1.25 \times P_{\text{pico}}(\text{dBm})$ . It demonstrates that both the revised and the original Algorithm 1 perform better than the heuristic method at all the SNRs. When the SNR is low, the achievable  $R$  is also small. It means the backhaul is not crowded and therefore the revise step is not that helpful. However, when SNR is high,  $R$  becomes large. The revise step happens more frequently and thus the performance improvement is more pronounced.

## 6. CONCLUSION

In this paper, we combined bisection and ADMM for the BS clustering in HetNets with rate-limited backhaul links. We aimed at maximizing the minimum rate under the power and the individual backhaul capacity constraints. Inspired by practical scenarios, we consider a tree structure network. The rate on a link was formulated as the summation of 0-1 functions associated with the beamformer. The problem was nonsmooth and nonconvex. We adopted bisection search for the minimum rate. For a given minimum rate, ADMM, which boils down to using an SOCP for the beamformer design and a closed-form update for the BS clustering, was utilized. Since ADMM cannot guarantee satisfaction of the constraints, a revise step was

<sup>1</sup>In the simulation, it shows best to pick 3 BSs under our network setup.

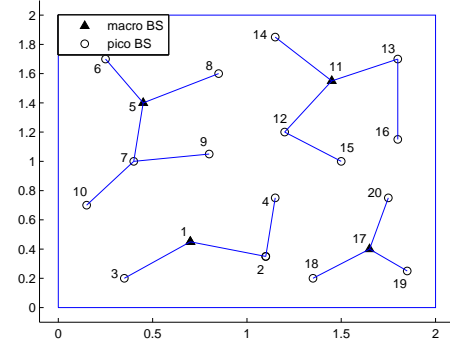


Fig. 2. simulation scenario

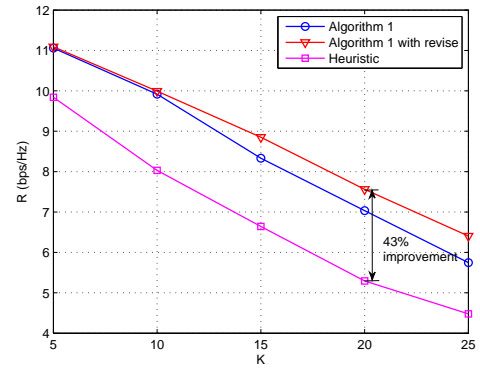


Fig. 3. max-min rate performance verse the number of users

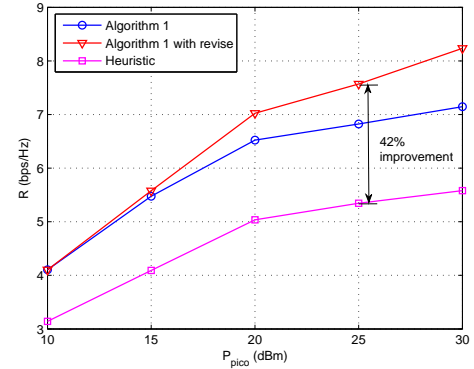


Fig. 4. max-min rate performance verse the SNR

presented to deal with the case when the clustering result showed some users were not served by any BSs. The simulation results validated the superior performance of the proposed algorithm as well as the revise step.

## 7. REFERENCES

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