OPTIMAL BASE STATION DENSITIES FOR COST-EFFICIENT MULTI-TIER HETEROGENEOUS CELLULAR NETWORKS

Ran Cai⋆*, Wei Zhang*§*, P. C. Ching*⋆

⋆ Department of Electronic Engineering, The Chinese University of Hong Kong, Hong Kong
§ School of Electrical Engineering & Telecommunications, The University of New South Wales, Australia

ABSTRACT

Heterogeneous cellular networks have emerged to be the primary solution for overwhelming traffic demands. This study addresses the architecture optimization of multi-tier open access downlink heterogeneous cellular networks to achieve the maximum network spatial throughput under practical constraints on the network deployment cost and traffic loads of individual tiers. In particular, with the use of a stochastic geometry-based network model, the problem of optimizing the densities of base stations (BSs) in different tiers is shown to be a linear-fractional programming that can be further transformed into a linear programming by employing Charnes–Cooper transformation. Such programming can, in turn, be solved efficiently. Furthermore, for the special case of a two-tier network, the optimal BS densities are derived in closed form using the graphical method. Simulation results demonstrate significant throughput gain from the optimal deployment of multi-tier heterogeneous cellular networks.

***Index Terms*—** Heterogeneous cellular networks, network architecture optimization, network economics, stochastic geometry.

1. INTRODUCTION

Heterogeneous cellular networks have been introduced to address the exponential growth of mobile data traffic [1]. A typical heterogeneous cellular network consists of traditional base stations (BSs) for long-range coverage, operator-managed picocells for blind angle elimination [2], and femtocells for short-range coverage [3, 4]. Deploying dense BSs enhances network throughput and mobile quality-of-service [5] but increases network deployment-and-operation cost because of the consumption of BS hardware, backhual cables, and energy [6]. This study optimizes the BS densities of multi-tier heterogeneous cellular networks by balancing extension of throughput and reduction of expenses.

Stochastic geometry has recently been considered as a tractable tool for modeling wireless networks [5, 7–15]. Stochastic geometry-based network models have been applied to analyze and design resource allocation algorithms for heterogeneous cellular networks. In particular, joint and disjoint sub-channel allocation schemes have been studied in [11], whereas partial spectrum reuse factor has been analyzed in [13]. In this paper, we design BS density for multi-tier open access downlink heterogeneous cellular networks. The multi-tier heterogeneous cellular networks are modeled as independent Poisson point processes (PPPs), as proposed in [8].

From the perspective of network designers and operators, it is important to constrain the deployment-and-operation cost in network designs and analyzing performance to avoid yielding impractical and misleading insight [16, 17]. For this reason, cost-effective strategies have been studied for single-tier cellular networks in [6, 18] and two-tier heterogeneous cellular networks on the basis of lattice network models [19, 20]. However, few results are known on the optimization of multi-tier heterogenous cellular network architectures under constraints on the deployment cost. Building on the coverage-and throughput results from [8], in this paper we study a problem of optimizing BS densities of multi-tier heterogeneous cellular networks for maximizing the network throughput under a set of practical constraints on the deployment-and-operation cost, traffic loads for different tiers, and received signal power. The optimization problem is equivalent to a linear-fractional programming (LFP) and it can be efficiently solved using a linear programming (LP) by employing Charnes–Cooper transformation. Furthermore, for a two-tier network, the optimal BS densities are derived in closed form by analyzing the polygon geometry of the feasibility region for the aforementioned optimization problem. This approach provides useful guidelines for the deployment of heterogenous cellular networks.

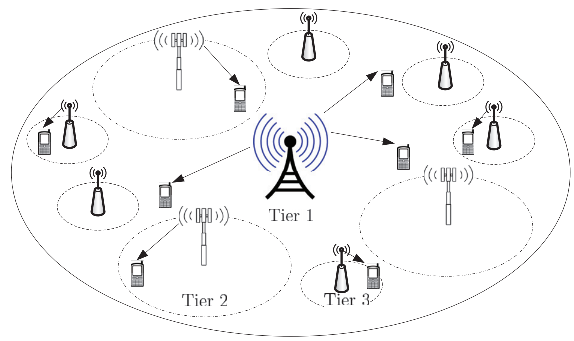

Fig. 1: Multi-tier heterogeneous cellular networks.

2. NETWORK MODEL AND METRICS

As illustrated in Fig. 1, a K-tier ($K \geq 2$) open access downlink heterogeneous cellular network comprises different types of BSs, such as macrocell, picocell, and femtocell BSs. BSs of the same type are collectively called a tier and characterized by corresponding transmission power, density, and target data rate, which are denoted for the k-th tier ($k = 1, \ldots, K$) as P_k, λ_k, and $\log(1+\beta_k)$, respectively, with $\beta_k > 1$ being the target signal-to-interference ratio (SIR). Different tiers are modeled as independent and homogeneous PPPs, denoted as $\{\Psi_k\}_{k=1}^{K}$, in the horizontal plane as in [7, 8]. Mobile users are modeled by another independent PPP, denoted as Φ, with density λ_u. Each BS is assumed to deploy a single antenna. Propagation is characterized by both path loss and small-scale fading. In particular, the received power at a receiver $Z \in \mathbb{R}^2$ attributed to transmission from a BS in the k-th tier at $Y \in \mathbb{R}^2$ is given by $P_k h_{YZ}|Y-Z|^{-\alpha}$, where $\alpha > 2$ is the path-loss exponent, $|\cdot|$ is the transmission distance, and h_{YZ} is an $\exp(1)$ random variable modeling Rayleigh fading. All channel coefficients $\{h_{YZ}\}$ are assumed to be i.i.d.

Given the stationarity of the network model, network performance can be characterized by considering the throughput of a typical mobile user located at the origin represented by O. For simplicity, the network is assumed to be interference limited and furthermore enables *open access* (i.e., a typical mobile user is allowed to connect to a BS in any tier. In particular, each mobile user is served by the BS providing the highest received SIR [8].) Given that the typical mobile user is connected to a BS X in the k-th tier, the received SIR can be written as

$$\text{SIR}(X) = \frac{P_k h_{XO}|X|^{-\alpha}}{\sum_{k=1}^{K} \sum_{Y \in \Psi_k \setminus \{X\}} P_k h_{YO}|Y|^{-\alpha}}, \qquad X \in \Psi_k. \tag{1}$$

To justify the assumption of dominant interference over noise and to avoid reaching any misleading conclusion, the received power should be ensured to be sufficiently large by applying the following constraint [21]:

Constraint 1 (Signal power). The received signal power at a typical mobile user, denoted as S_0, has to exceed a positive threshold δ, except for a small constant probability ε; i.e., $\mathbb{P}[S_0 < \delta] \leq \varepsilon$.

This work builds on several key results from [8]. The data transmitted by a BS in the k-th tier is successfully received at the intended mobile user if and only if the received SIR is no smaller than the target value β_k. A mobile user is *within coverage* if at least one BS of any tier can provide the mobile user with reliable transmission. The *coverage probability* of the typical mobile user, denoted as P_{cv}, is given as

$$P_{cv} = \mathbb{P}\left[\bigcup_{X \in \Psi_k, k \in \{1,2,\ldots,K\}} (\text{SIR}(X) > \beta_k)\right]. \tag{2}$$

From [8], we obtain

$$P_{cv} = \frac{\alpha \sin(2\pi/\alpha) \sum_{k=1}^{K} \beta_k^{-2/\alpha} P_k^{2/\alpha} \lambda_k}{2\pi \sum_{k=1}^{K} P_k^{2/\alpha} \lambda_k}. \tag{3}$$

Definition 1. The *fractional load* of the k-th tier, denoted by η_k, is defined as the fraction of time during which a typical mobile user is connected to a BS in the k-th tier:

$$\eta_k = \frac{\mathbb{P}\left[\bigcup_{X \in \Psi_k} (\text{SIR}(X) > \beta_k)\right]}{P_{cv}}. \tag{4}$$

From [8], we obtain

$$\eta_k = \frac{\beta_k^{-2/\alpha} P_k^{2/\alpha} \lambda_k}{\sum_{k=1}^{K} \beta_k^{-2/\alpha} P_k^{2/\alpha} \lambda_k}. \tag{5}$$

With the definition of fractional load, we have the following constraint that guarantees a given minimum level of utilization and profit for each tier.

Constraint 2 (Tier traffic load). The fractional loads of different tiers are constrained as $\eta_k \geq \theta_k, k = 1, 2, \ldots, K$, where $\{\theta_k\}$ are given constants satisfying $0 \leq \theta_k \leq 1$ and $\sum_k \theta_k \leq 1$.

Furthermore, the average rate R achieved by a randomly located mobile user when it is within coverage can be expressed as [8]

$$R = \mathbb{E}\left[\log\left(1 + \max_{X \in \Psi_k, k \in \{1,2,\ldots,K\}} \text{SIR}(X)\right) \Bigg| \bigcup_{X \in \Psi_k, k \in \{1,2,\ldots,K\}} (\text{SIR}(X) > \beta_k)\right], \tag{6}$$

$$= \log(1 + \beta_{\min}) + \frac{\sum_{k=1}^{K} \int_{\beta_{\min}}^{\infty} \frac{\max(\beta_k, t)^{-2/\alpha}}{1+t} \mathrm{d}t P_k^{2/\alpha} \lambda_k}{\sum_{k=1}^{K} \beta_k^{-2/\alpha} P_k^{2/\alpha} \lambda_k}, \tag{7}$$

where $\beta_{\min} = \min_k \{\beta_k\}$. According to the above definitions, the performance metric (i.e., spatial throughput) can be readily defined as follows.

Definition 2. The *spatial throughput* of the K-tier heterogeneous cellular network is

$$T = \lambda_u R P_{cv}, \tag{8}$$

where P_{cv} and R are defined in (3) and (7), respectively.

Last, in this study we consider the network deployment-and-operational cost which results from factors including BS hardware, backhaul cables connecting BSs to switching centers, and power consumption. From [6], the cost can be expressed as a linear function of base station density. Specifically, we have the following definition and constraint.

Definition 3. The *deployment-and-operation cost* per unit area for the k-th tier, denoted as $\mathcal{C}_k$, is defined as $\mathcal{C}_k = c_k \lambda_k$, where c_k is the total price for laying a cable of unit length, onsite BS hardware, and power consumption for a k-tier BS.

Constraint 3 (Deployment cost). The deployment-and-operation cost for the tiers are constrained as $\mathcal{C}_k \leq \phi_k, k = 1, 2, \ldots, K$, where $\{\phi_k\}$ are given positive constraints.

3. NETWORK DENSITY OPTIMIZATION

In this section we design the densities of K-tier heterogenous cellular networks considering the tradeoff between performance and cost. The optimization problem is first formulated as a *linear-fractional programming* (LFP) and then solved by two methods.

3.1. Problem Formulation

The BS density optimization problem for throughput maximization under Constraints 1–3 is formulated as follows:

$$(\mathbf{P0}) \quad \begin{aligned} \max_{\{\lambda_k\}} \quad & T(\lambda_1, \lambda_2, \cdots, \lambda_K) \\ \text{s.t.} \quad & \mathbb{P}[S_0 < \delta] \leq \varepsilon, \\ & \eta_k \geq \theta_k, \quad \forall k, \\ & \mathcal{C}_k \leq \phi_k, \quad \forall k, \\ & \lambda_k \geq 0, \quad \forall k. \end{aligned}$$

Next, we show that $(\mathbf{P0})$ can be expressed as an LFP. According to (3), (7), (8), the objective function in $(\mathbf{P0})$ can be rewritten as the ratio between two linear functions of $\mathbf{x} = [\lambda_1, \lambda_2, \ldots, \lambda_K]^T$,

$$T(\mathbf{x}) = \frac{\mathbf{a}^T\mathbf{x}}{\mathbf{b}^T\mathbf{x}}, \tag{9}$$

where $\mathbf{a} = [a_1, a_2, \ldots, a_K]^T$, and $\mathbf{b} = [b_1, b_2, \ldots, b_K]^T$ with

$$\begin{aligned} a_k =& \lambda_u \alpha \sin(2\pi/\alpha)\bigg[\log(1+\beta_{\min})\beta_k^{-2/\alpha} + \\ & \int_{\beta_{\min}}^{\infty} \frac{\max(\beta_k, t)^{-2/\alpha}}{1+t}\mathrm{d}t\bigg] P_k^{2/\alpha}, \quad (10) \\ b_k =& 2\pi P_k^{2/\alpha}. \quad (11) \end{aligned}$$

for $k = 1, 2, \cdots, K$. By the application of Marking Theorem [22], the first constraint in $(\mathbf{P0})$ is transformed into one on the tier densities as follows:

$$\sum_{k=1}^{K} P_k^{2/\alpha}\lambda_k \geq \zeta, \tag{12}$$

where the constraint ζ is

$$\zeta = \frac{\alpha\delta^{2/\alpha}\log(1/\varepsilon)}{2\pi\Gamma(2/\alpha)}, \tag{13}$$

and Γ denotes the Gamma function. Furthermore, all the linear constraints on BS densities are combined by introducing the following $(3K+1)\times K$ matrix:

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \mathbf{C}_3 \\ -I \end{bmatrix}, \tag{14}$$

where $\mathbf{C}_1$ is a row vector $\left[-P_1^{\frac{2}{\alpha}} \quad -P_2^{\frac{2}{\alpha}} \quad \ldots \quad -P_K^{\frac{2}{\alpha}}\right]$, $\mathbf{C}_2$ denotes a $K\times K$ sub-matrix in which the element of the m-th row and n-th column is

$$[\mathbf{C}_2]_{m,n} = \begin{cases} (1-1/\theta_n)(P_n/\beta_n)^{2/\alpha}, & m = n, \\ (P_n/\beta_n)^{2/\alpha}, & m \neq n, \end{cases} \tag{15}$$

$\mathbf{I}$ is a $K\times K$ identity matrix, and $\mathbf{C}_3$ is a $K\times K$ diagonal matrix with the diagonal elements taken from the vector $[c_1, c_2, \cdots, c_K]$. Moreover, we define the $(3K+1)\times 1$ vector $\mathbf{d}$ as

$$\mathbf{d} = [-\zeta, \underbrace{0\ldots,0}_{K}, \phi_1, \phi_2, \ldots, \phi_K, \underbrace{0\ldots,0}_{K}]^T. \tag{16}$$

With (9), (14) and (16), the density optimization problem can be rewritten as the following LFP:

$$(\mathbf{P1}) \qquad \max_{\mathbf{x}} \quad \frac{\mathbf{a}^T\mathbf{x}}{\mathbf{b}^T\mathbf{x}} \qquad \text{s.t.} \quad \mathbf{C}\mathbf{x} \leq \mathbf{d}$$

The solution is discussed in the remainder of this section.

3.2. Solution

In the following two subsections, we show that the density optimization problem can be solved by two methods for a general case and a special case, respectively. In particular, for networks with many tiers, the problem can be solved efficiently by *linear programming* (LP) transformation; whereas for networks with two tiers, a closed-form solution can be obtained using the *graphical method* [23].

3.2.1. Solution by Linear Programming

Under the assumption that the feasible region of $(\mathbf{P1})$ is non-empty and bounded, $(\mathbf{P1})$ can be converted into an LP using Charnes–Cooper transformation [23]. We define $\mathbf{y} = \mathbf{x}/(\mathbf{b}^T\mathbf{x})$, and $z = 1/(\mathbf{b}^T\mathbf{x})$, such that $(\mathbf{P1})$ is equivalent to

$$(\mathbf{P1.1}) \qquad \begin{aligned} \max_{\mathbf{y},z} \quad & \mathbf{a}^T\mathbf{y} \\ \text{s.t.} \quad & \mathbf{C}\mathbf{y} \leq \mathbf{d}z, \\ & \mathbf{b}^T\mathbf{y} = 1, \\ & z \geq 0. \end{aligned}$$

The LP can be efficiently solved using standard algorithms, such as the simplex method [24]. Let $\mathbf{y}^*$ and z^* denote the resultant solution. Then, the solution to $(\mathbf{P1})$, $\mathbf{x}^*$, is given as $\mathbf{x}^* = \mathbf{y}^*/z^*$. Hence, the maximum throughput is given by (9) substituted with $\mathbf{x}^*$.

Remark 1. The maximum throughput T^* is unique, but infinite combinations of tier densities $\{\lambda_k^*\}$ may exist to achieve this throughput. In particular, the domain for the optimal tier densities of two-tier networks can be a line segment, as elaborated in the following subsection.

3.2.2. Solution by the Graphical Method

For the simple case of two-tier networks, $(\mathbf{P1})$ reduces to

$$(\mathbf{P1.2}) \qquad \begin{aligned} \max_{\lambda_1,\lambda_2} \quad & \frac{a_1\lambda_1 + a_2\lambda_2}{b_1\lambda_1 + b_2\lambda_2} \\ \text{s.t.} \quad & (\lambda_1, \lambda_2) \in \mathcal{F}. \end{aligned}$$

where $\mathcal{F}$ denotes the feasibility region of $(\mathbf{P1.2})$

$$\begin{aligned} \mathcal{F} = \{(\lambda_1,\lambda_2) \in \mathbb{R}_+^2 \mid\ & P_1^{2/\alpha}\lambda_1 + P_2^{2/\alpha}\lambda_2 \geq \zeta, \\ & \lambda_2/\lambda_1 \leq (P_1\beta_2/P_2/\beta_1)^{2/\alpha}(1-\theta_1)/\theta_1, \\ & \lambda_2/\lambda_1 \geq (P_1\beta_2/P_2/\beta_1)^{2/\alpha}\theta_2/(1-\theta_2), \\ & \lambda_1 \leq \phi_1/c_1, \lambda_2 \leq \phi_2/c_2\}. \end{aligned} \tag{17}$$

From (17), the feasibility region is a polygon that is characterized as follows: First, it is obtained from the two constraints on the ratio λ_2/λ_1 that the feasibility region for $(\mathbf{P1.2})$ is nonempty only if $(1-\theta_1)/\theta_1 \geq \theta_2/(1-\theta_2)$. This equivalent condition $\theta_1 + \theta_2 \leq 1$ is satisfied from the definition. Next, the edges of $\mathcal{F}$ correspond to the lines defined by setting equalities in the first five constraints of $(\mathbf{P1.2})$ and denoted in order as $\ell_1, \ell_2, \ldots, \ell_5$. In particular, these edges are given as $\ell_1 : \lambda_2 = -(P_1/P_2)^{2/\alpha}\lambda_1 + \zeta/P_2^{2/\alpha}$, $\ell_2 : \lambda_2 = (P_1\beta_2/P_2/\beta_1)^{2/\alpha}(1-\theta_1)/\theta_1\lambda_1$, $\ell_3 : \lambda_2 = (P_1\beta_2/P_2/\beta_1)^{2/\alpha}\theta_2/(1-\theta_2)\lambda_1$, $\ell_4 : \lambda_1 = \phi_1/c_1$ and $\ell_5 : \lambda_2 = \phi_2/c_2$. Given that $\mathcal{F}$ is nonempty, the optimal tier densities are characterized as follows:

Proposition 1. Assuming that $\mathcal{F}$ is nonempty, the solution for $(\mathbf{P1.2})$, denoted as $(\lambda_1^*, \lambda_2^*)$, has the following properties:

1. If $a_1/b_1 < a_2/b_2$, $(\lambda_1^*, \lambda_2^*) = \arg\min_{(\lambda_1,\lambda_2)\in\mathcal{F}} \lambda_1/\lambda_2$.
2. If $a_1/b_1 > a_2/b_2$, $(\lambda_1^*, \lambda_2^*) = \arg\max_{(\lambda_1,\lambda_2)\in\mathcal{F}} \lambda_1/\lambda_2$.
3. If $a_1/b_1 = a_2/b_2$, every feasible point (λ_1, λ_2) yields the same maximal throughput $T^* = a_1/b_1 = a_2/b_2$.

Table 1: Optimal BS density for throughput maximization subject to cost constraint in two-tier heterogeneous cellular networks ($\varepsilon = 0.1$)

Maximum deployment cost constraints (ϕ_1, ϕ_2)	Optimal densities $(\lambda_1^*, \lambda_2^*)$ by LP method	Optimal densities $(\lambda_1^*, \lambda_2^*)$ by the graphical method	Maximum network throughput T^*	Deployment-and-operation costs $(\mathcal{C}_1, \mathcal{C}_2)$
$(1.0, 0.6)$	$(0.38, 0.6)$	$(0.38, 0.6)$	0.50	$(0.76, 0.6)$
$(1.0, 0.8)$	$(0.34, 0.8)$	$(0.34, 0.8)$	0.54	$(0.68, 0.8)$
$(1.0, 1.0)$	$(0.30, 1.0)$	$(0.30, 1.0)$	0.58	$(0.59, 1.0)$
$(1.0, 1.2)$	$(0.25, 1.2)$	$(0.25, 1.2)$	0.63	$(0.51, 1.2)$

Sketch of proof: Essentially, the simple proof is based on rewriting the objective function of $(\mathbf{P1.2})$ as $\frac{a_1\lambda_1/\lambda_2+a_2}{b_1\lambda_1/\lambda_2+b_2}$ and observing the function to grow with the decreasing ratio λ_1/λ_2 if $a_1/b_1 < a_2/b_2$; otherwise, the function grows with the increasing ratio. $\square$

We consider the case of $a_1/b_1 < a_2/b_2$ and obtain the solution for $(\mathbf{P1.2})$ in closed form. The analysis for the case of $a_1/b_1 > a_2/b_2$ is similar and is thus omitted for brevity. For ease of notation, let the intersection points between a pair of lines (ℓ_m, ℓ_n) be specified by $(\lambda_1, \lambda_2) = (\lambda_1(\ell_m, \ell_n), \lambda_2(\ell_m, \ell_n))$.

Corollary 1. Assuming that $\mathcal{F}$ is nonempty, and $a_1/b_1 < a_2/b_2$.

1. If $\phi_2/c_2 > \lambda_2(\ell_1, \ell_2)$, $(\lambda_1^*, \lambda_2^*)$ is an arbitrary point in the following line segment:

$$\big\{(\lambda_1, \lambda_2) \in \mathbb{R}_+^2 \mid \lambda_2 = (P_1\beta_2/P_2/\beta_1)^{2/\alpha}(1-\theta_1)/\theta_1\lambda_1, \quad \lambda_2 \in [\lambda_2(\ell_1, \ell_2), \min(\lambda_2(\ell_2, \ell_4), \phi_2/c_2)]\big\}. \tag{18}$$

2. If $\lambda_2(\ell_1, \ell_3) \leq \phi_2/c_2 \leq \lambda_2(\ell_1, \ell_2)$, $(\lambda_1^*, \lambda_2^*)$ is unique:

$$(\lambda_1^*, \lambda_2^*) = \left(\zeta/P_1^{2/\alpha} - (P_2/P_1)^{2/\alpha}\phi_2/c_2, \phi_2/c_2\right) \tag{19}$$

The proof is on the basis of the analysis of the polygon geometry of the feasibility region. The details are omitted for brevity.

Remark 2. For the first case in Corollary 1, although all points in the said line segment are optimal, the point with minimum tier densities is desirable to use in practice to minimize the deployment cost (e.g., $(\lambda_1(\ell_1, \ell_2), \lambda_2(\ell_1, \ell_2))$ for case 1).

Remark 3. For a two-tier network, the density optimization problem can be solved using the graphical method for analyzing the geometry of the feasibility region, thus yielding a closed-form solution. However, for networks with many tiers, the number of edges/faces of the feasibility region grows exponentially with the number of tiers. For such cases, the graphical method is no longer efficient, and the use of the LP approach based on Charnes–Cooper transformation shown in $(\mathbf{P1.1})$ is preferred.

4. NUMERICAL RESULT

In this section, we evaluate the throughput gain of a two-tier heterogeneous cellular network with the use of the proposed density optimization. The evaluations in multi-tier networks show similar results. The following settings are used in the simulation: The mobile user density is $\lambda_u = 1$ per unit area, and the path-loss exponent is $\alpha = 3$. The transmission powers, target SIRs, minimum fractional loads, and deployment prices of each tier are chosen as $(P_1, P_2) = (10, 1), (\beta_1, \beta_2) = (10, 1), (\theta_1, \theta_2) = (0.1, 0.6)$, and $(c_1, c_2) = (2, 1)$, respectively. The maximum deployment cost for tier 1 is $\phi_1 = 1$ per unit area, whereas that for tier 2, ϕ_2, is a variable. The parameter ε for Constraint 1 is specified in the table and figure.

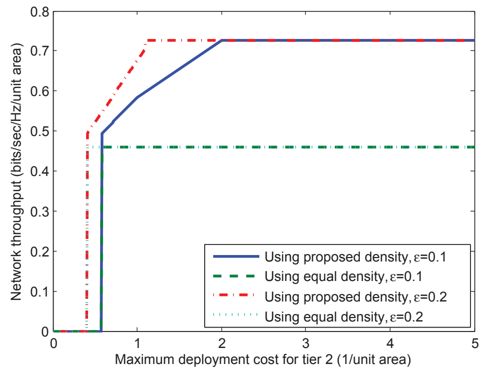

Fig. 2: Maximum network throughput T^* versus the maximum permissible deployment cost ϕ_2 for tier 2.

Table 1 demonstrates the optimal BS densities in two-tier heterogenous cellular networks under different deployment cost constraints. We can see that the two proposed methods, namely LP method and the graphical method, provide the same optimal solution that maximizes the network throughput. The resultant deployment cost also satisfies Constraint 3; i.e., $(\mathcal{C}_1, \mathcal{C}_2) \leq (\phi_1, \phi_2)$.

In Fig. 2, the network throughput is plotted against the constrained deployment cost ϕ_2 for both the case of optimized deployment and the baseline case of equal tier densities. When ϕ_2 is small, tier densities are no longer supportable without violating Constraint 3. Thus, the feasibility region becomes empty, and throughput reduces to zero. Given the intermediate values of ϕ_2, the throughput increases with ϕ_2 because Constraint 3 shapes the feasibility region. When ϕ_2 is so large that Constraint 3 becomes redundant in terms of shaping the feasibility region, the maximum throughput is always achievable. By contrast, the throughput is $(a_1+a_2)/(b_1+b_2)$ when the equal density approach is used. As a result, the proposed density increases the throughput by more than 50% as opposed to the equal density approach. Moreover, as shown in Fig. 2, when Constraint 1 is activated, substantial throughput gain can be obtained by increasing ε. This finding is attributed to the fact that more transmission can be supported if higher outage requirement is allowed.

5. CONCLUSION

In this study, we have considered the tradeoff between the network throughput and deployment cost when designing BS density in multi-tier heterogeneous cellular networks. The optimization problem is formulated as an LFP that can be solved efficiently with the use of Charnes–Cooper transformation. Moreover, a closed-form solution has been derived for two-tier networks. The numerical results have demonstrated that the proposed BS density significantly enhances throughput given the constraint on deployment cost.

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