ON THE BROADBAND EFFECT OF REMOTE STATIONS IN DPD ALGORITHM

Cyrile Delestre[1,2], *Anne Ferréol*[1,2], *Alon Amar*[3] *and Pascal Larzabal*[2]

[1]Thales Communications & Security, 4, Avenue des Louvresses 92230 Gennevilliers Cedex, France
[2]Laboratoire SATIE - ENS Cachan - CNRS - UniverSud, 61 av du President Wilson 94235 CACHAN Cedex, France
[3]Signal Processing Department, Acoustics Research Center, Rafael, Haifa, P.O.B. 2250, Israel

ABSTRACT

This paper addresses the direct geolocation of sources in one step via a multi-base (or multi-array) context. The 1-step methods such as DPD and LOST are working on a global array composed of all the sensors of each base. However, even if these algorithms introduce a narrowband decomposition (unfortunately imperfect), these recent powerful algorithms can be disturbed by the residual broadband effect due to the partial coherence of signals (even incoherent signals) between stations. The main contribution of this work is to study the DPD performance in presence of a residual array-broadband effect.

***Index Terms*—** Geolocation - Parameter bias - DPD

1. INTRODUCTION

The context addressed in this article is source localization using multi-sensor remote stations (the multi-sensor station will be called array or base). On each array, the inter-sensors distances are small enough so that the arrays are considered narrowband *vis-a-vis* the radiations they receive (*i.e.*, the received signals on all the sensors of the array are spatially coherent). The transmitters (or sources) locations are traditionally estimated in 2-steps with conventional algorithms. For instance, the Angles of Arrival (AoA) of sources are estimated by each station independently, exploiting the spatial coherence on an array [2] in the first step and, in a second step, the locations of sources are computed from the AoA (*e.g.*, by triangulation) [1]. In practice the signals observed by different sensors are partially coherent and previous works discussed AoA estimation with imperfect coherence across the array (*e.g.* [3]). The performance of geolocation with 2-step methods in the presence of imperfect coherence between the arrays is considered in [11].

The 1-step methods [4]-[8] have been introduced in order to improve the performances of conventional algorithms [9]. These methods are based on a direct estimation of the geographical coordinates of the transmitters, working on the global array, sometimes called "array of arrays" [11] in the literature. This global-array is composed of all the bases of the multi-base context. Unfortunately, due to the broadband signals and the remoteness of arrays, a spatial incoherence effect appears between the bases. Consequently, some works consider the total spatial-incoherence between arrays [4] or a total spatial-coherence between all the sensors of the global array [5], [6]. In [11] the performance of the 1-step method which not counteracts the broadband effect in presence of the partial spatial coherence between the bases is treated. Then, A.Amar and A.J.Weiss introduced the DPD (Direct Position Determination) algorithm [7] and J.Bosse *et al* the LOST (LOcalization by Space-Time) algorithm [8] in order to counteract the partial-coherence effect. The aim of both methods is to decompose the signals in K narrowband sub-signals assuming a spatial coherence inside each array of the global array. More precisely, a signal of bandwidth B is processed as multiple signals of bandwidth $\frac{B}{K}$. Then, these algorithms (DPD and LOST) assume that the signal of bandwidth $\frac{B}{K}$ is narrowband, which retrieves the spatial coherence of signals between the bases.

The performance of direct geolocation which counteracts the broadband effect has not been discussed to date in case of partially coherent signals, except in [10]. In this last reference, the authors focused on the geolocation bias of LOST algorithm where the complex attenuations of stations are assumed to be known. Hence, our purpose is to evaluate the performances of the DPD algorithm when the narrowband assumption is not verified for the sub-band of bandwidth $\frac{B}{K}$ and the complex attenuations are unknown. More precisely, the purpose is to give a closed form expression of the geolocation bias when the covariance matrix of the received signals is perfectly known. In order to do this, we first provide a closed form expression of the error on the covariance matrix due to the narrowband hypothesis and deduce the bias.

Notations: $\mathbf{A}$ or $(a_{ij})_{1\leq i\leq I, 1\leq j\leq J}$ $\forall (I,J)\in\mathbb{N}_*^2$ is a matrix of dimension $I\times J$ and $[\mathbf{A}]_{i,j}=a_{i,j}$ is the ij-th element of the $\mathbf{A}$ matrix, $\mathbf{a}$ or $(a_i)_{1\leq i\leq I}$ $\forall I\in\mathbb{N}_*$ is a column vector of dimension I, $\mathbf{I}_I$ is the identity matrix of dimension I, a or A is a scalar, $(\cdot)^H$ is the Hermitian of a matrix or a vector, $(\cdot)^T$ is the transpose of a matrix or vector, $(\cdot)^*$ is the conjugate of a scalar, $\mathbb{E}[\cdot]$ is mathematical expectation, $[\![a,b]\!]$ is the set defined by $\left\{x\in\mathbb{Z} : a\leq x\leq b, \forall(a,b)\in\mathbb{Z}^2\right\}$, for all commutative ring or semiring $\mathbb{K}$ we have $\mathbb{K}_*=\mathbb{K}\backslash\{0\}$ and $\mathbb{K}_+=\{x\in\mathbb{K} : 0\leq x<+\infty\}$.

2. ASSUMPTIONS AND MODEL

2.1. Assumptions about the system

The global geolocation system is composed of L remote stations with the same reception band. Each of these bases are composed of M_l sensors. Thus, the system has M sensors ($\sum_{l=1}^{L} M_l = M$). In this paper, the number Q of sources is assumed to be known. The analytic sources signals $s_q(t)$ at the location $\mathbf{p}_q$ are statistically independent. These sources stem from a stationary and spectrally white signal modulated for the transmission. We define the autocorrelation of the signal by:

$$r_q(\tau) \quad = \quad \mathbb{E}\left[s_q(t)s_q^*(t-\tau)\right] \tag{1}$$

The bandwidth at the output of each station is B and T_e is the sampling time of all the bases. Moreover, all the bases are perfectly time and frequency synchronized with each other. Finally, a stationary additive white noise at the output of the base station, with zero mean and variance σ^2, is added. Fig.1 represents the propagation of one source to the remote stations.

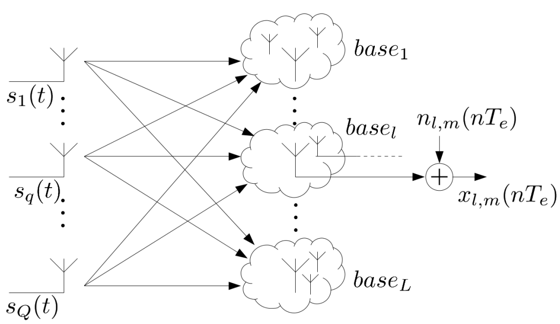

Fig. 1. System diagram

2.2. Signal modeling

In this paper, we are interested in the multi-base geolocation methods in one step where the signals, at the output of all stations, are concatenated into a vector:

$$\mathbf{y}(t) = \left[\mathbf{x}_1^T(t), ..., \mathbf{x}_l^T(t), ..., \mathbf{x}_L^T(t)\right]^T \tag{2}$$

In the Line of Sight (LoS) assumption, the M sensors of the global system observe the direct paths of the sources. Then, the output of the l-th station is:

$$\mathbf{x}_l(t) = \sum_{q=1}^{Q} \rho_{l,q}\mathbf{a}_l(\theta_l(\mathbf{p}_q))s_q(t-\tau_l(\mathbf{p}_q)) + \mathbf{n}_l(t) \tag{3}$$

where $\rho_{l,q}$, $\mathbf{a}_l(\theta_l(\mathbf{p}_q))$ (noted $\mathbf{a}_l(\mathbf{p}_q)$ in the remainder of the paper), $\theta_l(\mathbf{p}_q)$ and $\tau_l(\mathbf{p}_q)$ are the complex attenuation, the steering vector, the Angle of Arrival (AoA) and the Time of Arrival (ToA) respectively associated to the q-th source and the l-th base and $\mathbf{n}_l(t)$ the additive noise of the l-th base.

3. A CENTRALIZED METHOD: DPD

The DPD algorithm [7] filters the received signal at the sensors of the stations by a filter bank composed of K FIR filters, centered at $f_k = \frac{k}{KT_e}$, of bandwidth $\frac{B}{K}$ and composed of J coefficients $\mathbf{w}_k = [w_k[1], ..., w_k[J]]^T$. After this filter bank, the signals are assumed narrowband in each sub-band. Moreover, the narrowband hypothesis is verified on the global array if and only if the Time-Bandwidth (TB) product fulfills:

$$\max_{q\in[\![1,Q]\!],(l,j)\in[\![1,L]\!]^2} T_{l,j}(\mathbf{p}_q) \times \frac{B}{K} \ll 1 \tag{4}$$

with $T_{l,j}(\mathbf{p}_q) = \left|\frac{||\mathbf{b}_l-\mathbf{p}_q||-||\mathbf{b}_j-\mathbf{p}_q||}{c}\right|$, where $\mathbf{b}_l$ is the l-th base location and c is the light speed in vacuum. Observing Eq.(4) we note that the higher the number of decompositions K is, the smaller the TB product is. However, the Q signals have to be spectrally present on each sub-band. The DPD assumes that the bandwidth $\frac{B}{K}$ is small enough to consider the narrowband hypothesis on the global array. If we note $\mathbf{x}_{l,k}(t)$ and $s_{q,k}(t)$ as the received signal and the transmitted signal at the output of the k-th filter of the filter bank respectively, the model of the received signal is:

$$\mathbf{y}_k(t) = \begin{bmatrix}\mathbf{x}_{1,k}(t)\\ \vdots \\ \mathbf{x}_{L,k}(t)\end{bmatrix} \approx \sum_{q=1}^{Q}\mathbf{U}(\mathbf{p}_q, f_k)\boldsymbol{\rho}_q s_{q,k}(t) + \mathbf{n}_k(t) \tag{5}$$

where $\mathbf{n}_k(t)$ is the noise filtered by the k-th filter of the filter bank, the vector $\boldsymbol{\rho}_q$ is the vector of all the attenuations $(\rho_{l,q})_{1\leq l\leq L}$ of the q-th source, let $\mathbf{U}(\mathbf{p}_q, f_k)$ be the matrix such as:

$$\mathbf{U}(\mathbf{p}_q, f_k) = \left(\delta_{l,j}\mathbf{a}_l(\mathbf{p}_q)e^{-2i\pi f_k\tau_l(\mathbf{p}_q)}\right)_{1\leq(l,j)\leq L} \tag{6}$$

with $\delta_{l,j}$ the Kronecker delta. Then, the covariance matrix of the DPD model at the output of each frequency sub-band is:

$$\begin{aligned}\mathbf{R}_k &= \mathbb{E}\left[\mathbf{y}_k(t)\mathbf{y}_k^H(t)\right] \qquad\qquad \forall k\in[\![1;K]\!]\\ &= \sum_{q=1}^{Q}\mathbb{E}\left[|s_{q,k}(t)|^2\right]\mathbf{U}(\mathbf{p}_q, f_k)\boldsymbol{\rho}_q\boldsymbol{\rho}_q^H\mathbf{U}^H(\mathbf{p}_q, f_k)+\sigma_k^2\mathbf{I}_M\end{aligned} \tag{7}$$

where σ_k^2 is the noise power on the k-th channel. The power of the q-th signal with the k-th filter of the filter bank is:

$$\mathbb{E}\left[|s_{q,k}(t)|^2\right] = \mathbf{w}_k^H\boldsymbol{\Gamma}_q\mathbf{w}_k \tag{8}$$

where, according to Eq.(1), $\boldsymbol{\Gamma}_q = \left(r_q((v-j)T_e)\right)_{1\leq(v,j)\leq J}$ is the *cross-energy* matrix of the shaping filter of the q-th source.

According to the MUSIC algorithm [2], we deduce the projector onto the noise subspace, named $\boldsymbol{\Pi}_k^{\perp}$, associated to the covariance matrix $\mathbf{R}_k$. Then, thanks to Rayleigh's quotient [12], the DPD criterion [7] is an incoherent sum of the MUSIC criteria in each sub-band and permits us to deduce the sources parameters $\mathbf{p}_q$

$$J_{DPD}(\mathbf{p}) = \lambda_1\left\{\mathbf{Q}(\mathbf{p})\right\} \tag{9}$$

with

$$\mathbf{Q}(\mathbf{p}) = \sum_{k=1}^{K}\mathbf{U}^H(\mathbf{p}, f_k)\boldsymbol{\Pi}_k^{\perp}\mathbf{U}(\mathbf{p}, f_k) \tag{10}$$

where, if we consider the eigenvalue decomposition of the matrix $\mathbf{A}$ with $\lambda_1 \leq \lambda_2 \leq ...$, then, $\lambda_1 \{\mathbf{A}\}$ designates the smallest eigenvalue of the eigenvalue decomposition of $\mathbf{A}$. In the remainder of this paper, let the eigenvectors $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, ...]$ (*i.e.*, $\mathbf{A}\mathbf{v}_1 = \lambda_1 \mathbf{v}_1$) associated to the eigenvalues. Finally, the position of the Q sources can be estimated by searching the Q minimum of the cost function:

$$\{\mathbf{p}_1, ..., \mathbf{p}_Q\} = \arg\min_{\mathbf{p} \in \mathbb{R}^2} J_{DPD}(\mathbf{p}) \tag{11}$$

4. PARTIALLY COHERENT SIGNALS

In this section, we will consider the partially coherent case after an imperfect narrowband decomposition of the DPD (where K is not high enough). We will establish the real covariance matrix of the system, which will allow us to establish a disturbance on the projector in order to deduce the error generated on the estimation of interest parameters.

Presently, the sub-signals of the DPD process do not respect the narrowband assumption. Therefore, the approximation made in Eq.(5) is no longer satisfied. Consequently, the signal filtered by the k-th filter is:

$$\mathbf{y}_k(t) = \sum_{q=1}^{Q} \tilde{\mathbf{U}}(\boldsymbol{\rho}_q, \mathbf{p}_q) \Big(s_{q,k}(t - \tau_l(\mathbf{p}_q)) \Big)_{1 \leq l \leq L} + \mathbf{n}_k(t) \tag{12}$$

where

$$\tilde{\mathbf{U}}(\boldsymbol{\rho}_q, \mathbf{p}_q) = \Big(\delta_{l,j} \rho_{l,q} \mathbf{a}_l(\mathbf{p}_q) \Big)_{1 \leq (l,j) \leq L} \tag{13}$$

We recall that the sources are uncorrelated with each other. Then, the theoretical broadband covariance matrix can be written as follows:

$$\begin{aligned} \hat{\mathbf{R}}_k &= \mathbb{E}\Big[\mathbf{y}_k(t)\mathbf{y}_k^H(t)\Big] \quad \forall k \in [\![1; K]\!] &(14)\\ &= \sum_{q=1}^{Q} \tilde{\mathbf{U}}(\boldsymbol{\rho}_q, \mathbf{p}_q) \mathbf{R}_{s_k}(\mathbf{p}_q) \tilde{\mathbf{U}}^H(\boldsymbol{\rho}_q, \mathbf{p}_q) + \sigma_k^2 \mathbf{I}_M &(15)\end{aligned}$$

The matrix $\mathbf{R}_{s_k}(\mathbf{p}_q)$ is the covariance matrix of the transmitted signals filtered by the k-th filter of the filter bank.

$$\mathbf{R}_{s_k}(\mathbf{p}_q) = \begin{pmatrix} r_{q,k}(0) & \cdots & r_{q,k}(\Delta\tau_{1,L}(\mathbf{p_q})) \\ \vdots & \ddots & \vdots \\ r_{q,k}^*(\Delta\tau_{1,L}(\mathbf{p_q})) & \cdots & r_{q,k}(0) \end{pmatrix} \tag{16}$$

with

$$\begin{cases} r_{q,k}(\tau) = \mathbb{E}\left[s_{q,k}(t) s_{q,k}^*(t-\tau)\right] \\ \Delta\tau_{l,v}(\mathbf{p}_q) = \tau_l(\mathbf{p}_q) - \tau_v(\mathbf{p}_q) \quad \forall (l,v) \in [\![1, L]\!]^2 \end{cases} \tag{17}$$

where $\Delta\tau_{l,v}$ is the Time Differential of Arrival (TDoA) between the l-th and v-th bases and the q-th source. The term $r_{q,k}(\Delta\tau_{l,v}(\mathbf{p_q}))$ depends on the coefficients of the k-th filter and on the shaping filter associated to the q-th source:

$$r_{q,k}(\Delta\tau_{l,v}(\mathbf{p_q})) = \mathbf{w}_k^H \hat{\mathbf{\Gamma}}_q(\Delta\tau_{l,v}(\mathbf{p_q}))\mathbf{w}_k \tag{18}$$

where, according to Eq.(1),

$$\hat{\mathbf{\Gamma}}_q(\Delta\tau_{l,v}) = r_q\big((i-j)T_e - \Delta\tau_{l,v}\big)_{1 \leq (i,j) \leq J} \tag{19}$$

is the matrix of *cross-energy* of the shaping filter applied to the q-th source as in Eq.(8) and where the TDoA is now taken into account. In other words, Eq.(18) generalizes the DPD result in Eq.(8) in the sense that the time difference of the signals at pairs of arrays is taken into account.

In the remainder of this article we will build on this result to observe some consequences due to the broadband effect.

5. BIAS CLOSED FORM EXPRESSION

We will give a closed form expression of the asymptotic bias (asymptotic in the number of snapshots) on the sources localization, named $\Delta\mathbf{p}_q = \mathbf{p}_q - \hat{\mathbf{p}}_q$, where $\mathbf{p}_q$ is the source position deduced from the DPD model (Sec.3), and $\hat{\mathbf{p}}_q$ the source position when we have partial coherence between the bases (Sec.4.1). *In fine*, this error is caused by the difference between the covariance matrix model of DPD and the broadband one: $\Delta\mathbf{R}_k = \hat{\mathbf{R}}_k - \mathbf{R}_k$. We express here the bias of the projector where, in [13], it is shown that the first-order relationship between $\mathbf{\Pi}_k^{\perp}$ and $\hat{\mathbf{\Pi}}_k^{\perp}$ is:

$$\hat{\mathbf{\Pi}}_k^{\perp} = \mathbf{\Pi}_k^{\perp} - \Delta\mathbf{\Pi}_k^{\perp} \tag{20}$$

with

$$\Delta\mathbf{\Pi}_k^{\perp} \approx \mathbf{\Pi}_k^{\perp} \Delta\mathbf{R}_k (\mathbf{R}_k)^{+} + (\mathbf{R}_k)^{+} \Delta\mathbf{R}_k \mathbf{\Pi}_k^{\perp} \tag{21}$$

where $(\cdot)^{+}$ is the MoorePenrose pseudoinverse. We note the estimated quadratic form for the DPD (Eq.(10)) as:

$$\hat{\mathbf{Q}}(\mathbf{p}) = \sum_{k=1}^{K} \mathbf{U}^H(\mathbf{p}, f_k) \hat{\mathbf{\Pi}}_k^{\perp} \mathbf{U}(\mathbf{p}, f_k) \tag{22}$$

and its derivative with $i \in \{x, y\}$:

$$\frac{\partial \hat{\mathbf{Q}}(\mathbf{p})}{\partial p_i} = 2 \sum_{k=1}^{K} \Re\left\{ \frac{\partial \mathbf{U}^H(\mathbf{p}, f_k)}{\partial p_i} \hat{\mathbf{\Pi}}_k^{\perp} \mathbf{U}(\mathbf{p}, f_k) \right\} \tag{23}$$

Since the Rayleigh's quotient of Eq.(9) was used so that the criteria only depends on the position $\mathbf{p}$, the eigenvector $\mathbf{v}_1$ is independent of the position parameters. Indeed the complex attenuations does not depend on the position of the sources. If we note $\mathbf{v}_1 = \boldsymbol{\rho}_q + \partial\mathbf{v}_1$ and $J_{DPD}(\mathbf{v}_1, \hat{\mathbf{p}}_q) = \mathbf{v}_1^H \hat{\mathbf{Q}}(\hat{\mathbf{p}}_q)\mathbf{v}_1$ the estimated criteria, its Taylor-Young expansion can be written as [15]:

$$\begin{aligned} J_{DPD}(\mathbf{v}_1, \hat{\mathbf{p}}_q) = J_{DPD}(\boldsymbol{\rho}_q, \mathbf{p}_q) + \begin{bmatrix} \Delta\mathbf{p}_q \\ \partial\mathbf{v}_1 \end{bmatrix}^T \nabla J_{DPD}(\boldsymbol{\rho}_q, \mathbf{p}_q) + \\ \frac{1}{2}\Big(\Delta\mathbf{p}_q^T \mathbf{H}(J_{DPD}(\boldsymbol{\rho}_q, \mathbf{p}_q)) \Delta\mathbf{p}_q + \\ \partial\mathbf{v}_1^T \mathbf{H}_{\mathbf{v}_1}(J_{DPD}(\boldsymbol{\rho}_q, \mathbf{p}_q)\partial\mathbf{v}_1 \Big) + o\left(\Delta\mathbf{p}_q^2 + \partial\mathbf{v}_1^2\right) \end{aligned} \tag{24}$$

where ∇ is the gradient operator, $\mathbf{H}$ (respectively $\mathbf{H}_{\mathbf{v}_1}$) is the Hessian matrix function of the position (respectively the eigenvector). As one could see in Eq.(24), the parameters $\hat{\mathbf{p}}_q$ and $\mathbf{v}_1$ are totally decorrelated, so we can optimize the position parameter of the source independently of $\mathbf{v}_1$. Then, the expression of the bias on the parameters is given by the 2^{nd} order approximation as follows:

$$\Delta\mathbf{p}_q \approx -\tilde{\mathbf{H}}^{-1}(\lambda_1\{\hat{\mathbf{Q}}(\mathbf{p}_q)\})\nabla\lambda_1\{\hat{\mathbf{Q}}(\mathbf{p}_q)\} \tag{25}$$

If we define the gradient as $\nabla^T(\mathbf{p}) = \left(\frac{\partial}{\partial p_x}, \frac{\partial}{\partial p_y}\right)$, we have for each element of the gradient in the DPD [15], [16]:

$$\frac{\partial\lambda_1\{\hat{\mathbf{Q}}(\mathbf{p}_q)\}}{\partial p_i} = \mathbf{v}_1^H \frac{\partial\hat{\mathbf{Q}}(\mathbf{p}_q)}{\partial p_i}\mathbf{v}_1 \tag{26}$$

and, for a mathematical tractability issue, we use the following approximation of the Hessian:

$$\tilde{\mathbf{H}}(\mathbf{p}) = \begin{pmatrix} \tilde{H}_{xx}(p_x) & \tilde{H}_{xy}(p_x, p_y) \\ \tilde{H}_{yx}(p_x, p_y) & \tilde{H}_{yy}(p_y) \end{pmatrix} \tag{27}$$

we have for each element of the Hessian approximation [15], [16]:

$$\tilde{H}_{ij}(\lambda_1\{\hat{\mathbf{Q}}(\mathbf{p}_q)\}) = 2\mathbf{v}_1^H \sum_{k=1}^{K} \Re\left\{\frac{\partial\mathbf{U}^H(\mathbf{p}_q, f_k)}{\partial p_i}\hat{\mathbf{\Pi}}_k^{\perp}\frac{\partial\mathbf{U}(\mathbf{p}_q, f_k)}{\partial p_j}\right\}\mathbf{v}_1 \tag{28}$$

with $(i,j) \in \{x,y\}^2$. Using Eq.(20) of the projector and performing a first order Taylor expansion of the bias $\Delta\mathbf{p}_q$ with respect to the matrix $\Delta\mathbf{R}_k$ [14], the expressions of $\frac{\partial\lambda_1\{\hat{\mathbf{Q}}(\mathbf{p}_q)\}}{\partial p_i}$ and $\tilde{H}_{ij}(\lambda_1\{\hat{\mathbf{Q}}(\mathbf{p}_q)\}) = \tilde{H}_{ij}$ are:

$$\frac{\partial\lambda_1\{\hat{\mathbf{Q}}(\mathbf{p}_q)\}}{\partial p_i} \approx -2\mathbf{v}_1^H \sum_{k=1}^{K} \Re\left\{\frac{\partial\mathbf{U}^H(\mathbf{p}_q, f_k)}{\partial p_i}\mathbf{\Pi}_k^{\perp}\Delta\mathbf{R}_k \mathbf{R}_k^{\dagger}\mathbf{U}(\mathbf{p}_q, f_k)\right\}\mathbf{v}_1 \tag{29}$$

$$\tilde{H}_{ij} \approx 2\mathbf{v}_1^H \sum_{k=1}^{K} \Re\left\{\frac{\partial\mathbf{U}^H(\mathbf{p}_q, f_k)}{\partial p_i}\mathbf{\Pi}_k^{\perp}\frac{\partial\mathbf{U}(\mathbf{p}_q, f_k)}{\partial p_j}\right\}\mathbf{v}_1 \tag{30}$$

6. SIMULATIONS

In this part, we will consider a single source case ($Q = 1$) and a dual sources case ($Q = 2$), with two bases ($L = 2$). We will also consider a zero noise ($\sigma^2 = 0$), the goal being to eliminate any disturbance other than the broadband effect. In a Cartesian coordinate system, we place the first base at (-400m,-400m), and the second at (400m,-400m). The bases are composed of six sensors where five are in a circular formation around a sixth in the center. The bases radius is 0.8m. The two bases are perfectly synchronized with the sampling frequency $T_e = \frac{1}{500 \cdot 10^3}$[sec]. The first source location (presents in the two cases) is moved along the abscissa axis with the position (d,0) with $d \in \mathbb{R}_+$. In the two sources case, we consider that the second source follows the first source with the position (d,-100m). Thus, in the two cases, the sources go from the narrowband case to the broadband case.

We consider the sources with a carrier frequency $f_0 = 900$MHz, a Nyquist shaping filter and boxcar as filters of the filter bank (FFT filter bank). The bandwidth of the sources is $B_1 = 426$KHz. We note that, thanks to the triangular inequality, the Time-Bandwidth product defined as in Eq.(4) is bounded: $0 \leq TB < 1.14$. The upper bound is reached when $d \to \infty$. When d increases, the system becomes broadband.

We plot the bias with respect to the TB product normalized in number of decomposition ($K \times TB$) at Fig.2. The real bias is given by the minimization of the broadband theoretical criterion determined thanks to the covariance matrix of Eq.(15) (blue and green squares), and the closed form is given by Eq.(25) (dashed line with cross). Thus, we observe that for $K = 2$ the more the TB product increases the greater the bias is and for $K = 8$ we have a smaller bias. We also show that the closed form bias fits well with the real bias. We note that a gap is created between the real and the closed form bias due to the second order approximation on the parameters bias and to the first order approximation on the error on the projector.

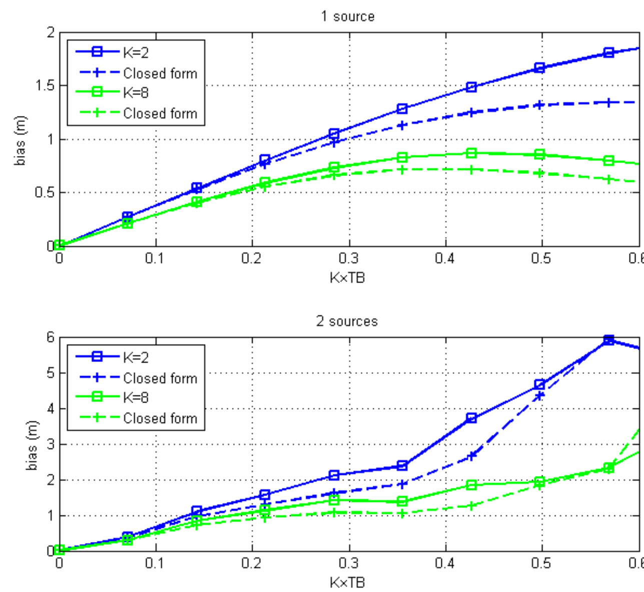

Fig. 2. Visualization of the bias

We plot the bias for the one source case (respectively the two sources case) at the top (respectively at the bottom) of Fig.2. We observe that the bias increases as the number of sources increases.

7. CONCLUSION

The obtained results in this paper should sensitize the designers of geolocation systems using one step methods to the influence on performances of the number K of channels they employ. More precisely, we observed that the broadband effect due to the remote stations has an impact on the localization performance. Consequently, the estimation methods must now necessarily work in the broadband case and the recently appeared solutions in the literature such as DPD are based on a division of the spectral band in K channels. We have established the bias of this method with respect to K. This last relation will permit to predetermine K relatively to an expected precision or a calibration quality of the antenna. This work is preliminary and must now be deepened relatively to the effective use of these results on an operational implementation.

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