COOPERATIVE SELF-LOCALIZATION IN ASYNCHRONOUS SENSORS NETWORKS BASED ON TOA FROM TRANSMITTERS AT UNKNOWN LOCATIONS

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ABSTRACT

We consider self-localization in an ad-hoc, asynchronous sensors network. A mobile beacon transmits a short wideband signal from a few locations, unknown to the sensors. Each of the sensors receives the transmissions and estimates their Times of Arrival (TOAs) relative to its own timebase, which has an unknown relative synchronization offset. If the positions of the beacon were known, each sensor could estimate its own time-offset and position. Since the beacon's positions are unknown, the sensors need to collaborate in order to estimate these positions along with their own. We propose a collaborative iterative scheme, where in each iteration each sensor announces its current estimate of the beacon's positions, along with an associated uncertainty covariance matrix. This information is received by neighboring sensors, and each sensor exploits the received information to refine its own estimates of the beacon's positions, as well as of its own time-offset and position. We show simulation results indicating successful self-localization using this scheme.

Index Terms— ad-hoc sensor network, decentralized self localization, TOA, beacons.

1. INTRODUCTION

Self localization in wireless sensors networks has attracted considerable research efforts in recent years (see, e.g., [12], [13] for overviews). Many existing methods (e.g., [7], [4], [3], [11], [15]) consider range and/or direction measurements between the sensors as the basis for the positions estimation. These measurements are usually extracted from Time of Arrival (TOA), Received Signal Strength (RSS) or Angle of Arrival (AOA) estimation. Operating with TOA measurements usually requires the sensors to be able to transmit wideband signals to each other (so as to facilitate accurate TOA estimation), which may sometimes be too expensive. Thus, alternative methods (e.g., [2], [17], [10], [8]) rely on an auxiliary, mobile "assisting transmitter" ("beacon"), or several static beacons, transmitting beacon signals from different positions. These beacon signals are received by the sensors, and by extracting range and/or direction information, each sensor can individually estimate its own position - as long as the positions of the beacon(s) at the time of transmission are known to the sensors.

Assuming a 2-Dimensional (2D) geometry for the area of deployment of the sensors (e.g., assuming that all sensors are located on flat grounds), if the ranging is TOA-based and the sensors are synchronized to the beacon(s), transmissions from (at least) three different known positions are sufficient for all sensors to determine their own positions, without any need for centralized processing or for collaboration between sensors. The same is true for the case of asynchronous sensors (in fact, such a scenario can be regarded as the inverse problem of localizing an unknown transmitter by asynchronous sensors at known positions, as considered, e.g., in [5]), however in that case the accuracy can still be improved by centralized or collaborative processing, because TOA measurements at different sensors carry information on their mutual time-offsets.

Nevertheless, this mode of operation heavily relies on the assumption that the beacons' positions are known to the sensors. Indeed, such an assumption may be quite reasonable in various practical scenarios (these positions may either be fixed and known in advance, or may be transmitted by the beacons to the sensors). However, some scenarios may come to mind, in which such information would not be available to the sensors. This may be the case, e.g., when the flying beacon is an opportunistic, non-cooperating airborne radar which transmits wideband pulsed signals along its flight trajectory, but would not transmit any information about its positions. Such beacons are completely external to the network and cannot be simply regarded as additional nodes - since they cannot collaborate with or pass information to / from the other sensors in any way.

A scenario in which the positions of the beacon(s) are unknown to the sensors, and moreover, the sensors are not synchronized (neither to the beacon(s), nor among themselves) is much more challenging, and, to the best of our knowledge, has not been considered before in literature. Evidently, in order to enable any localization, some "anchor sensors" with known positions should be included in the network. When a sufficient number of such anchor sensors exists, a simple two-stage scenario may be employed: In the first stage, the anchor sensors would estimate the positions of the beacons, and would announce these positions to all other sensors; Then in a second stage, the other sensors would estimate their own positions utilizing the information regarding the beacons' positions. However, such a scheme requires full connectivity between the anchor sensors, which might not be available in practice. Moreover, while being relatively simple, such a scheme would be significantly sub-optimal, as it fails to exploit the additional information that the non-anchor sensors' TOA measurements contain regarding the beacons' positions.

In this work we propose a collaborative scheme, in which an *ad-hoc* network of sensors with limited connectivity collaborates in an iterative message-passing protocol to jointly estimate the positions of the beacons, together with the positions of the sensors and with their timing-offsets. The message-passing protocol is reminiscent of a "Belief Propagation" (BP) scheme (e.g., [1] in a localization context), but is essentially different from BP, since the messages between the sensors relate only to position estimates of the beacons, and is not directly related to the unknown positions of the sensors passing the messages. We consider topologies in which the anchor sensors are not sufficiently connected among themselves to enable them to estimate the beacons' positions. Yet, through the collaboration of the other sensors, whose positions are not known *a-priori*, gradual refinement of the global information regarding the beacons' positions is attained, leading to successful self-localization of the entire network - as we shall demonstrate by simulation.

Our proposed scheme is "free running", in the sense that it does not require any pre-assignment of different roles to different sensors, or any advance knowledge of the network's connectivity - and thus accommodates *ad-hoc* configurations. No "hand-shaking" transmission procedure between sensors is needed and no fine synchronization is required. In each iteration, each sensor transmits (in its pre-assigned time-slots) its own current estimates of the positions of the beacons, along with an associated uncertainty covariance matrix, and the localization estimation is based on the "opportunistic" receptions of these messages by neighboring sensors. The relative timing-offsets of the timebases of all sensors are regarded as additional (nuisance) parameters, which are estimated along with the sensors' positions - as considered, e.g., in [9], [14], [1], [5]. Our estimation process basically relies on Weighted Least Squares (WLS) estimation, but at the same time takes a Bayesian flavor, in which arbitrary initial positions and time-offsets of most (non-anchor) nodes, as well as positions of the beacons, are initially provided as "fictitious measurements" with large uncertainty (poor accuracy), whereas just a few initial positions of (anchor) nodes are provided with low uncertainty (high accuracy). As the process evolves, the positioning accuracies of the non-anchor nodes improves (and, as by-products, so do the positioning accuracies of the beacons and the estimated time-offsets of all sensors).

2. PROBLEM FORMULATION

Consider a wireless sensors network consisting of N sensors (nodes), which are arbitrarily positioned at $\boldsymbol{q}_1, ..., \boldsymbol{q}_N \in \mathbb{R}^2$, most of which are unknown. An additional set of K beacons is arbitrarily positioned at unknown positions $\boldsymbol{p}_1, ..., \boldsymbol{p}_K \in \mathbb{R}^2$. These beacons are possibly a single flying beacon, transmitting from K unknown positions along its flight trajectory. In such a case the beacon's positions are actually in $\mathbb{R}^3$, however, we shall assume that its altitude is roughly known and is constant throughout the trajectory, and therefore only its 2D ground coordinates are assumed unknown.

The beacons transmit a sequence of wideband (possibly pulsed) signals at known intervals, synchronized to each other. For example, the beacon can be an airborne radar transmitting pulses at a known Pulse Repetition Interval (PRI). These pulses are transmitted in the course of flight of the beacon, and are thus transmitted form different positions. We denote the transmission time of the k-th beacon signal as t_k, where all $\{t_k\}_{k=1}^K$ are measured relative to the beacons' own timebase, thus, without loss of generality we may assume, e.g., $t_1 = 0$. Since the repetition intervals are assumed fixed and known, the relative transmission times $t_1, ..., t_K$ can be assumed known to the sensors. Each sensor measures the time of arrival of each beacon signal (relative to its own, unsynchronized timebase), and we denote the raw measured TOAs as

$$\widehat{t}^o_{n,k} = t_k + \tfrac{1}{c}\|\boldsymbol{q}_n - \boldsymbol{p}_k\| - \tau_n + v_{n,k} \quad {\scriptstyle n=1,...,N \;\; k=1,...,K}, \quad (1)$$

where c denotes the propagation speed, $\|\cdot\|$ denotes the Euclidean norm, τ_n denotes the unknown timing offset of the n-th sensor relative to the beacons' time-origin and $\{v_{n,k}\}$ denote zero-mean estimation-errors. We further assume that M of the N sensors (normally with $M \ll N$) are anchor sensor, with some prior knowledge available regarding their positions. Without loss of generality we shall assume that the anchor sensors are the first M sensors, so their positions are $\boldsymbol{q}_1, ..., \boldsymbol{q}_M$, which are known in advance up to some pre-specified precision. It is desired to estimate the positions of the sensors based on the NK TOA measurements, using the known positions of the M anchor sensors.

Since the estimation process involves the exchange of messages between the sensors, we need to assume some model for the connectivity within the network. To this end, let us assume a simple model, where each sensor has its own transmission power, so that messages transmitted by sensor n can be received up to range r_n from that sensor. For convenience let us denote a *connectivity indicators matrix* $\boldsymbol{G} \in \{0,1\}^{N\times N}$, such that $G_{m,n} = 1$ iff $\|\boldsymbol{q}_m - \boldsymbol{q}_n\| \le r_n$, $m \neq n$, indicating that sensor m can receive the messages transmitted by sensor n. Note that since different sensors may have different ranges, $\boldsymbol{G}$ is not necessarily symmetric.

3. COOPERATIVE ESTIMATION

Each sensor is characterized by three unknown parameters - its time-offset and 2D position, which we concatenate into vectors $\boldsymbol{\theta}_n \triangleq [\tau_n \;\; \boldsymbol{q}_n^T]^T \in \mathbb{R}^3$ for $n = 1, ..., N$. Each beacon is characterized by its 2D position (taking its possible altitude as known) $\boldsymbol{p}_k \in \mathbb{R}^2$ for $k = 1, ..., K$. We shall denote by $\boldsymbol{\phi} = [\boldsymbol{p}_1^T \;\cdots\; \boldsymbol{p}_K^T]^T \in \mathbb{R}^{2K}$ the concatenation of all the unknown beacons' positions.

By subtracting the known relative transmission times of the beacons $t_1, ..., t_K$ from the raw TOA measurements, each sensor can obtain a shift-eliminated measurement

$$\widehat{t}_{n,k} \triangleq \widehat{t}^o_{n,k} - t_k = \tfrac{1}{c}\|\boldsymbol{q}_n - \boldsymbol{p}_k\| - \tau_n + v_{n,k}. \quad (2)$$

Note that if a fixed, known altitude A of the beacons is involved, then $\|\boldsymbol{q}_n - \boldsymbol{p}_k\|$ should be interpreted as $\|\tilde{\boldsymbol{q}}_n - \tilde{\boldsymbol{p}}_k\|$, where $\tilde{\boldsymbol{q}}_n = [\boldsymbol{q}_n^T \;\; 0]^T$ and $\tilde{\boldsymbol{p}}_k = [\boldsymbol{p}_k^T \;\; A]^T$. Defining the function

$$h(\boldsymbol{\theta}_n, \boldsymbol{p}_k) \triangleq \tfrac{1}{c}\|\boldsymbol{q}_n - \boldsymbol{p}_k\| - \tau_n, \quad (3)$$

each sensor can model its K shift-eliminated TOA measurements as

$$\underbrace{\begin{bmatrix} \widehat{t}_{n,1} \\ \widehat{t}_{n,2} \\ \vdots \\ \widehat{t}_{n,K} \end{bmatrix}}_{\boldsymbol{t}_n} = \underbrace{\begin{bmatrix} h(\boldsymbol{\theta}_n, \boldsymbol{p}_1) \\ h(\boldsymbol{\theta}_n, \boldsymbol{p}_2) \\ \vdots \\ h(\boldsymbol{\theta}_n, \boldsymbol{p}_K) \end{bmatrix}}_{\boldsymbol{h}(\boldsymbol{\theta}_n, \boldsymbol{\phi})} + \underbrace{\begin{bmatrix} v_{n,1} \\ v_{n,2} \\ \vdots \\ v_{n,K} \end{bmatrix}}_{\boldsymbol{v}_n} \in \mathbb{R}^K \quad {\scriptstyle n=1,...,N}. \quad (4)$$

A plausible approach for each sensor n for estimating the unknown parameters $\boldsymbol{\theta}_n$ and $\boldsymbol{\phi}$ would be to seek the Least-Squares (LS) solution, minimizing $C_{LS}(\boldsymbol{\theta}_n, \boldsymbol{\phi}) \triangleq (\boldsymbol{t}_n - \boldsymbol{h}(\boldsymbol{\theta}_n, \boldsymbol{\phi}))^T(\boldsymbol{t}_n - \boldsymbol{h}(\boldsymbol{\theta}_n \boldsymbol{\phi}))$ with respect to (w.r.t.) $\boldsymbol{\theta}_n$ and $\boldsymbol{\phi}$, but obviously this would be an ill-posed problem (admitting infinitely many solutions) with K equations in $3 + 2K$ unknowns. Nevertheless, we note that if centralized processing (using a fusion center) were possible, all these measurements (from the N sensors combined) could be concatenated into KN equations in $3N + 2K$ unknowns, but these would still be ill-posed as long as they do not involve any additional information regarding the positions of the anchor sensors. An apparently straightforward remedy would then be to plug these $2M$ known parameters into the equations (4) for $n = 1, ..., M$, thereby reducing the number of remaining unknown parameters to $3N + 2K - 2M$ and hoping to gain uniqueness of the global minimum.

However (still in the framework of hypothetical centralized processing), an alternative approach, such as the one taken in [15], is also possible: Rather than regard the prior information on the anchors as known parameters, we would assume that additional "fictitious measurements" are available, directly providing the values of all unknown parameters, but with varying levels of uncertainty: the uncertainty of the position "measurements" would be arbitrarily large for most sensors, but negligibly small for the anchors. Likewise, we would assume fictitious measurements of the timing offsets and of the beacons' positions, initially with very large uncertainty. We would then seek a Weighted LS (WLS) solution, which would attribute proper relative weights to all the available measurements, "true" and "fictitious". The uncertainly levels would be expressed by the variances (or covariances) attributed to the fictitious measurements' errors, and the weighting matrix for the WLS solution would be the inverse of the overall covariance matrix. In fact, this approach can be regarded as a regularization strategy with a Bayesian flavor.

The very same approach can also be applied in our decentralized processing scheme: For each sensor n, the set of TOA measurements would be augmented with a set of direct "fictitious measurements" of the sensor's parameters $\boldsymbol{\theta}_n$, as well as with a set of fictitious measurements of the beacons' positions, obtained from estimates thereof, as announced by neighboring sensors. The various measurements will be associated with errors-covariance matrices, reflecting the attributed or computed uncertainty.

Upon initialization, these fictitious measurements and their associated variances / covariances would be the following:

- For the timing offsets τ_n - the fictitious measurements are denoted $\overline{\tau}_n$ and are all taken as zeros. Their associated variances are denoted σ_τ^2, taken to be of the (squared) order of the assumed range of synchronization errors.
- For the sensors' positions $\boldsymbol{q}_n$ - the fictitious measurements are denoted $\overline{\boldsymbol{q}}_n$. For anchor nodes we take $\overline{\boldsymbol{q}}_n$ to be their true, known positions, whereas for the other nodes we take $\overline{\boldsymbol{q}}_n$ to be at the center of the area of deployment of the sensors. The associated covariance matrices are denoted $\boldsymbol{C}_n$ and are taken to be of the form $\boldsymbol{C}_n = \overline{\sigma}_n^2 \boldsymbol{I}_{2\times 2}$, where $\boldsymbol{I}_{2\times 2}$ denotes an Identity matrix and where $\overline{\sigma}_n$ is small for the anchors, but large relative to the diameter of the deployment area for all other nodes. Note that using this strategy we may also allow "soft anchors" - sensors for which some *a-priori* position information is available, but with some level of uncertainty (as expressed by their covariance matrices).
- For the k-th beacon's position as viewed by the n-th sensor we take $\overline{\boldsymbol{p}}_n^{(k)}$ to be located at the center of the area of deployment. The associated covariance matrix is denoted $\overline{\boldsymbol{P}}_n^{(k)} = \overline{\sigma}_p^2 \boldsymbol{I}_{2\times 2}$, where $\overline{\sigma}_p$ is large relative to the diameter of the deployment area.

For convenience we define the following augmented fictitious measurements vectors and associated error-covariance matrices:

$$\overline{\boldsymbol{\theta}}_n \triangleq \begin{bmatrix} \overline{\tau}_n \\ \overline{\boldsymbol{p}}_n \end{bmatrix} \in \mathbb{R}^3 \quad \overline{\boldsymbol{C}}_n \triangleq \begin{bmatrix} \sigma_\tau^2 & 0 \\ \boldsymbol{0} & \boldsymbol{C}_n \end{bmatrix} \in \mathbb{R}^{3\times 3}, \tag{5}$$

$$\overline{\boldsymbol{\phi}}_n \triangleq \begin{bmatrix} \overline{\boldsymbol{p}}_n^{(1)} \\ \vdots \\ \overline{\boldsymbol{p}}_n^{(K)} \end{bmatrix} \in \mathbb{R}^{2K} \quad \overline{\boldsymbol{P}}_n \triangleq \begin{bmatrix} \overline{\boldsymbol{P}}_n^{(1)} & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \overline{\boldsymbol{P}}_n^{(K)} \end{bmatrix} \in \mathbb{R}^{2K\times 2K}. \tag{6}$$

Each iteration begins in a sequence of timed announcements by each of the sensors. On its turn to announce, sensor n announces its current estimate $\overline{\boldsymbol{\phi}}_n$ of the beacons' location vector $\boldsymbol{\phi}$, as well as the associated errors covariance matrix $\overline{\boldsymbol{P}}_n$. For ease of reference let us denote by $\mathcal{T}_n$ the set of all sensors whose transmissions can be received by sensor n, namely $\mathcal{T}_n = \{m | G_{n,m} = 1\}$, and let T_n denote the cardinality of this set. We may then denote the identities of these sensors as $m_n(1), m_n(2), ..., m_n(T_n)$, all comprising $\mathcal{T}_n$.

We are now ready to reformulate an augmented version of the basic measurement model (4) for the n-th sensor, accounting also for the fictitious measurements and for information regarding the beacons as received by this sensor from its T_n neighbors:

$$\underbrace{\begin{bmatrix} \boldsymbol{t}_n \\ \overline{\boldsymbol{\theta}}_n \\ \overline{\boldsymbol{\phi}}_n \\ \overline{\boldsymbol{\phi}}_{m_n(1)} \\ \overline{\boldsymbol{\phi}}_{m_n(2)} \\ \vdots \\ \overline{\boldsymbol{\phi}}_{m_n(T_n)} \end{bmatrix}}_{\boldsymbol{y}_n} = \underbrace{\begin{bmatrix} \boldsymbol{h}(\boldsymbol{\theta}_n, \boldsymbol{\phi}) \\ \boldsymbol{\theta}_n \\ \boldsymbol{\phi} \\ \boldsymbol{\phi} \\ \boldsymbol{\phi} \\ \vdots \\ \boldsymbol{\phi} \end{bmatrix}}_{\boldsymbol{g}(\boldsymbol{\theta}_n, \boldsymbol{\phi})} + \underbrace{\begin{bmatrix} \boldsymbol{v}_n \\ \boldsymbol{u}_n \\ \boldsymbol{e}_n \\ \boldsymbol{e}_{m_n(1)} \\ \boldsymbol{e}_{m_n(2)} \\ \vdots \\ \boldsymbol{e}_{m_n(T_n)} \end{bmatrix}}_{\boldsymbol{\epsilon}_n} \in \mathbb{R}^{K+3+2+2T_n}. \tag{7}$$

Here $\boldsymbol{u}_n$ and $\boldsymbol{e}_m$ denote the errors in $\overline{\boldsymbol{\theta}}_n$ and in $\overline{\boldsymbol{\phi}}_m$, respectively. In each iteration, each sensor attempts to solve a WLS problem, minimizing $C_{WLS}(\boldsymbol{\theta}_n, \boldsymbol{\phi}) \triangleq (\boldsymbol{y}_n - \boldsymbol{g}(\boldsymbol{\theta}_n, \boldsymbol{\phi}))^T \boldsymbol{W}_n (\boldsymbol{y}_n - \boldsymbol{g}(\boldsymbol{\theta}_n \boldsymbol{\phi}))$ w.r.t. $\boldsymbol{\theta}_n$ and $\boldsymbol{\phi}$, where $\boldsymbol{W}_n$ is an optimal weight matrix, given by the inverse of the covariance matrix of $\boldsymbol{\epsilon}_n$,

$$\boldsymbol{\Lambda}_n \triangleq \mathrm{Bdiag}\{\boldsymbol{\Gamma}_n, \overline{\boldsymbol{C}}_n, \overline{\boldsymbol{P}}_n, \overline{\boldsymbol{P}}_{m_n(1)}, ..., \overline{\boldsymbol{P}}_{m_n(T_n)}\}. \tag{8}$$

Here $\mathrm{Bdiag}\{\cdot\}$ denotes a block-diagonal matrix, and $\boldsymbol{\Gamma}_n$ denotes the covariance of the vector $\boldsymbol{v}_n$ of TOA measurements errors, which can usually be assumed uncorrelated and with equal variance $\forall n, k$.

This nonlinear problem can be solved using the Gauss-Newton (GN) method (e.g., [6]). The Jacobian of $\boldsymbol{g}(\boldsymbol{\theta}_n, \boldsymbol{\phi})$ is given by

$$\boldsymbol{G}_n(\boldsymbol{\theta}_n, \boldsymbol{\phi}) \triangleq \frac{\partial \boldsymbol{g}(\boldsymbol{\theta}_n, \boldsymbol{\phi})}{\partial(\boldsymbol{\theta}_n, \boldsymbol{\phi})} = \begin{bmatrix} \partial \boldsymbol{h}(\boldsymbol{\theta}_n, \boldsymbol{\phi})/\partial \boldsymbol{\theta}_n & \partial \boldsymbol{h}(\boldsymbol{\theta}_n, \boldsymbol{\phi})/\partial \boldsymbol{\phi} \\ \boldsymbol{I}_{3\times 3} & \boldsymbol{0}_{3\times 2K} \\ \boldsymbol{0}_{2K\times 3} & \boldsymbol{I}_{2K\times 2K} \\ \boldsymbol{0}_{2K\times 3} & \boldsymbol{I}_{2K\times 2K} \\ \vdots & \vdots \\ \boldsymbol{0}_{2K\times 3} & \boldsymbol{I}_{2K\times 2K} \end{bmatrix} \tag{9}$$

where

$$\frac{\partial \boldsymbol{h}(\boldsymbol{\theta}_n, \boldsymbol{\phi})}{\partial \boldsymbol{\theta}_n} = \begin{bmatrix} \partial h(\boldsymbol{\theta}_n, \boldsymbol{p}_1)/\partial \boldsymbol{\theta}_n \\ \vdots \\ \partial h(\boldsymbol{\theta}_n, \boldsymbol{p}_K)/\partial \boldsymbol{\theta}_n \end{bmatrix} \tag{10}$$

$$\frac{\partial \boldsymbol{h}(\boldsymbol{\theta}_n, \boldsymbol{\phi})}{\partial \boldsymbol{\phi}} = \begin{bmatrix} \partial h(\boldsymbol{\theta}_n, \boldsymbol{p}_1)/\partial \boldsymbol{p}_1 & & \boldsymbol{0}_{1\times 2} \\ & \ddots & \\ \boldsymbol{0}_{1\times 2} & & \partial h(\boldsymbol{\theta}_n, \boldsymbol{p}_K)/\partial \boldsymbol{p}_K \end{bmatrix} \tag{11}$$

$$\frac{\partial h(\boldsymbol{\theta}_n, \boldsymbol{p}_k)}{\partial \boldsymbol{\theta}_n} = \begin{bmatrix} -1 & \frac{1}{c} \cdot \frac{(\boldsymbol{q}_n - \boldsymbol{p}_k)^T}{\|\boldsymbol{q}_n - \boldsymbol{p}_k\|} \end{bmatrix} \tag{12}$$

$$\frac{\partial h(\boldsymbol{\theta}_n, \boldsymbol{p}_k)}{\partial \boldsymbol{p}_k} = -\frac{1}{c} \cdot \frac{(\boldsymbol{q}_n - \boldsymbol{p}_k)^T}{\|\boldsymbol{q}_n - \boldsymbol{p}_k\|}. \tag{13}$$

Given estimates $\widehat{\boldsymbol{\theta}}_n^{(i)}$ and $\widehat{\boldsymbol{\phi}}^{(i)}$ at the i-th GN iteration, the estimate at the next GN iteration would be given by

$$\begin{bmatrix} \widehat{\boldsymbol{\theta}}_n^{(i+1)} \\ \widehat{\boldsymbol{\phi}}^{(i+1)} \end{bmatrix} = \begin{bmatrix} \widehat{\boldsymbol{\theta}}_n^{(i)} \\ \widehat{\boldsymbol{\phi}}^{(i)} \end{bmatrix} + \left[\boldsymbol{G}_n^T(\widehat{\boldsymbol{\theta}}_n^{(i)}, \widehat{\boldsymbol{\phi}}^{(i)}) \boldsymbol{\Lambda}_n^{-1} \boldsymbol{G}_n^T(\widehat{\boldsymbol{\theta}}_n^{(i)}, \widehat{\boldsymbol{\phi}}^{(i)}) \right]^{-1} \cdot \boldsymbol{G}_n^T(\widehat{\boldsymbol{\theta}}_n^{(i)}, \widehat{\boldsymbol{\phi}}^{(i)}) \boldsymbol{\Lambda}_n^{-1} \left(\boldsymbol{y}_n - \boldsymbol{g}(\widehat{\boldsymbol{\theta}}_n^{(i)}, \widehat{\boldsymbol{\phi}}^{(i)}) \right). \tag{14}$$

As an initial guess, each sensor can use its available prior information, $\widehat{\boldsymbol{\theta}}_n^{(0)} = \overline{\boldsymbol{\theta}}_n$ and $\widehat{\boldsymbol{\phi}}^{(0)} = \overline{\boldsymbol{\phi}}_n$. After running a pre-defined number of GN iterations, each sensor has a refined estimate $\widehat{\boldsymbol{\theta}}_n$ of its own parameters (time offset and position), as well as an updated estimate $\widehat{\boldsymbol{\phi}}$ of the beacons' positions, along with updated uncertainties in the form of the respective blocks in the Mean Square Error (MSE) matrix, known (e.g., [6]) to be given (approximately) by

$$\boldsymbol{R}_n = \left[\boldsymbol{G}_n^T(\widehat{\boldsymbol{\theta}}_n, \widehat{\boldsymbol{\phi}})\boldsymbol{\Lambda}_n^{-1}\boldsymbol{G}_n^T(\widehat{\boldsymbol{\theta}}_n, \widehat{\boldsymbol{\phi}})\right]^{-1} \in \mathbb{R}^{(3+2K)\times(3+2K)}. \tag{15}$$

To share the updated estimates of the beacons' positions with its neighboring sensors, each sensor will then announce a new fictitious measurement $\overline{\boldsymbol{\phi}}_n = \widehat{\boldsymbol{\phi}}$, along with an associated updated covariance matrix $\overline{\boldsymbol{P}}_n$, given by the lower-right $2K \times 2K$ block of $\boldsymbol{R}_n$. In addition, each sensor would update its own fictitious measurement of its own parameters, $\overline{\boldsymbol{\theta}}_n = \widehat{\boldsymbol{\theta}}_n$, with its associated updated covariance $\overline{\boldsymbol{C}}_n$, given by the upper-left 3×3 block of $\boldsymbol{R}_n$. Consequently, once all messages are announced and received, each sensor will have a new set of measurements $\boldsymbol{y}_n$ and associated covariance matrix $\boldsymbol{\Lambda}_n$, so as to run a new sequence of GN iterations using the new $\overline{\boldsymbol{\theta}}_n$ and $\overline{\boldsymbol{\phi}}_n$ as an initial guess. The algorithm is summarized below.

Algorithm 1 Cooperative Self Localization with Unknown Beacons

Inputs: TOA measurements $\widehat{t}_{n,k}$ as per (2); Anchors positions $\boldsymbol{q}_m$.
Initialization: Initialize $\overline{\boldsymbol{\theta}}_n$ and $\overline{\boldsymbol{\phi}}_n$ and the associated covariance matrices $\overline{\boldsymbol{C}}_n$ and $\overline{\boldsymbol{P}}_n$ as in (5), (6), where all $\{\overline{\tau}_n\}_{n=1}^N$ are set to zeros, all $\{\overline{\boldsymbol{q}}_n\}_{n=1}^M$ take the anchors' positions, all $\{\overline{\boldsymbol{q}}_n\}_{n=M+1}^N$ and $\{\overline{\boldsymbol{p}}_k\}_{k=1}^K$ are set to the middle of the deployment zone.
  for $j = 1, 2, ...$ Number of desired global iterations **do**
    for all sensors $n = 1, ..., N$ **do**
      Announce (transmit) your id n, together with $\overline{\boldsymbol{\phi}}_n$ and $\overline{\boldsymbol{P}}_n$;
      Record $\overline{\boldsymbol{\phi}}_{m_n(1)}, ..., \overline{\boldsymbol{\phi}}_{m_n(T_n)}$ and $\overline{\boldsymbol{P}}_{m_n(1)}, ..., \overline{\boldsymbol{P}}_{m_n(T_n)}$ from all T_n sensors $m_n(1), ..., m_n(T_n) \in \mathcal{T}_n$ that you heard (received);
      Construct the measurements vector $\boldsymbol{y}_n$ as per (7);
      Construct the covariance matrix $\boldsymbol{\Lambda}_n$ as per (8);
      Set $\widehat{\boldsymbol{\theta}}_n^{(0)} := \overline{\boldsymbol{\theta}}_n$ and $\widehat{\boldsymbol{\phi}}^{(0)} := \overline{\boldsymbol{\phi}}_n$ and run a fixed number I of GN iterations using (14);
      Set $\overline{\boldsymbol{\theta}}_n := \widehat{\boldsymbol{\theta}}_n^{(I)}$ and $\overline{\boldsymbol{\phi}}_n := \widehat{\boldsymbol{\phi}}_n^{(I)}$;
      Compute $\boldsymbol{R}_n$ using (15) and set $\overline{\boldsymbol{C}}_n$ and $\overline{\boldsymbol{P}}_n$ to its upper-left 3×3 and lower-right $2K \times 2K$ blocks, respectively.
    end for
  end for
Outputs: Each sensor n has in $\overline{\boldsymbol{\theta}}_n$ its own estimates $\overline{\tau}_n$ and $\overline{\boldsymbol{q}}_n$ of its time-shift and position (resp.);
As a by-product, each sensor n has in $\overline{\boldsymbol{\phi}}_n$ its own estimates $\overline{\boldsymbol{p}}_1, ..., \overline{\boldsymbol{p}}_K$ of the beacons' positions.

To understand how this form of collaboration operates, we note that in the beginning of the process the only reliable data are the TOA measurements and the anchors' positions. As already mentioned, if the anchors were fully connected, they could collaborate directly and use this data to determine the beacons' position. Fortunately, even when they cannot communicate directly with each other, while none of them can determine the beacons' positions on its own, each can still obtain rough estimates of these positions up to differently-oriented uncertainties (estimation covariance): For example, given the TOA measurements, the uncertainties in the radial directions from each anchor are much smaller than those in the tangent directions. But the radial directions from different anchors are different, and therefore regular (non-anchor) sensors, which may receive information containing the uncertainty (covariance) matrices from several anchors, can apply proper geometrical weighting in fusing the data from these anchors (and from other neighboring sensors), so as to narrow-down the uncertainties in all directions. As the information from different anchors and sensors propagates within the network, the covariance matrices $\overline{\boldsymbol{C}}_n$ and $\overline{\boldsymbol{P}}_n$, which were initially diagonal, take different forms which reflect the reduction of uncertainties in the relevant directions for each sensor, so as to be properly exploited by the other sensors.

4. SIMULATION RESULTS

We simulated N sensors, four of which were anchor sensors placed at the four corners of a 20[Km] $\times$ 20[Km] square, and the rest uniformly and independently deployed within the square. K beacons were evenly deployed on a circle of radius 10[Km], centered about the square's center, at a fixed (known) altitude of 1[Km]. The transmission ranges $r_1, ..., r_4$ of the anchor sensors were all set to $19[Km]$, whereas the ranges $r_5, ..., r_N$ of each of the other sensors were uniformly and independently drawn between $5[Km]$ and $10[Km]$. This disables direct communication between the anchor sensors (each is 20[Km] away from the nearest anchor), and yields (empirically) an average connectivity level of approximately 32%. The synchronization offsets $\tau_1, ..., \tau_N$ of all sensors were independently drawn as $\tau_n \sim \mathcal{N}(0, \sigma_n^2)$ with $\sigma_n = 1$[Km]/c $\approx 3.3[\mu$S] $\forall$n. The TOA estimation errors $v_{m,n}$ were drawn as $v_{m,n} \sim \mathcal{N}(0, \sigma_v^2)$ with $\sigma_v = 30$[m]/c ≈ 100[nS].

We noted that sometimes the estimated covariance matrices tend to shrink too fast, thereby impeding the convergence. To circumvent this problem, we incorporated two heuristics into the algorithm: In forming $\overline{\boldsymbol{P}}_n$ we bounded its eigenvalues from below by 30^2[m^2]; And we incorporated a "refresh" stage once every 100 iterations, in which all $\overline{\boldsymbol{C}}_n$ and $\overline{\boldsymbol{P}}_n$ are reinstated to their initial values.

A short animated video clip (a Matlab® Movie from a simulation run), demonstrating the convergence of the position estimates to their true values, can be accessed online [16].

Table 1 shows numerical results taken from 100 independent trials, running 400 iterations per trial, with $I = 5$ GN iterations per each global iteration. We note that comparison to alternative algorithms could not be applied here in a meaningful way, due to incompatibility of the presumed operational scenario.

	N=25		N=50		N=100	
K	P	M	P	M	P	M
3	73%	235[m]	97%	158[m]	100%	163[m]
4	67%	87[m]	99%	85[m]	100%	124[m]

Table 1: P: percentage of successful trials (A successful trial is counted when for at least 90% of the sensors the error distance $\|\widehat{\boldsymbol{q}}_n - \boldsymbol{q}_n\|$ is smaller than 1[Km]); M: median error distance, averaged over the successful trials.

5. CONCLUSION

We proposed a collaborative, iterative estimation scheme for an asynchronous *ad-hoc* sensors' network measuring TOAs from beacons at unknown locations. The scheme is based on simple message-passing sequences, interlaced by iterative solutions of a WLS problem by each sensor. The opportunistic nature of the approach allows pre-programming of all sensors, with no need for prior knowledge of the eventual network topology.

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