LOW-RANK APPROXIMATION-BASED DISTRIBUTED NODE-SPECIFIC SIGNAL ESTIMATION IN A FULLY-CONNECTED WIRELESS SENSOR NETWORK

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ABSTRACT

In this paper, we consider the problem of distributed estimation of node-specific signals in a fully-connected wireless sensor network with multi-sensor nodes. The estimation relies on a data-driven design of a spatial filter, referred to as the generalized eigenvalue decomposition (GEVD)-based multichannel Wiener filter (MWF). In non-stationary or low-SNR conditions, this GEVD-based MWF has been demonstrated to be more robust than the original MWF due to an inherent GEVD-based low-rank approximation of the sensor signal correlation matrix. In a centralized realization where a fusion center has access to all the nodes' sensor signal observations, the network-wide sensor signal correlation matrix and its low-rank approximation can be directly estimated from the sensor signals. However, in this paper we aim to avoid centralizing the sensor signal observations, in which case this network-wide correlation matrix cannot be estimated. We introduce a distributed algorithm which is able to significantly compress the broadcast signals while still converging to the centralized GEVD-based MWF as if each node would have access to all sensor signal observations.

Index Terms— wireless sensor networks (WSNs), distributed estimation, low rank approximation, generalized eigenvalue decomposition (GEVD).

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1. INTRODUCTION

Most spatial filtering or beamforming techniques use a fixed sensor array with a limited number of (often closely-spaced) wired sensors, resulting in only a local sampling [1], [2]. An alternative could be to deploy a wireless sensor network (WSN) [3], [4], with a larger number of sensor nodes, to collect more diverse information of the spatial field.

To process the sensor signal observations of a WSN, one possibility is to collect them in a fusion center, which we refer to as a centralized approach. However, this centralized processing requires a large communication bandwidth and computational workload. In this paper, we aim for a distributed approach, in which the nodes cooperate to solve an estimation task by sharing compressed sensor signal observations, and by distributing the computational burden amongst them.

A large class of estimation problems in WSNs deals with the estimation of a common network-wide parameter or signal of interest [5–7], which basically means that all nodes collaborate to attain a global goal. In other estimation problems however, the parameters or signals of interest differ at each node, i.e., they are node-specific [8–11]. In some cases, these node-specific desired signals are related across the different nodes, e.g., when the signals of interest must be estimated as they are observed at a local sensor of each node to preserve the spatial properties in the signals [12–15].

The distributed adaptive node-specific signal estimation (DANSE) algorithm [16] is originally designed to estimate a node-specific desired signal at each node in a fully-connected WSN in a distributed fashion. In essence, DANSE can be viewed as a distributed realization of the centralized MWF and it considers the case where the node-specific desired signals share a common (unknown) latent signal subspace. By exploiting this common interest of the nodes, DANSE significantly compresses the broadcast signals while still converging to the centralized linear minimum mean square error (MMSE) estimators as if each node would have access to all sensor signal observations of the WSN.

Originally MWF has been designed based on a low-rank approximation of the signal correlation matrix with a so-called column decomposition [17], [18]. However, in low-SNR conditions, and for highly non-stationary noise in particular, the signal correlation matrix is often estimated poorly, which leads to suboptimal or even unstable filters [18]. Alternatively, either an eigenvalue decomposition (EVD)-based or a generalized EVD (GEVD)-based low-rank approximation of the signal correlation matrix can be applied to improve the estimation performance in such cases. MWF with GEVD-based low-rank approximation has been shown to deliver the best performance, as it effectively selects the "mode" corresponding to the highest SNR [18]. The resulting spatial filter is referred to as the GEVD-based MWF.

In this paper, the objective is to design a DANSE-like algorithm that computes the GEVD-based MWF in a distributed fashion in a fully-connected[1] WSN. We will refer to this as the GEVD-based DANSE algorithm. The proposed GEVD-based DANSE algorithm compresses the multi-sensor signals at each node into a smaller number of signal observations which are then broadcast to the other nodes. Remarkably, even though the GEVD-based DANSE algorithm is not able to compute the network-wide signal correlation matrix (and its GEVD) from these compressed signal observations, the algorithm does converge to the centralized GEVD-based MWF as if each node would have access to all (uncompressed) sensor signal observations.

The paper is organized as follows. The data model is presented in Section 2. The centralized GEVD-based MWF is explained in Section 3. The GEVD-based DANSE algorithm and its convergence analysis is addressed in Section 4. Numerical simulations are presented in Section 5. Finally conclusions are drawn in Section 6.

2. DATA MODEL AND MOTIVATION

We consider a fully-connected WSN with K multi-sensor nodes. Each node $k \in \mathcal{K} = \{1, \dots, K\}$ is assumed to collect observations of a complex-valued M_k-channel sensor signal $\mathbf{y}_k$. Note that this also allows for a hierarchical WSN where K master nodes collect sensor signal observations from M_k slave nodes with a single sensor. The sensor signal $\mathbf{y}_k$ can be modeled as

$$\mathbf{y}_k = \mathbf{d}_k + \mathbf{n}_k = \mathbf{A}_k \mathbf{s} + \mathbf{n}_k \qquad (1)$$

where $\mathbf{s}$ is a latent S-channel signal defining S latent source signals, $\mathbf{A}_k$ is an unknown $M_k \times S$ complex-valued steering matrix, and $\mathbf{n}_k$ is additive noise. The sensor signal $\mathbf{y}_k$ is assumed to satisfy short-term stationarity and ergodicity conditions. By stacking all $\mathbf{y}_k$, $\mathbf{n}_k$ and $\mathbf{d}_k$, we obtain the network-wide M-channel sensor signals $\mathbf{y}$, $\mathbf{d}$ and $\mathbf{n}$, respectively, where $M = \sum_{k=1}^{K} M_k$ and hence $\mathbf{y} = \mathbf{d} + \mathbf{n}$.

The goal for each node $k \in \mathcal{K}$ is to denoise all M_k channels of $\mathbf{y}_k$. Hence the desired signal to be estimated at each node is the M_k-channel signal $\mathbf{d}_k$. This means that the estimation procedure will preserve the node-specific spatial information in $\mathbf{d}_k$ while reducing the noise $\mathbf{n}_k$.

[1]It is noted that all results in this paper can be extended to tree topology networks, using similar strategies as in [19].

3. CENTRALIZED GEVD-BASED MWF

We first consider the centralized estimation problem. Therefore the objective for each node k is to estimate a complex-valued node-specific unknown M_k-channel desired signal $\mathbf{d}_k$, from the observations of all sensor signals in $\mathbf{y}$. Node k uses an $M \times M_k$ linear estimator $\hat{\mathbf{W}}_k$ to estimate $\mathbf{d}_k$ as $\hat{\mathbf{d}}_k = \hat{\mathbf{W}}_k^H \mathbf{y}$, where superscript H denotes the conjugate transpose operator and where the hat $\hat{(.)}$ refers to the fact that the centralized solution is considered. The MWF [17] computes $\hat{\mathbf{W}}_k$ based on the minimum mean square error (MMSE) criterion, such that

$$\hat{\mathbf{W}}_k^{\text{MMSE}} = \underset{\mathbf{W}_k}{arg\,min}\, E\Big\{ \big\| \mathbf{d}_k - \mathbf{W}_k^H \mathbf{y} \big\|^2 \Big\} \qquad (2)$$

where $E\{.\}$ is the expected value operator. Assuming $\mathbf{R}_{yy} = E\{\mathbf{y}\mathbf{y}^H\}$ has full rank, the unique solution of (2) is [17]:

$$\hat{\mathbf{W}}_k^{\text{MMSE}} = \mathbf{R}_{yy}^{-1} \mathbf{R}_{dd} \qquad (3)$$

where $\mathbf{R}_d = E\{\mathbf{d}\mathbf{d}^H\}$. We also define the network-wide noise covariance matrix $\mathbf{R}_{nn} = E\{\mathbf{n}\mathbf{n}^H\}$, where it is assumed that $\mathbf{R}_{nn}$ is either known a-priori or can be estimated from noise-only segments in the sensor signal observations. The latter can be performed in applications where the target signal has an on-off behavior, such as in speech enhancement where $\mathbf{R}_{yy}$ and $\mathbf{R}_{nn}$ can be estimated during "speech-and-noise" and "noise-only" segments, respectively, using a voice activity detection [17], [13]. The estimated correlation matrices will be denoted as $\bar{\mathbf{R}}_{yy}$, $\bar{\mathbf{R}}_{nn}$ and $\bar{\mathbf{R}}_{dd}$.

Assuming $\mathbf{d}$ and $\mathbf{n}$ are uncorrelated, the signal correlation matrix $\mathbf{R}_{dd}$ can be estimated as $\mathbf{R}_{yy} - \mathbf{R}_{nn}$. Note that in theory $\mathbf{R}_{dd}$ is a rank-S matrix, which can be verified by considering

$$\mathbf{R}_{dd} = E\{\mathbf{d}\mathbf{d}^H\} = \mathbf{A}\mathbf{\Psi}\mathbf{A}^H \qquad (4)$$

where $\mathbf{A}$ is the stacked version of all $\mathbf{A}_k$ steering matrices, and where $\mathbf{\Psi} = \text{diag}\{\psi_1, \dots, \psi_S\}$ is an S-dimensional diagonal matrix, where $\psi_s = E\{|s_t|^2\}$, with $t \in \{1, \dots, S\}$. In practice, however, the estimated $\bar{\mathbf{R}}_{dd}$ has generally a rank greater than S, and it may even not be positive semi-definite due to the subtraction $\bar{\mathbf{R}}_{yy} - \bar{\mathbf{R}}_{nn}$. In this case, it has been demonstrated in [18] that incorporating a low rank approximation based on either the eigenvalue decomposition (EVD) of $\bar{\mathbf{R}}_{dd}$ or the generalized eigenvalue decomposition (GEVD) of $\bar{\mathbf{R}}_{yy}$ and $\bar{\mathbf{R}}_{nn}$ enhances the estimation performance of the MWF, especially in low-SNR conditions. The GEVD-based low-rank approximation has been shown to deliver the best performance, as it effectively selects the "mode" corresponding to the highest SNR [18]. In the rest of this section, the GEVD-based MWF solution is explained in detail.

In order to perform a GEVD of the ordered matrix pair $(\bar{\mathbf{R}}_{yy}, \bar{\mathbf{R}}_{nn})$, each generalized eigenvector (GEVC) and its corresponding generalized eigenvalue (GEVL), $\mathbf{x}_m$ and λ_m $(m = 1 \dots M)$, respectively, must be computed such that $\bar{\mathbf{R}}_{yy}\mathbf{x}_m = \lambda_m \bar{\mathbf{R}}_{nn}\mathbf{x}_m$ [20], or equivalently

$$\bar{\mathbf{R}}_{yy}\mathbf{X} = \bar{\mathbf{R}}_{nn}\mathbf{X}\mathbf{\Lambda} \tag{5}$$

where $\mathbf{X} = [\mathbf{x}_1 ... \mathbf{x}_M]$ and $\mathbf{\Lambda} = \text{diag}\{\lambda_1 \dots \lambda_M\}$. Note that when $\bar{\mathbf{R}}_{nn}$ is invertible, (5) can be written as a non-symmetric EVD as

$$\bar{\mathbf{R}}_{nn}^{-1}\bar{\mathbf{R}}_{yy} = \mathbf{X}\mathbf{\Lambda}\mathbf{X}^{-1}. \tag{6}$$

In the sequel, we assume w.l.o.g. that the GEVLs in $\mathbf{\Lambda}$ are sorted in descending order. Since the GEVCs are defined up to a scaling, we assume w.l.o.g. that all $\mathbf{x}_m$'s are scaled such that $\mathbf{X}^H\bar{\mathbf{R}}_{nn}\mathbf{X} = \mathbf{I}_M$ where $\mathbf{I}_M$ denotes the $M \times M$ identity matrix. It is noted that the GEVD is equivalent to a joint diagonalization of $\bar{\mathbf{R}}_{yy}$ and $\bar{\mathbf{R}}_{nn}$, i.e., it can be verified from (6) that

$$\bar{\mathbf{R}}_{yy} = \mathbf{Q}\mathbf{\Sigma}\mathbf{Q}^H, \;\; \bar{\mathbf{R}}_{nn} = \mathbf{Q}\mathbf{\Gamma}\mathbf{Q}^H \tag{7}$$

where $\mathbf{Q} = \mathbf{X}^{-H}$ is a full-rank $M \times M$ matrix (not necessarily orthogonal), and where $\mathbf{\Sigma} = \text{diag}\{\sigma_1, \dots, \sigma_M\}$ and $\mathbf{\Gamma} = \text{diag}\{\gamma_1, \dots, \gamma_M\}$ are diagonal matrices. Note that (6) then implies that the GEVLs are equal to $\lambda_m = \frac{\sigma_m}{\gamma_m}$. Reconsidering $\mathbf{y} = \mathbf{d} + \mathbf{n}$ and (7)), it follows that

$$\bar{\mathbf{R}}_{dd} \;\; = \;\; \bar{\mathbf{R}}_{yy} - \bar{\mathbf{R}}_{nn} = \mathbf{Q}(\mathbf{\Sigma} - \mathbf{\Gamma})\mathbf{Q}^H = \mathbf{Q}\mathbf{\Delta}\mathbf{Q}^H \tag{8}$$

where $\mathbf{\Delta} = \text{diag}\{\delta_1, ..., \delta_M\}$ with $\delta_m = \sigma_m - \gamma_m$. The rank-$R$ approximation of $\bar{\mathbf{R}}_{dd}$ becomes $\mathbf{Q}\mathbf{\Delta}_R\mathbf{Q}^H$ with $\mathbf{\Delta}_R$ denoting the diagonal matrix $\mathbf{\Delta}$ with the $M - R$ smallest diagonal entries set to zero. Ideally (but not necessarily), R is set to $R = S$, which is motivated by (4). By replacing $\bar{\mathbf{R}}_{dd}$ with its rank-R approximation in (3), the GEVD-based MWF is defined as

$$\hat{\mathbf{W}}_k = \bar{\mathbf{R}}_{yy}^{-1}\mathbf{Q}\mathbf{\Delta}_R\mathbf{Q}^H\mathbf{E}_k \tag{9}$$

where $\mathbf{E}_k$ is a $M \times M_k$ matrix which selects the M_k columns corresponding to node k. The next section explains how the GEVD-based DANSE algorithm obtains the signal estimates $\hat{\mathbf{d}}_k = \hat{\mathbf{W}}_k^H\mathbf{y}$, i.e., the outputs of (9) in a decentralized fashion.

4. GEVD-BASED DANSE

In this section, we briefly introduce the GEVD-based DANSE algorithm to obtain the same node-specific solution (9) at each node $k \in \mathcal{K}$, without accessing to the full signal $\mathbf{y}$.

In GEVD-based DANSE, each node $k \in \mathcal{K}$ first optimally fuses its M_k-channel signal $\mathbf{y}_k$ into a J-channel signal $\mathbf{z}_k = \mathbf{F}_k^H\mathbf{y}_k$ with an $M_k \times J$ fusion matrix $\mathbf{F}_k$ (which will be defined later, see (13)), and then broadcasts observations of $\mathbf{z}_k$ to all other nodes. Consequently and compared to the centralized GEVD-based MWF, the algorithm reduces the required per-node communication bandwidth by a factor of $max\{(M_k/J), 1\}$.

Considering $\mathbf{z} = [\mathbf{z}_1^T \dots \mathbf{z}_K^T]^T$, $\mathbf{z}_{-k}$ denotes the vector $\mathbf{z}$ with $\mathbf{z}_k$ omitted. Each node k in GEVD-based DANSE has access to a P_k-channel signal $\widetilde{\mathbf{y}}_k$ which is defined as $\widetilde{\mathbf{y}}_k = [\mathbf{y}_k^T \;\; \mathbf{z}_{-k}^T]^T$, with $P_k = M_k + J(K-1)$. We use a similar notation for the desired and the noise component of $\widetilde{\mathbf{y}}_k$, i.e., $\widetilde{\mathbf{d}}_k$ and $\widetilde{\mathbf{n}}_k$.

In the DANSE algorithm [16], at iteration i, node q is the updating node where the local MMSE problem and its solution take the form (the iteration index i is omitted for conciseness):

$$\widetilde{\mathbf{W}}_q^{\text{MMSE}} = \underset{\widetilde{\mathbf{W}}_q}{arg\,min}\, E\left\{\left\|\mathbf{d}_q - \widetilde{\mathbf{W}}_q^H\widetilde{\mathbf{y}}_q\right\|^2\right\} \tag{10}$$

$$\widetilde{\mathbf{W}}_q^{\text{MMSE}} = (\bar{\mathbf{R}}_{\tilde{y}_q\tilde{y}_q})^{-1}\bar{\mathbf{R}}_{\tilde{d}_q\tilde{d}_q}\widetilde{\mathbf{E}} \tag{11}$$

(compare with (2)-(3)) where $\bar{\mathbf{R}}_{\tilde{y}_q\tilde{y}_q}$, $\bar{\mathbf{R}}_{\tilde{n}_q\tilde{n}_q}$ and $\bar{\mathbf{R}}_{\tilde{d}_q\tilde{d}_q}$ are the P_k-dimensional correlation matrices corresponding respectively to $\tilde{\mathbf{y}}_q$, $\tilde{\mathbf{n}}_q$ and $\tilde{\mathbf{d}}_q$ signals, and where $\widetilde{\mathbf{E}}$ is a $P_k \times M_k$ matrix which selects the first M_k columns of $\bar{\mathbf{R}}_{\tilde{d}_q\tilde{d}_q}$.

Similar to (5)-(8), here we locally perform a GEVD at node q on the matrix pair $\bar{\mathbf{R}}_{\tilde{y}_q\tilde{y}_q}$ and $\bar{\mathbf{R}}_{\tilde{n}_q\tilde{n}_q}$. This leads to the corresponding local P_k-dimensional matrices $\widetilde{\mathbf{X}}_q$, $\widetilde{\mathbf{\Lambda}}_q$, $\widetilde{\mathbf{Q}}_q$, $\widetilde{\mathbf{\Sigma}}_q$, $\widetilde{\mathbf{\Gamma}}_q$ and $\widetilde{\mathbf{\Delta}}_q$, where $\widetilde{\mathbf{Q}}_q = \widetilde{\mathbf{X}}_q^{-H}$. When replacing $\bar{\mathbf{R}}_{\tilde{d}_q\tilde{d}_q}$ by its GEVD-based rank-R approximation, solution (11) becomes

$$\widetilde{\mathbf{W}}_q = (\bar{\mathbf{R}}_{\tilde{y}_q\tilde{y}_q})^{-1}\widetilde{\mathbf{Q}}_q\widetilde{\mathbf{\Delta}}_{q_R}\widetilde{\mathbf{Q}}_q^H\widetilde{\mathbf{E}} \tag{12}$$

(compare with (9)) where $\widetilde{\mathbf{\Delta}}_{q_R}$ is the P_k-dimensional diagonal matrix $\widetilde{\mathbf{\Delta}}_q$ with the $P_k - R$ smallest diagonal entries set to zero. The aforementioned fusion rule $\mathbf{F}_q$ at node q is then chosen as

$$\mathbf{F}_q = \;\; \begin{bmatrix}\mathbf{I}_{M_q} & \mathbf{0}\end{bmatrix} \widetilde{\mathbf{W}}_q \begin{bmatrix}\mathbf{I}_J \\ \mathbf{0}\end{bmatrix}. \tag{13}$$

Finally node q estimates its node-specific M_k-channel desired signal as $\bar{\mathbf{d}}_q = \widetilde{\mathbf{W}}_q^H\widetilde{\mathbf{y}}_q$. The resulting GEVD-based DANSE algorithm is described in Table 1.

Note that the fusion rule defined in (13) is the same as in the original DANSE algorithm, but now $\widetilde{\mathbf{W}}_q$ is the result of a low-rank approximation-based MWF instead of a full-rank MWF. Due to this modification, the convergence proof of the original DANSE algorithm in [16] is not applicable anymore. Nevertheless, convergence of the GEVD-based DANSE algorithm can be proven under some technical conditions, as given by the following theorem:

Theorem I: *If $J = R$ and under some technical conditions, the GEVD-based DANSE algorithm converges for any initialization of its parameters to the centralized GEVD-based MWF solution, i.e., when $i \to \infty$, $\bar{\mathbf{d}}_k = \hat{\mathbf{d}}_k$.*

Proof: Omitted.

Remark I: The conditions stated above are as follows: 1) $\bar{\mathbf{R}}_{yy}$ is rank-M 2) $\widetilde{\mathbf{W}}_k^i, \forall k \in \mathcal{K}$ is rank-R in each iteration i of the GEVD-based DANSE algorithm. In practice, these conditions are usually satisfied . It is noted that both conditions are also required for the convergence of the original DANSE algorithm. However, the latter also requires strict conditions on the data model, i.e., the node-specific desired

Table 1. GEVD-based DANSE algorithm

1. Set $i \leftarrow 0$, $q \leftarrow 1$, and initialize all $\mathbf{F}_k^0$ and $\widetilde{\mathbf{W}}_k^0$, $\forall\, k \in \mathcal{K}$, with random entries.
2. Each node $k \in \mathcal{K}$ broadcasts the J-channel fused signal of its N new observations

$$\mathbf{z}_k[iN+j] = \mathbf{F}_k^{i\,H}\mathbf{y}_k[iN+j], \qquad j = 1 \ldots N \quad (14)$$

where the notation [.] denotes a sample index.

3. At node q:
   - Compute $\bar{\mathbf{R}}^i_{\tilde{y}_q\tilde{y}_q}$ and $\bar{\mathbf{R}}^i_{\tilde{n}_q\tilde{n}_q}$ via sample averaging.
   - Compute $\widetilde{\mathbf{Q}}^i_q$ and $\widetilde{\mathbf{\Delta}}^i_q$ from the GEVD of $(\bar{\mathbf{R}}^i_{\tilde{y}_q\tilde{y}_q}, \bar{\mathbf{R}}^i_{\tilde{n}_q\tilde{n}_q})$ similar to (5)-(8).
   - Compute the local MWF with rank-R approximation of $\bar{\mathbf{R}}^i_{\tilde{d}_q\tilde{d}_q}$ as follows:

$$\widetilde{\mathbf{W}}_q^{i+1} = (\bar{\mathbf{R}}^i_{\tilde{y}_q\tilde{y}_q})^{-1}\widetilde{\mathbf{Q}}^i_q\widetilde{\mathbf{\Delta}}^i_{q_R}\widetilde{\mathbf{Q}}^{i\,H}_q\widetilde{\mathbf{E}} \quad (15)$$

   - Update the fusion rule as

$$\mathbf{F}_q^{i+1} = \left[\mathbf{I}_{M_q} \;\; \mathbf{0}\right]\widetilde{\mathbf{W}}_q^{i+1}\begin{bmatrix}\mathbf{I}_J \\ \mathbf{0}\end{bmatrix} \quad (16)$$

4. Other nodes $k \in \mathcal{K} \setminus q$ update their parameters as $\widetilde{\mathbf{W}}_k^{i+1} = \widetilde{\mathbf{W}}_k^i$ and $\mathbf{F}_k^{i+1} = \mathbf{F}_k^i$.
5. Each node $k \in \mathcal{K}$ estimates its M_k-channel signal $\mathbf{d}_k$, as

$$\bar{\mathbf{d}}_k[iN+j] = \widetilde{\mathbf{W}}_k^{i+1}\tilde{\mathbf{y}}_k[iN+j] \quad (17)$$

6. $i \leftarrow i+1$ and $q \leftarrow (q \mod K)+1$ and return to step 2.

signals $\mathbf{d}_k$ should share a common latent signal subspace. Although the existence of such a subspace motivates the low-rank approximation of $\mathbf{R}_{dd}$ (see (4)), it is not a requirement as such for the GEVD-based DANSE algorithm to converge to the centralized GEVD-based MWF.

Remark II: It should be emphasized that for any choice of $J = R$, the GEVD-based DANSE algorithm converges to the centralized GEVD-based MWF, while only both are optimal in MMSE-sense when $J = R = S$. However note that it has been shown in [16] that the original DANSE algorithm only converges to the centralized MMSE-based MWF solution if $J = S$.

Remark III: It should be mentioned that GEVD-based DANSE can be shown to be equivalent (up to specific per-node transformations) to the DACGEE algorithm [21], based on an invariance-property of the GEVD with respect to row transformations. As a results, convergence of the latter can be exploited to prove convergence of the former. Although the proof of this relationship between both algorithms is not trivial, the mere fact that they are related may not be a complete surprise, since the GEVD-based MWF with rank-R approximation implicitly also computes a GEVD.

5. NUMERICAL SIMULATIONS

A Monte-Carlo (MC) simulation scenario with $K = 10$ nodes and $M_k = 15, \forall k \in \mathcal{K}$ is considered. The observations of the latent S-channel signal $\mathbf{s}$, and the entries of the $M_k \times S$ steering matrix $\mathbf{A}_k$, $\forall k \in \mathcal{K}$ are both independently drawn from a uniform distribution over the interval $[-0.5; 0.5]$. Two target sources ($S = 2$) as well as two localized noise sources are assumed, where the target sources have an on-off behavior, while the noise sources are continuously active. In order to model sensor noise as well (spatially uncorrelated components), $\mathbf{n}_k$ also contains an additive stochastic signal from which the observations are independently drawn from a uniform distribution over the interval $[-\sqrt{0.2}/2; \sqrt{0.2}/2]$.

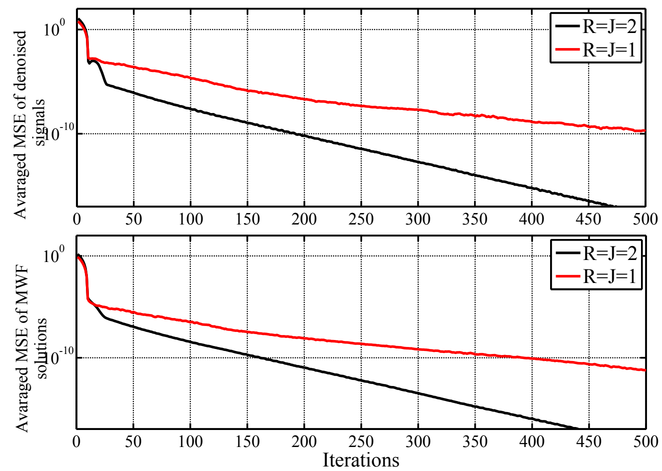

Fig. 1. Convergence of GEVD-based DANSE

Fig.1 illustrates the convergence results for two cases: 1) $J = R = 2$ and 2) $J = R = 1$, averaged over 200 MC runs ($S = 2$ in both cases). In the upper part, the mean squared errors (MSEs) between the entries of $\hat{\mathbf{d}}_k$ and $\bar{\mathbf{d}}_k$ (averaged over the nodes) are shown over the different iterations of the GEVD-based DANSE. Similarly, the bottom part illustrates the MSE between $\hat{\mathbf{W}}_k$ and the corresponding filters in the case of GEVD-based DANSE. It is observed that for both cases, the GEVD-based DANSE algorithm converges (with a random initialization of its parameters) to the centralized GEVD-based MWF solution. It is noted that the case $J = R = 2$ converges faster than the case $J = R = 1$, which can be explained by the larger number of degrees of freedom in each update step in the case of the former.

6. CONCLUSION

In this paper, we have proposed a distributed algorithm for the estimation of node-specific desired signals in a fully-connected wireless sensor network. The estimation has been based on a GEVD-based low-rank approximation of the correlation matrices within the MWF that is locally computed at each node. The resulting GEVD-based DANSE algorithm significantly compresses the broadcasting signals compared to a centralized approach with a fusion center. We have stated (without a proof) that the GEVD-based DANSE algorithm converges to the centralized GEVD-based MWF as if each node would have access to all the sensor signal observations.

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