A SIMPLE METHOD FOR DOA ESTIMATION IN THE PRESENCE OF UNKNOWN NONUNIFORM NOISE

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ABSTRACT

When considering the problem of direction-of-arrival (DOA) estimation, uniform noise is often assumed and hence, the corresponding noise covariance matrix is diagonal and has identical diagonal entries. However, this does not always hold true since the noise is nonuniform in certain applications and a model of arbitrary diagonal noise covariance matrix should be adopted. To this end, a simple approach to handling the unknown nonuniform noise problem is proposed. In particular, an iterative procedure is developed to determine the signal subspace and noise covariance matrix. As a consequence, existing subspace-based DOA estimators such as MUSIC can be applied. Furthermore, the proposed method converges within very few iterations, in each of which closed-form estimates of the signal subspace and noise covariance matrix can be achieved. Hence, it is much more computationally attractive than conventional methods which rely on multi-dimensional search. It is shown that the proposed method enjoys good performance, simplicity and low computational cost, which are desirable in practical applications.

***Index Terms*—** Direction-of-arrival (DOA) estimation, subspace estimation, nonuniform noise.

1. INTRODUCTION

Very often it is explicitly or implicitly assumed in direction-of-arrival (DOA) estimation that the background noise is unknown uniform white. Therefore, the noise covariance matrix is diagonal and has identical diagonal entries. In this case, the signal and noise subspaces can be simply separated according to the eigenvalues of the array covariance matrix. However, it has been shown that in certain practical applications, e.g., when the sensors are sparsely deployed, though the sensor noise is spatially white, the variances are not identical to each other [1]–[4]. This is referred to as nonuniform noise environment where the noise covariance matrix is still diagonal but the diagonal entries are no longer identical. Without appropriate preprocessing, the conventional subspace-based methods [8], [9] as well as the maximum-likelihood (ML) method [10] cannot offer satisfactory performance. As a result, much attention has been given to the problem of DOA estimation in nonuniform noise environments [1]–[7].

For instance, a deterministic nonuniform ML estimator was studied in [1]. This method is implemented by the so-called stepwise concentration of the log-likelihood function with respect to the signal and noise nuisance parameters. It was shown that this method converges fast, but needs to solve a highly nonlinear optimization problem in each iteration. As a result, a time-consuming multi-dimensional search should be performed in general. To reduce the complexity, another iterative algorithm, which can avoid the estimation of nuisance parameters, is proposed in [2]. However, this method is still time-consuming due to the optimization of the nonlinear problem in the power domain (PD). Unlike the above-mentioned methods, a computationally efficient method is presented in [3]. This methods has the ability to estimate the noise covariance matrix in closed-form and hence, the observations can be prewhitened. Nevertheless, it is limited to specific scenarios where the sensor number is larger than three times of the source number.

In this paper, a new approach to dealing with the unknown nonuniform noise is presented. The proposed method starts from an least-squares (LS) minimization problem with respect to the signal subspace and noise covariance matrix. A simple iterative procedure is then designed to achieve the solution. On one hand, different from the earlier iterative methods such as nonuniform ML estimator [1] and PD approach [2] which involve nonlinear optimization problems, the resultant problem in each iteration of the proposed method can be solved analytically. On the other hand, the proposed method is free of the limitation imposed by the method [3] and therefore can resolve much more sources. More precisely, for an array with M sensors, the proposed method can resolve $M-1$ sources while the prewhitening method [3] resolves $\lfloor M/3 \rfloor$ sources at most, where $\lfloor \cdot \rfloor$ denotes the floor function. Numerical examples are provided to evaluate its performance.

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2. PROBLEM FORMULATION

Consider an array with M sensors and L noncoherent narrowband signals income from far-field. It is assumed that $L < M$. The array output can be expressed as

$$\mathbf{x}(t) = \sum\nolimits_{l=1}^{L} \mathbf{a}(\theta_l)s_l(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where $\mathbf{a}(\theta_l) \in \mathbb{C}^M$ is the steering vector corresponding to the DOA of the lth source, i.e., θ_l, and the array geometry which is governed by the steering matrix $\mathbf{A} \in \mathbb{C}^{M\times L}$ composing of the steering vectors as follows

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_L)]. \quad (2)$$

In (1), $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_L(t)]^T \in \mathbb{C}^L$ is the vector of the signal waveforms and $\mathbf{n}(t) \in \mathbb{C}^M$ is the noise vector. The array covariance matrix is given by

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}\mathbf{P}\mathbf{A}^H + \mathbf{Q} \quad (3)$$

where $E\{\cdot\}$ denotes statistical expectation, $(\cdot)^H$ denotes Hermitian transpose, $\mathbf{P} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\} \in \mathbb{C}^{L\times L}$ and $\mathbf{Q} = E\{\mathbf{n}(t)\mathbf{n}^H(t)\} \in \mathbb{C}^{M\times M}$ are respectively the signal covariance matrix and noise covariance matrix. In practice, we have $\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N} \mathbf{x}(t)\mathbf{x}^H(t)$, where N is the snapshot number.

In this paper, nonuniform noise is considered. More precisely, it is assumed that the sensor noise is a spatially and temporally uncorrelated zero-mean Gaussian process with covariance matrix given by

$$\mathbf{Q} = \mathrm{diag}\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_M^2\} \quad (4)$$

where $\mathrm{diag}\{\cdot\}$ denotes a diagonal matrix composed of the bracketed elements, and σ_m^2 denotes the noise variance of the mth sensor. In particular, when $\sigma_1^2 = \sigma_2^2 = \cdots = \sigma_M^2 = \sigma^2$, we have $\mathbf{Q} = \sigma^2\mathbf{I}$ and the above nonuniform noise model is reduced to uniform noise model. In the following section, the problem of estimating the DOAs given the nonuniform noise model in (4) is addressed.

3. PROPOSED DOA ESTIMATOR

In a uniform noise environment, it is well known that the signal and noise subspaces can be obtained through the eigendecomposition of the array covariance matrix $\mathbf{R}$. However, this is not directly applicable to the nonuniform case, where the eigenvectors associated with the L largest eigenvalues do not span the signal subspace any more. To tackle this problem, a simple method is derived in this section to estimate the signal subspace and noise covariance matrix, which are then utilized for DOA estimation.

Under the assumption of noncoherent signals, the signal covariance matrix $\mathbf{P}$ has full rank and hence can be factorized, say using Cholesky decomposition, as follows

$$\mathbf{P} = \mathbf{L}\mathbf{L}^H \quad (5)$$

where $\mathbf{L} \in \mathbb{C}^{L\times L}$ is a nonsingular matrix. As a result, the array covariance matrix in (3) can be rewritten as

$$\mathbf{R} = \mathbf{A}\mathbf{L}\mathbf{L}^H\mathbf{A}^H + \mathbf{Q} = \mathbf{B}\mathbf{B}^H + \mathbf{Q} \quad (6)$$

where $\mathbf{B} = \mathbf{A}\mathbf{L} \in \mathbb{C}^{M\times L}$. Note that $\mathbf{L}$ is nonsingular and therefore $\mathbf{B}$ spans the same column space as the steering matrix $\mathbf{A}$, which corresponds to the signal subspace. This implies that $\mathbf{B}$ also spans the signal subspace, and hence, if it has been obtained, the conventional subspace-base methods can be applied directly to estimate the DOAs.

Based on the above observation, once we have obtained the array covariance matrix estimate $\hat{\mathbf{R}}$, an LS minimization problem with respect to $\mathbf{B}$ and $\mathbf{Q}$ can be formulated as follows

$$\begin{aligned} &\min_{\mathbf{B},\mathbf{Q}} \|\hat{\mathbf{R}} - \mathbf{B}\mathbf{B}^H - \mathbf{Q}\|_F^2 \\ &\text{s.t. } \mathbf{Q} \in \mathbb{D}^{L\times L} \end{aligned} \quad (7)$$

where $\|\cdot\|_F$ denotes the Frobenius norm and $\mathbb{D}$ denotes the set of diagonal matrices. It should be mentioned that the optimization problem in (7) has no unique solution. This is because for any nonsingular matrix $\mathbf{T} \in \mathbb{C}^{L\times L}$ which satisfies $\mathbf{T}\mathbf{T}^H = \mathbf{I}$, we have

$$\mathbf{B}\mathbf{B}^H = \mathbf{B}\mathbf{T}\mathbf{T}^H\mathbf{B}^H = \tilde{\mathbf{B}}\tilde{\mathbf{B}}^H, \tilde{\mathbf{B}} \triangleq \mathbf{B}\mathbf{T} \quad (8)$$

and hence either $(\mathbf{B}, \mathbf{Q})$ or $(\tilde{\mathbf{B}}, \mathbf{Q})$ is the solution to (7). Fortunately, though there exists a certain rotation between $\mathbf{B}$ and $\tilde{\mathbf{B}}$, they span the same column space. Thus, such a rotation does not affect the DOA estimation. To avoid the problem of infinite many solutions, an additional constraint, i.e., $\mathbf{B}^H\mathbf{B}$ is diagonal, is imposed. Consequently, the problem of signal subspace and noise covariance matrix estimation is given by

$$\begin{aligned} &\min_{\mathbf{B},\mathbf{Q}} \|\hat{\mathbf{R}} - \mathbf{B}\mathbf{B}^H - \mathbf{Q}\|_F^2 \\ &\text{s.t. } \mathbf{Q} \in \mathbb{D}^{L\times L},\ \mathbf{B}^H\mathbf{B} \in \mathbb{D}^{L\times L}. \end{aligned} \quad (9)$$

In order to solve the minimization problem in (9) subject to the constraints that $\mathbf{B}^H\mathbf{B}$ and $\mathbf{Q}$ are diagonal matrices, we first rewrite the objective function as

$$\begin{aligned} f(\mathbf{B}, \mathbf{Q}) &= \|\hat{\mathbf{R}} - \mathbf{B}\mathbf{B}^H - \mathbf{Q}\|_F^2 \\ &= \mathrm{tr}\{(\hat{\mathbf{R}} - \mathbf{B}\mathbf{B}^H - \mathbf{Q})(\hat{\mathbf{R}} - \mathbf{B}\mathbf{B}^H - \mathbf{Q})^H\} \\ &= \mathrm{tr}\{\hat{\mathbf{R}}^2\} - 2\mathrm{tr}\{\mathbf{B}^H\hat{\mathbf{R}}\mathbf{B}\} - 2\mathrm{tr}\{\mathbf{Q}\hat{\mathbf{R}}\} \\ &\quad + 2\mathrm{tr}\{\mathbf{B}^H\mathbf{Q}\mathbf{B}\} + \mathrm{tr}\{\mathbf{B}\mathbf{B}^H\mathbf{B}\mathbf{B}^H\} + \mathrm{tr}\{\mathbf{Q}^2\} \end{aligned} \quad (10)$$

where $\mathrm{tr}\{\cdot\}$ denotes the matrix trace operator. In (10), we have used the identities $\|\mathbf{X}\|_F^2 = \mathrm{tr}\{\mathbf{X}\mathbf{X}^H\}$, $\mathrm{tr}\{\mathbf{X}\mathbf{Y}\} = \mathrm{tr}\{\mathbf{Y}\mathbf{X}\}$, $\hat{\mathbf{R}} = \hat{\mathbf{R}}^H$, and $\mathbf{Q} = \mathbf{Q}^H$. It can be derived that the partial derivatives of $f(\mathbf{B}, \mathbf{Q})$ with respect to $\mathbf{Q}$ and $\mathbf{B}^*$ are respectively given by

$$\frac{\partial f(\mathbf{B}, \mathbf{Q})}{\partial \mathbf{Q}} = -2\mathcal{D}\{\hat{\mathbf{R}}\} + 2\mathcal{D}\{\mathbf{B}\mathbf{B}^H\} + 2\mathbf{Q} \quad (11a)$$

$$\frac{\partial f(\mathbf{B}, \mathbf{Q})}{\partial \mathbf{B}^*} = -2\hat{\mathbf{R}}\mathbf{B} + 2\mathbf{Q}\mathbf{B} + 2\mathbf{B}\mathbf{B}^H\mathbf{B} \quad (11b)$$

where $\mathcal{D}\{\mathbf{X}\}$ denotes a diagonal matrix composed of the diagonal elements of $\mathbf{X}$, and $(\cdot)^*$ is the conjugate operator. Due to space limitation, a detailed derivation of the above partial derivatives is omitted.

Let the derivative in (11a) be zero, i.e.,

$$-2\mathcal{D}\{\hat{\mathbf{R}}\} + 2\mathcal{D}\{\mathbf{B}\mathbf{B}^H\} + 2\mathbf{Q} = \mathbf{0} \tag{12}$$

and take the constraint that $\mathbf{Q}$ is diagonal into account, one gets the solution to (12) with respect to $\mathbf{B}$ as follows

$$\mathbf{Q} = \mathcal{D}\{\hat{\mathbf{R}} - \mathbf{B}\mathbf{B}^H\}. \tag{13}$$

As a result, for a given $\mathbf{B}$, the objective function f is minimized if $\mathbf{Q}$ is given by (13).

On the other hand, to minimize the objective function f for a given $\mathbf{Q}$, we let the derivative in (11b) be zero, i.e.,

$$-2\hat{\mathbf{R}}\mathbf{B} + 2\mathbf{Q}\mathbf{B} + 2\mathbf{B}\mathbf{B}^H\mathbf{B} = \mathbf{0} \tag{14}$$

which is equivalent to

$$(\hat{\mathbf{R}} - \mathbf{Q})\mathbf{B} = \mathbf{B}(\mathbf{B}^H\mathbf{B}). \tag{15}$$

Recalling the constraint that $\mathbf{B}^H\mathbf{B}$ is an $L \times L$ diagonal matrix, it can be concluded that $\mathbf{B}$ is a matrix consisting of L eigenvectors of $\hat{\mathbf{R}} - \mathbf{Q}$, and the diagonal entries of $\mathbf{B}^H\mathbf{B}$ are the corresponding eigenvalues. Let us define

$$\mathbf{\Sigma} \triangleq \mathbf{B}^H\mathbf{B} \tag{16}$$

such that it is a diagonal matrix containing L eigenvectors of $\hat{\mathbf{R}} - \mathbf{Q}$. Then, substituting (15) and (16) into (10), one gets

$$\begin{aligned} f(\mathbf{B}) =& \mathrm{tr}\{\hat{\mathbf{R}}^2 + \mathbf{Q}^2 - 2\mathbf{Q}\hat{\mathbf{R}}\} \\ &- \mathrm{tr}\{2\mathbf{B}^H\hat{\mathbf{R}}\mathbf{B} - 2\mathbf{B}^H\mathbf{Q}\mathbf{B} - \mathbf{B}\mathbf{B}^H\mathbf{B}\mathbf{B}^H\} \\ =& \mathrm{tr}\{\hat{\mathbf{R}}^2 + \mathbf{Q}^2 - 2\mathbf{Q}\hat{\mathbf{R}}\} - \mathrm{tr}\{\mathbf{\Sigma}^2\}. \end{aligned} \tag{17}$$

From (17), it can be seen that f is minimized if and only if $\mathbf{\Sigma}$ contains the L largest (principal) eigenvalues of $\hat{\mathbf{R}} - \mathbf{Q}$.

To proceed, we express the eigendecomposition of $\hat{\mathbf{R}} - \mathbf{Q}$ as $\hat{\mathbf{R}} - \mathbf{Q} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_M]$ is composed of the eigenvectors and $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, ..., \lambda_M\}$ is a diagonal matrix composed of the eigenvalues with $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_M$. Moreover, we have $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. Since $\mathbf{\Sigma}$ contains the L principal eigenvalues, we denote

$$\mathbf{\Sigma} = \mathrm{diag}\{\lambda_1, \lambda_2, ..., \lambda_L\} \tag{18}$$

and

$$\mathbf{U}_P = [\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_L]. \tag{19}$$

From (15)–(19), one gets $(\hat{\mathbf{R}} - \mathbf{Q})\mathbf{U}_P = \mathbf{U}_P\mathbf{\Sigma}$ and hence, $(\hat{\mathbf{R}} - \mathbf{Q})(\mathbf{U}_P\mathbf{\Sigma}^{1/2}) = (\mathbf{U}_P\mathbf{\Sigma}^{1/2})(\mathbf{U}_P\mathbf{\Sigma}^{1/2})^H(\mathbf{U}_P\mathbf{\Sigma}^{1/2})$. As a result, if we take

$$\mathbf{B} = \mathbf{U}_P\mathbf{\Sigma}^{1/2} \tag{20}$$

Table 1. Proposed method for subspace estimation

1.	compute $\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}(t)\mathbf{x}^H(t)$ from N snapshots
2.	initialization: $k = 0$, $\mathbf{Q}_{(0)} = (\mathcal{D}(\hat{\mathbf{R}}^{-1}))^{-1}$
3.	**do**
4.	$\quad \mathbf{B}_{(k)} = \mathbf{U}_{P(k)}\mathbf{\Sigma}_{(k)}^{1/2}$
5.	$\quad \mathbf{Q}_{(k)} = \mathcal{D}\{\hat{\mathbf{R}} - \mathbf{B}_{(k)}\mathbf{B}_{(k)}^H\}$
6.	$\quad \mathbf{U}_{(k)}\mathbf{\Lambda}_{(k)}\mathbf{U}_{(k)}^H = \hat{\mathbf{R}} - \mathbf{Q}_{(k)}$
7.	$\quad \mathbf{\Sigma}_{(k)} = \mathrm{diag}\{\lambda_{1(k)}, \lambda_{2(k)}, ..., \lambda_{L(k)}\}$
8.	$\quad \mathbf{U}_{P(k)} = [\mathbf{u}_{1(k)}, \mathbf{u}_{2(k)}, ..., \mathbf{u}_{L(k)}]$
9.	$\quad k = k + 1$
10.	**while** $\|\|\mathbf{Q}_{(k)} - \mathbf{Q}_{(k-1)}\|\|_F^2 \leq \epsilon$
11.	return $\mathbf{B}_{(k)}$ and $\mathbf{Q}_{(k)}$

then (15) and (16) are satisfied and, subsequently, f is minimized. This implies that, for a given $\mathbf{Q}$, the solution which minimizes f is given by (20). From the above analysis, it can be found that $\mathbf{B}$ and $\mathbf{Q}$ can be estimated in an iterative manner. The proposed algorithm is summarized in Table 1.

It should be noticed that in Table 1 the iteration procedure is terminated when the condition $||\mathbf{Q}_{(k)} - \mathbf{Q}_{(k-1)}||_F^2 \leq \epsilon$ is reached and ϵ is a prescribed value. Furthermore, the algorithm can be initialized by other values such as $\mathbf{Q}_{(0)} = \mathbf{I}$ instead of $\mathbf{Q}_{(0)} = (\mathcal{D}(\hat{\mathbf{R}}^{-1}))^{-1}$. However, it is experimentally found that the choice of these initializations only slightly affects the convergence speed and hence, the choice $\mathbf{Q}_{(0)}$ becomes less crucial. It should be also mentioned that, theoretically, the algorithm presented above converges to a local solution. Fortunately, in our extensive simulations, it is observed that the proposed method can always provide a satisfactory solution regardless of the initializations above-mentioned.

Now, let $\hat{\mathbf{B}}$ and $\hat{\mathbf{Q}}$ be the estimated signal subspace and noise covariance matrix, it is known that the MUSIC algorithm can be applied to determine the DOAs. Note that, $\hat{\mathbf{B}}$ is given by $\hat{\mathbf{B}} = \hat{\mathbf{U}}_P\hat{\mathbf{\Sigma}}^{1/2}$ and $\hat{\mathbf{U}}_P$ is an orthonormal matrix. Therefore, the following spatial spectrum can be employed

$$G_1(\theta) = \Big(\mathbf{a}(\theta)(\mathbf{I} - \hat{\mathbf{U}}_P\hat{\mathbf{U}}_P^H)\mathbf{a}^H(\theta)\Big)^{-1}. \tag{21}$$

Alternatively, if the estimate of noise covariance matrix is available, it can used for prewhitening. In other words, we define a prewhitened array covariance matrix and steering vector as $\mathbf{R}_W = \mathbf{Q}^{-1/2}\hat{\mathbf{R}}(\mathbf{Q}^{-1/2})^H = \mathbf{Q}^{-1/2}\mathbf{A}\mathbf{P}(\mathbf{Q}^{-1/2}\mathbf{A})^H + \mathbf{I}$ and $\mathbf{a}_W(\theta) = \mathbf{Q}^{-1/2}\mathbf{a}(\theta)$, respectively. The noise subspace can thus be computed from $\mathbf{R}_W$ and the DOAs can be found from the spatial spectrum

$$G_2(\theta) = \|\hat{\mathbf{U}}_{WN}^H\mathbf{a}_W(\theta)\|^{-2} \tag{22}$$

where $\hat{\mathbf{U}}_{WN}$ represents the noise subspace which is obtained from $\hat{\mathbf{R}}_W = \hat{\mathbf{Q}}^{-1/2}\hat{\mathbf{R}}\hat{\mathbf{Q}}^{-1/2}$. It is worth mentioning that (21) and (22) may lead to slightly different results as shown in the next section. One possible explanation is that $\hat{\mathbf{U}}_P$ and $\hat{\mathbf{Q}}$ do not have exactly same level of accuracy in general.

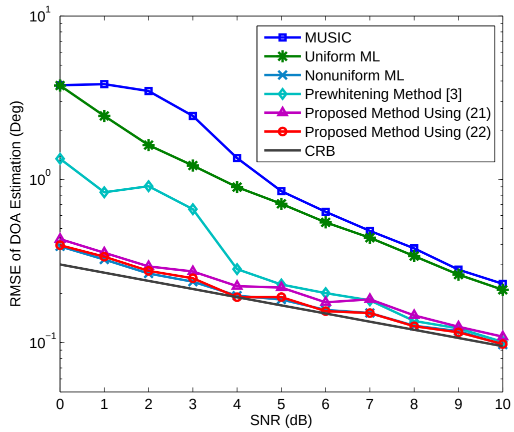

Fig. 1. RMSE of DOA estimation versus SNR.

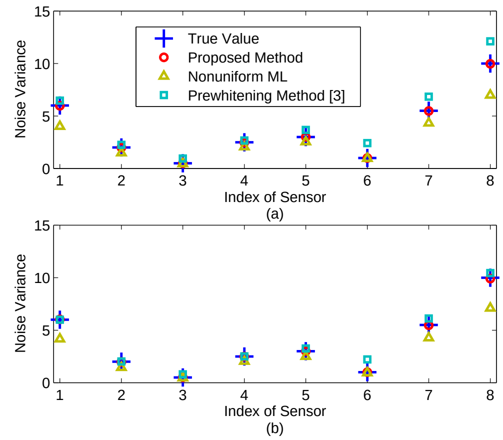

Fig. 2. Averaged noise variance estimation when (a) $\mathrm{SNR} = 0\mathrm{dB}$ and (b) $\mathrm{SNR} = 5\mathrm{dB}$.

4. SIMULATION RESULTS

In this section, the performance of the proposed method is evaluated. A uniform linear array with $M = 8$ sensors separated by half wavelength was performed. It is assumed that two uncorrelated narrowband signals with equal power impinge on the array from far-field. The DOAs of them are assumed to be $\theta_1 = -3^\circ$ and $\theta_2 = 6^\circ$. The nonuniform noise covariance matrix is $\mathbf{Q} = \mathrm{diag}\{6.0, 2.0, 0.5, 2.5, 3.0, 1.0, 5.5, 10.0\}$ and the signal-to-noise ratio (SNR) is defined as [1] $\mathrm{SNR} = \frac{\sigma_s^2}{M}\sum_{m=1}^{M}\frac{1}{\sigma_m^2}$, where σ_s^2 denotes the signal power.

In our simulations, the proposed method is initialized with $\mathbf{Q}^{(0)} = (\mathcal{D}(\hat{\mathbf{R}}^{-1}))^{-1}$, and the termination criteria is $\epsilon = 10^{-3}$. To examine the convergence speed, assume that $N = 500$ and $\mathrm{SNR} = 5\mathrm{dB}$. Moreover, the algorithm is also initialized with $\mathbf{Q}^{(0)} = \mathbf{I}$ for comparison. It is found that for these two choices, the algorithm converges within 3 and 6 iterations, respectively. This implies that though, to some extent, the choice of initialization affects the convergence speed, the algorithm converges very fast in general.

Next, we keep the number of snapshots to 500 and evaluate the performance of the proposed method at different SNRs. A total of 100 Monte Carlo experiments are run at each SNR and the corresponding root-mean-square error (RMSE) of DOA estimation is calculated as

$$\mathrm{RMSE} = \sqrt{\frac{1}{KL}\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\hat{\theta}_{k,l} - \theta_l\right)^2} \tag{23}$$

where $K = 100$ is the number of Monte Carlo experiments, and $\hat{\theta}_{k,l}$ is the estimated DOA of the lth signal in the kth experiment. For comparison, we tested several existing methods including the MUSIC algorithm, uniform ML method, nonuniform ML estimator, and prewhitening method [3]. The deterministic CRB (see (31), [1]) is also shown. The proposed method using (21) and (22) are both evaluated.

Fig. 1 shows the resultant RMSEs of DOA estimation using different estimators. It can be seen that neither the MUSIC algorithm nor the uniform ML estimator can offer satisfactory performance. However, the performance can be considerably improved by the nonuniform ML estimator and our proposed method. A careful examination also shows that the nonuniform ML estimator and our proposed method perform similarly. In particular, the proposed method using (22) performs almost the same as the nonuniform ML estimator. However, it is more computationally efficient. Additionally, though the prewhitening method [3] shows some improvements over the MUSIC and uniform ML estimator, it can only provide comparable performance to the proposed method at high SNRs.

Besides the performance of DOA estimation, we also evaluate the performance of noise variance estimation. Fig. 2 shows the estimated noise variances averaged from 100 experiments at two selected SNRs, i.e., $\mathrm{SNR} = 0\mathrm{dB}$ and $\mathrm{SNR} = 5\mathrm{dB}$. Interestingly, it is found that the proposed method also gives a better performance in noise variance estimation than the existing methods tested.

5. CONCLUSION

In this paper, a new and simple method for DOA estimation in the presence of unknown nonuniform noise is proposed. It is shown that the signal subspace and noise covariance matrix can be estimated in closed-form and the DOAs can be found by conventional computationally efficient subspace-based direction finding algorithms. Simulation results illustrate that on one hand the proposed method offers similar RMSE of DOA estimation to the nonuniform ML estimator but with a much lower complexity. on the other hand, it achieves higher accuracy in noise variance estimation than existing methods.

6. REFERENCES

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