ON TRANSMIT BEAMFORMING IN MIMO RADAR WITH MATRIX COMPLETION

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ABSTRACT

The paper proposes a matrix completion based colocated MIMO radar (MIMO-MC) approach that employs transmit beamforming. The transmit antennas transmit correlated waveforms to illuminate certain directions. Each receive antenna performs sub-Nyquist sampling of the target returns at uniformly random times, and forwards the samples to a fusion center along with information on the sampling times. Based on the forwarded samples, the fusion center partially fills a matrix, recovers the Nyquist rate samples via matrix completion, and subsequently proceeds with target estimation via standard techniques. The performance of matrix completion depends on the matrix coherence. The paper derives the relations between transmit waveforms and matrix coherence. Specifically, it is shown that, for a rank-1 beamformer, the coherence is optimal, i.e., 1, if and only if the waveforms are unimodular. For a multi-rank beamformer, the coherence of the row space of the data matrix is optimal if the waveform power is constant across each snapshot. Simulation results show that the proposed scheme achieves high resolution with a significantly reduced number of samples.

Index Terms— MIMO radar, matrix completion, transmit beamforming, coherence

1. INTRODUCTION

A colocated MIMO radar approach based on matrix completion (MC) [1] [2] [3] (MIMO-MC radar) has been recently proposed in [4] [5] to achieve the high resolution of MIMO radars while requiring significantly fewer samples to be collected and forwarded to a fusion center. Based In MIMO-MC radars, each receive antenna obtains samples at uniformly random times and forwards them to a fusion center, which partially fills a matrix, referred to as the data matrix. The matrix can be subsequently recovered via matrix completion techniques. As shown in [4] [6],[7] and [8],[9] the transmit/receive array configuration as well as transmit waveforms affect the matrix coherence and thus the MC performance.

Most of the work on MIMO radars assumes the transmission of uncorrelated waveforms from the transmit antennas. However, for MIMO radars operating in tracking mode, that considers correlated transmit waveforms [10] [11]. For example, in [10], the transmit waveforms correlation is designed so that a desired transmit beampattern is achieved. In [11], the authors proposed a phased-MIMO radar approach by dividing the transmit array into multiple sub-arrays, with each sub-array coherently transmitting a waveform which is orthogonal to waveforms transmitted by other sub-arrays. Thus, in each sub-array, beamforming is achieved. A multi-rank beamformer for MIMO radars has been recently proposed in [12], which, unlike [10] does not require solving a complicated optimization problem. The multi-rank beamformer is taken as the combination of rank-1 beamformers with the corresponding multiple waveforms chosen to be orthogonal.

In this paper we consider the same MIMO radar transmit beamforming framework as in [12]. When the number of illuminated targets is much smaller than the size of receive array, the data matrix formulated by the fusion center based on Nyquist-rate samples at the receive antennas is low-rank. Therefore, Nyquist sampling is not required at each receive antenna. Instead, the antenna can uniformly at random select samples and forward them to the fusion center, thus partially filling the data matrix. By applying MC, the fusion center can recover the full matrix. Based on the recovered data matrix, various methods, e.g., MUSIC [13], can be employed for target estimation. The advantages of sending fewer samples to the fusion center include power and bandwidth savings. The focus of this paper is to determine the suitability of MC in this scenario. For this purpose, we conduct matrix coherence analysis and derive the optimal waveform conditions for both rank-1 and multi-rank beamformers.

2. BACKGROUND

Consider a rank-K matrix $\mathbf{X} \in \mathbb{C}^{M_r \times N}$, with the singular value decomposition $\mathbf{X} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$.

Let U be the subspace spanned by the set singular vectors $\left\{\mathbf{u}_i \in \mathbb{C}^{M_r}\right\}_{i=1,\ldots,K}$, P_U the orthogonal projection onto U, i.e., $P_U = \sum\limits_{1\leq i\leq K} \mathbf{u}_i\mathbf{u}_i^H$, and $\mathbf{e}_i$ the standard basis vector whose i-th element is 1. The coherence of U is defined as $\mu(U) = \frac{M_r}{K} \max\limits_{1\leq i\leq M_r} \|P_U\mathbf{e}_i\|^2 \quad \in \left[1, \frac{M_r}{K}\right]$. The coherence $\mu(V)$ is defined in a similar way. The lower the coherence, the fewer entries of $\mathbf{X}$ are required to reconstruct it [1].

In the following, we consider a colocated MIMO radar

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system be based on uniform linear arrays (ULA) with M_t and M_r transmit/receive antennas, as well as $d_t = \lambda/2$ and $d_r = M_t\lambda/2$, inter-antenna spacing, respectively, where λ denotes the waveform length.

3. ON THE RANK-1 BEAMFORMING

Let $\mathbf{s} \in \mathbb{C}^{N\times 1}$ be the waveform sequence transmitted by each of the M_t antennas over one pulse. Let $\mathbf{w} \in \mathbb{C}^{M_t\times 1}$ denote the transmit beamformer. Then, according to [14], the rank-1 beamformer equals $\mathbf{w} = \mathbf{a}(\theta)/\|\mathbf{a}(\theta)\|$, where $\mathbf{a}(\theta)$ denotes the transmit steering vector corresponding to direction θ. High angle resolution can be achieved under the rank-1 beamformer [12] by doing joint transmit and receive beamforming, which shows great advantage of MIMO radar over phased-array radar for single target tracking.

Under the narrow-band assumption, the noise free receive data matrix collected at the fusion center that contains the samples of target reflections equals $\mathbf{X} = \mathbf{b}(\theta)\beta\zeta\mathbf{a}(\theta)^T\tilde{\mathbf{S}}$, where $\mathbf{b}(\theta)$ is the receive steering vector w.r.t. direction θ. The transmit signal matrix equals $\tilde{\mathbf{S}} = \mathbf{w}\mathbf{s}^T$. In addition, β and ζ are the target reflection coefficient and Doppler shift, respectively.

The matrix $\mathbf{X}$ is low rank, and as long as its left and right subspaces coherence is low, it can be recovered from a small number of its entries, selected uniformly at random.

In the following theorem we state the conditions so that the coherence of $\mathbf{X}$ achieves it smallest possible value of 1.

Theorem 1. *Under a ULA configuration, when the MIMO radar antennas transmit the same waveform and a rank-1 beamformer is used, i.e.,* $\mathbf{w} = \mathbf{a}(\theta)/\|\mathbf{a}(\theta)\|$, *the coherence of the matrix* $\mathbf{X}$ *achieves its lowest value, i.e.,* $\mu(U) = \mu(V) \equiv 1$ *if and only if the waveform sequence is unimodular.*

Proof. The data matrix $\mathbf{X} \in \mathbb{C}^{M_r\times N}$ as defined above is rank-1. Let its compact singular value decomposition (SVD) be $\mathbf{X} = \mathbf{u}\sigma\mathbf{v}^H$, where $\mathbf{u} \in \mathbb{C}^{M_r\times 1}, \mathbf{v} \in \mathbb{C}^{N\times 1}$ with $\mathbf{u}^H\mathbf{u} = 1, \mathbf{v}^H\mathbf{v} = 1$ and σ the corresponding singular value. Consider the QR decomposition of $\mathbf{b}(\theta)$ given by $\mathbf{b}(\theta) = \mathbf{q}_r r_r$, where

$$\mathbf{q}_r = \frac{1}{\sqrt{M_r}}\left[\begin{array}{cccc} 1 & e^{j\frac{2\pi}{\lambda}d_r\sin\theta} & \cdots & e^{j\frac{2\pi}{\lambda}(M_r-1)d_r\sin\theta}\end{array}\right]^T$$

such that $\mathbf{q}_r^H\mathbf{q}_r = 1$ and $r_r = \sqrt{M_r}$ where λ denotes the wavelength. Similarly, we consider the QR decomposition of $\tilde{\mathbf{S}}^T\mathbf{a}(\theta)$ given by $\tilde{\mathbf{S}}^T\mathbf{a}(\theta) = \frac{\mathbf{s}\mathbf{a}(\theta)^T\mathbf{a}(\theta)}{\|\mathbf{a}(\theta)\|} = \mathbf{q}_s r_s$, where $\mathbf{q}_s \in \mathbb{C}^{N\times 1}$ such that $\mathbf{q}_s^H\mathbf{q}_s = 1$, and r_s is a real number. Then, $\mathbf{X} = \mathbf{q}_r r_r\beta\zeta r_s\mathbf{q}_s^T$. The SVD of the complex number $r_r\beta\zeta r_s$ can be written as $r_r\beta\zeta r_s = q_1\rho q_2^*$, where $|q_1| = |q_2| = 1$ and ρ is a real number. Therefore, $\mathbf{X} = \mathbf{q}_r q_1\rho q_2^*\mathbf{q}_s^T = \mathbf{q}_r q_1\rho(\mathbf{q}_s^* q_2)^H$, which is a valid SVD of $\mathbf{X}$ since $(\mathbf{q}_r q_1)^H\mathbf{q}_r q_1 = 1, (\mathbf{q}_s^* q_2)^H\mathbf{q}_s^* q_2 = 1$. By the uniqueness of singular values of a matrix, it holds that $\sigma \equiv \rho$. Therefore, we can set $\mathbf{u} = \mathbf{q}_r q_1, \mathbf{v} = \mathbf{q}_s^* q_2$.

Let $\mathbf{q}_r^{(i)}$ denote the i-th element of $\mathbf{q}_r$. The coherence of the column space of $\mathbf{X}$ is

$$\mu(U) = \frac{M_r}{1}\sup_{i\in\mathbb{N}_{M_r}^+}\left|\mathbf{q}_r^{(i)}q_1\right|^2 = M_r\sup_{i\in\mathbb{N}_{M_r}^+}\left|\mathbf{q}_r^{(i)}\right|^2 \equiv 1. \quad (1)$$

Let $\mathbf{q}_s^{*(i)}$, s_i denote the i-th element of $\mathbf{q}_s^*$ and $\mathbf{s}$, respectively. The coherence of the row space of $\mathbf{X}$ is

$$\begin{aligned}\mu(V) &= \frac{N}{1}\sup_{i\in\mathbb{N}_N^+}\left|\mathbf{q}_s^{*(i)}q_2\right|^2 = N\sup_{i\in\mathbb{N}_N^+}\left|\mathbf{q}_s^{(i)}\right|^2\\ &= N\sup_{i\in\mathbb{N}_N^+}\left|\frac{s_i\mathbf{a}(\theta)^T\mathbf{a}(\theta)}{\|\mathbf{a}(\theta)\| r_s}\right|^2\\ &= N\sup_{i\in\mathbb{N}_N^+}\frac{\mathbf{a}(\theta)^H\mathbf{a}(\theta)^*|s_i|^2\mathbf{a}(\theta)^T\mathbf{a}(\theta)}{\|\mathbf{a}(\theta)\|^2 r_s^2}. \end{aligned} \quad (2)$$

Here, it holds that

$$\begin{aligned} r_s^2 &= r_s\mathbf{q}_s^H\mathbf{q}_s r_s = (\mathbf{q}_s r_s)^H\mathbf{q}_s r_s\\ &= \frac{\mathbf{a}(\theta)^H\mathbf{a}(\theta)^*\mathbf{s}^H\mathbf{s}\mathbf{a}(\theta)^T\mathbf{a}(\theta)}{\|\mathbf{a}(\theta)\|^2}. \end{aligned} \quad (3)$$

Consequently,

$$\mu(V) = N\sup_{i\in\mathbb{N}_N^+}\frac{|s_i|^2}{\mathbf{s}^H\mathbf{s}}. \quad (4)$$

Since $\sum_{i=1}^{N}|s_i|^2 = \mathbf{s}^H\mathbf{s}$ and $|s_i|^2 \geq 0$, the minimum possible value of $\mu(V)$ could achieve the minimum value, i.e., 1, if and only if $|s_i|^2 = \frac{1}{N}\mathbf{s}^H\mathbf{s}$ for any $i \in \mathbb{N}_N^+$. This condition suggests that the transmit power in each snapshot, i.e., $|s_i|^2$, should equal the total transmit power $\mathbf{s}^H\mathbf{s}$ divided by N. In other words, the transmit waveform should be unimodular. Consequently, it holds that $\mu(U) = \mu(V) \equiv 1$, which completes the proof. □

It is interesting to note that the coherence is optimal independent of the the beamforming vector.

4. ON THE MULTI-RANK BEAMFORMING

According to [12], to track multiple targets at directions $\{\theta_k\}_{k\in\mathbb{N}_K^+}$, a rank-$K$ beamformer $\mathbf{W} = [\mathbf{w}_1\cdots\mathbf{w}_K] \in \mathbb{C}^{M_t\times K}$ can be used, where $\mathbf{w}_k = \frac{\mathbf{a}(\theta_k)}{\|\mathbf{a}(\theta_k)\|}$ is the beamformer focussing on direction θ_k. The sampled transmitted signal matrix equals $\tilde{\mathbf{S}} = \sqrt{\frac{M_t}{K}}\mathbf{W}\mathbf{S}^T$, where $\sqrt{\frac{M_t}{K}}$ is a factor to satisfy that the total transmit energy is M_t; $\mathbf{S} \in \mathbb{C}^{N\times K}$ contains sampled orthogonal waveforms so that $\mathbf{S}^H\mathbf{S} = \mathbf{I}_K$. The transmit beampattern in direction ϕ is the sum of K rank-1 beampatterns, i.e.,

$$\begin{aligned} P_T(\phi) &= \frac{M_t}{K}\mathbf{a}(\phi)^H\mathbf{W}\mathbf{W}^H\mathbf{a}(\phi) \\ &= \frac{M_t}{K}\sum_{k=1}^{K}\mathbf{a}(\phi)^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{a}(\phi). \end{aligned} \tag{5}$$

Under the narrow-band assumption, the noise free receive data matrix is [4]

$$\mathbf{X} = \mathbf{B}\mathbf{D}\mathbf{A}^T\tilde{\mathbf{S}}, \tag{6}$$

where $\mathbf{A} \in \mathbb{C}^{M_t\times K}$ is the transmit steering matrix defined as $\mathbf{A} = [\mathbf{a}(\theta_1)\cdots\mathbf{a}(\theta_K)]$; $\mathbf{B} \in \mathbb{C}^{M_r\times K}$ is the receive steering matrix, and $\mathbf{D} \in \mathbb{C}^{K\times K}$ is a diagonal matrix containing target reflection coefficients and Doppler shifts. It can be shown that $\mathbf{X}$ is a low-rank matrix. Thus, depending on how low its coherence is, it can be recovered based on a small, uniformly sampled subset of its elements. On the coherence of $\mathbf{X}$, we have the following theorem.

Theorem 2. *Consider an ULA configuration and a MIMO radar applying the rank-K beamformer* $\mathbf{W} = [\mathbf{w}_1\cdots\mathbf{w}_K] \in \mathbb{C}^{M_t\times K}$ *to* K *orthogonal waveforms* $\mathbf{S} \in \mathbb{C}^{N\times K}$.

The coherence of the row space of $\mathbf{X}$ *is optimal, i.e.,* $\mu(V) \equiv 1$, *if and only if*

$$\mathbf{S}^{(i)}\left(\mathbf{S}^{(i)}\right)^H = \frac{K}{N},\ \forall\, i \in \mathbb{N}_N^+, \tag{7}$$

where $\mathbf{S}^{(i)} \in \mathbb{C}^{1\times K}$ *denotes the* i*-th row of* $\mathbf{S}$.

Proof. The compact SVD of $\mathbf{X}$ can be expressed as $\mathbf{X} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$, where $\mathbf{U} \in \mathbb{C}^{M_r\times K}$, $\mathbf{V} \in \mathbb{C}^{N\times K}$ such that $\mathbf{U}\mathbf{U}^H = \mathbf{I}_K, \mathbf{V}\mathbf{V}^H = \mathbf{I}_K$, and $\mathbf{\Lambda} \in \mathbb{R}^{K\times K}$ is a diagonal matrix containing the singular values of $\mathbf{X}$. Consider the QR decomposition of $\mathbf{B}$, i.e., $\mathbf{B} = \mathbf{Q}_r\mathbf{R}_r$, where $\mathbf{Q}_r \in \mathbb{C}^{M_r\times K}$ such that $\mathbf{Q}_r^H\mathbf{Q}_r = \mathbf{I}_K$, and $\mathbf{R}_r \in \mathbb{C}^{K\times K}$ is an upper triangular matrix. Similarly, we consider the QR decomposition of $\tilde{\mathbf{S}}^T\mathbf{A}$ given by $\tilde{\mathbf{S}}^T\mathbf{A} = \mathbf{Q}_s\mathbf{R}_s$, where $\mathbf{Q}_s \in \mathbb{C}^{N\times K}$ such that $\mathbf{Q}_s^H\mathbf{Q}_s = \mathbf{I}_K$ and $\mathbf{R}_s \in \mathbb{C}^{K\times K}$ is an upper triangular matrix. Then, $\mathbf{X} = \mathbf{Q}_r\mathbf{R}_r\mathbf{D}\mathbf{R}_s^T\mathbf{Q}_s^T$ and the matrix $\mathbf{R}_r\mathbf{D}\mathbf{R}_s^T \in \mathbb{C}^{K\times K}$ is rank-K whose SVD is given as $\mathbf{R}_r\mathbf{D}\mathbf{R}_s^T = \mathbf{Q}_1\mathbf{\Delta}\mathbf{Q}_2^H$. Here, $\mathbf{Q}_1 \in \mathbb{C}^{K\times K}$ is such that $\mathbf{Q}_1\mathbf{Q}_1^H = \mathbf{Q}_1^H\mathbf{Q}_1 = \mathbf{I}_K$ (the same holds for $\mathbf{Q}_2$) and $\mathbf{\Delta} \in \mathbb{R}^{K\times K}$ is non-zero diagonal, containing the singular values of $\mathbf{R}_r\mathbf{D}\mathbf{R}_s^T$. Therefore,

$$\mathbf{X} = \mathbf{Q}_r\mathbf{Q}_1\mathbf{\Delta}\mathbf{Q}_2^H\mathbf{Q}_s^T = \mathbf{Q}_r\mathbf{Q}_1\mathbf{\Delta}(\mathbf{Q}_s^*\mathbf{Q}_2)^H, \tag{8}$$

which is a valid SVD of $\mathbf{X}$ since $(\mathbf{Q}_r\mathbf{Q}_1)^H\mathbf{Q}_r\mathbf{Q}_1 = \mathbf{I}_K$ and $(\mathbf{Q}_s^*\mathbf{Q}_2)^H\mathbf{Q}_s^*\mathbf{Q}_2 = \mathbf{I}_K$. By the uniqueness of the singular values of a matrix, it holds that $\mathbf{\Lambda} \equiv \mathbf{\Delta}$. Therefore, we can set $\mathbf{U} = \mathbf{Q}_r\mathbf{Q}_1$ and $\mathbf{V} = \mathbf{Q}_s^*\mathbf{Q}_2$.

Let $\mathbf{Q}_s^{*(i)}, \mathbf{S}^{(i)} \in \mathbb{C}^{1\times K}$ denote the i-th row of $\mathbf{Q}_s^*$ and $\mathbf{S}$, respectively. Regarding the coherence of the row space of $\mathbf{X}$, we have

$$\begin{aligned} \mu(V) &= \frac{N}{K}\sup_{i\in\mathbb{N}_N^+}\left\|\mathbf{Q}_s^{*(i)}\mathbf{Q}_2\right\|^2 = \frac{N}{K}\sup_{i\in\mathbb{N}_N^+}\left\|\mathbf{Q}_s^{(i)}\right\|^2 \\ &= \frac{N}{K}\sup_{i\in\mathbb{N}_N^+}\left\|\sqrt{\frac{M_t}{K}}\mathbf{S}^{(i)}\mathbf{W}^T\mathbf{A}\mathbf{R}_s^{-1}\right\|^2 \\ &= \frac{N}{K}\sup_{i\in\mathbb{N}_N^+}\frac{M_t}{K}\mathbf{S}^{(i)}\mathbf{W}^T\mathbf{A}\mathbf{R}_s^{-1}\left(\mathbf{R}_s^{-1}\right)^H\mathbf{A}^H\mathbf{W}^*\left(\mathbf{S}^{(i)}\right)^H. \end{aligned}$$

Here, since the waveforms are orthogonal, i.e., $\mathbf{S}^H\mathbf{S} = \mathbf{I}_K$, it holds that

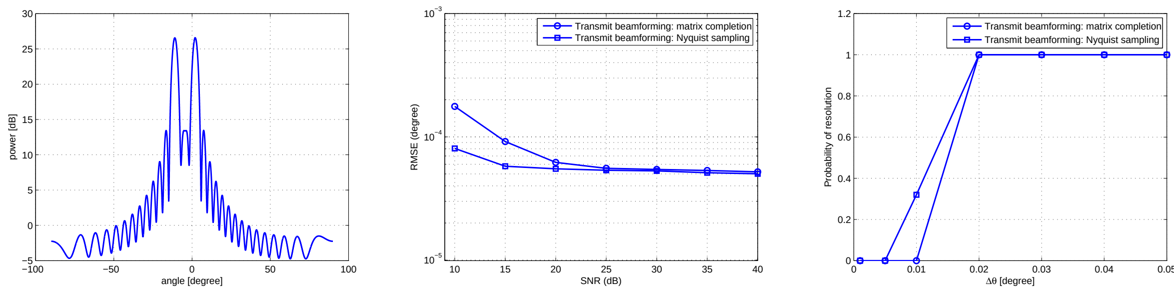
$$\begin{aligned} \mathbf{R}_s^{-1}\left(\mathbf{R}_s^{-1}\right)^H &= \left(\mathbf{R}_s^H\mathbf{R}_s\right)^{-1} = \left(\mathbf{R}_s^H\mathbf{Q}_s^H\mathbf{Q}_s\mathbf{R}_s\right)^{-1} \\ &= \frac{K}{M_t}\left(\mathbf{A}^H\mathbf{W}^*\mathbf{S}^H\mathbf{S}\mathbf{W}^T\mathbf{A}\right)^{-1} \\ &= \frac{K}{M_t}\left(\mathbf{A}^H\mathbf{W}^*\mathbf{W}^T\mathbf{A}\right)^{-1} \\ &= \frac{K}{M_t}\left(\mathbf{W}^T\mathbf{A}\right)^{-1}\left(\mathbf{A}^H\mathbf{W}^*\right)^{-1}. \end{aligned} \tag{9}$$

Consequently, $\mu(V) = \frac{N}{K}\sup_{i\in\mathbb{N}_N^+}\mathbf{S}^{(i)}\left(\mathbf{S}^{(i)}\right)^H$. In addition, it holds that $\sum_{i=1}^{N}\mathbf{S}^{(i)}\left(\mathbf{S}^{(i)}\right)^H = K$ for orthogonal waveforms. Therefore, to find the lowest possible value of $\mu(V)$, we solve the following optimization problem

$$\begin{aligned} &\min_{i\in\mathbb{N}_N^+}\left(\max_{i\in\mathbb{N}_N^+}\ \mathbf{S}^{(i)}\left(\mathbf{S}^{(i)}\right)^H\right) \\ &\text{s.t.}\ \sum_{i=1}^{N}\mathbf{S}^{(i)}\left(\mathbf{S}^{(i)}\right)^H = K. \end{aligned} \tag{10}$$

Since $\mathbf{S}^{(i)}\left(\mathbf{S}^{(i)}\right)^H \geq 0$, the solutions of the above problem are $\mathbf{S}^{(i)}\left(\mathbf{S}^{(i)}\right)^H = \frac{K}{N}, \forall i \in \mathbb{N}_N^+$. Consequently, the lowest possible value of row space of $\mathbf{X}$ is achieved as $\mu(V) \equiv 1$, which completes the proof. □

Theorem 2 indicates that the energy of the K orthogonal waveforms should be constant during each snapshot. It is interesting to note that the coherence of the row space of $\mathbf{X}$ is independent of the multi-rank beamformer. Therefore, the analysis results hold for all kinds of multi-rank beamformers obtained via different methods, e.g., multi-rank beamformer for the approximation of a desired beampattern, proposed in [15]. It should be noted that the coherence of the column space of $\mathbf{X}$, i.e., $\mu(U)$ coincides with the results in MIMO-MC radar and interested readers can refer to [7] for detail discussions.



(a) (b) (c)

Fig. 1: Simulations: (a) Transmit beampattern under rank-2 beamformer with $M_t = 30$ for directions $[-11^\circ, 2^\circ]$; (b) RMSE versus SNR with $M_r = 60, M_t = 30$; (c) Probability of target resolution with $M_r = 60, M_t = 20$ and $\text{SNR} = 25\text{dB}$. In figures (b) (c), the proposed approach is based on subsampling by 50%.

5. DOA ESTIMATION BASED ON MC

At the fusion center, for each pulse, the data matrix $\mathbf{X}$ is recovered via MC using a small portion of samples collected uniformly at random. Let $\tilde{\mathbf{X}}$ denote the recovered data matrix. Subsequently, matched filtering is applied on $\tilde{\mathbf{X}}$ to obtain

$$\mathbf{Y}_q = \tilde{\mathbf{X}}\mathbf{S}^* = \sqrt{\frac{M_t}{K}}\mathbf{B}\mathbf{D}\mathbf{A}^T\mathbf{W} + \mathbf{Z}_q, \tag{11}$$

where q is the pulse index and $\mathbf{Z}_q$ represents noise. Stacking the matrix (11) into a $KM_r \times 1$ vector, we get

$$\mathbf{y}_q = \text{vec}\left(\mathbf{Y}_q\right) = \sqrt{\frac{M_t}{K}}\sum_{k=1}^{K} d_k \left(\mathbf{W}^T\mathbf{a}\left(\theta_k\right)\right) \otimes \mathbf{b}\left(\theta_k\right) + \mathbf{z}_q,$$

where d_k denotes the reflection coefficient and Doppler shift w.r.t. the k-th target; $\mathbf{z}_q = \text{vec}\left(\mathbf{Z}_q\right)$. With Q pulses data, the sample covariance matrix is obtained as $\mathbf{R} = \frac{1}{Q}\sum_{q=1}^{Q}\mathbf{y}_q\mathbf{y}_q^H$. The pseudo-spectrum of MUSIC estimator is [13]

$$P\left(\theta\right) = \frac{1}{\mathbf{c}^H\left(\theta\right)\mathbf{E}_n\mathbf{E}_n^H\mathbf{c}\left(\theta\right)}, \tag{12}$$

where $\mathbf{c}\left(\theta\right) = \left(\mathbf{W}^T\mathbf{a}\left(\theta\right)\right) \otimes \mathbf{b}\left(\theta\right)$ and $\mathbf{E}_n \in \mathbb{C}^{KM_r \times (KM_r - K)}$ is a matrix containing the eigenvectors of the noise subspace of $\mathbf{R}$. The angle of the targets can be obtained by the finding the peak locations of the pseudo-spectrum (12).

In the following, we use simulation to test the performance of the proposed approach. Throughout the simulations, the transmit/receive arrays are configured as ULA with $d_t = \lambda/2$ and $d_r = M_t\lambda/2$. The first K waveforms of the Hadamard sequences are used for transmit beamforming. The number of pulses is set to $Q = 3$. The number of Nyquist samples in one pulse is $N = 128$. The data matrix $\mathbf{X}$ is recovered via the SVT algorithm [16] using only $p = 50\%$ of its entries. The obtained results are averaged over 100 independent runs. First, a rank-2 beamformer with $M_t = 30$ is applied to illuminate $K = 2$ targets at angles $\theta_1 = -11^\circ$ and $\theta_2 = 2^\circ$. The transmit beampattern of the beamformer is shown in Fig. 1 (a). The root mean square error (RMSE) of the direction of arrival (DOA) estimation for these two targets is plotted in Fig. 1(b) for $M_r = 60$. It can be found that the RMSE of DOA estimation using MC or Nyquist sampling decreases as the signal-to-noise ratio (SNR) becomes larger (the SNR is defined at the fusion center before the matched filtering operation). Interestingly, as $\text{SNR} \geq 25\text{dB}$, these two RMSE curves become almost identical. This is because the recovery error of $\mathbf{X}$ introduced by MC is quite small when the SNR is high [3]. Next, we access the capability of the proposed scheme to resolve two closely located targets. We take the first target to be in direction $\theta_1 = 10^\circ$ and the second in $\theta_2 = \theta_1 + \Delta\theta$. The targets are considered to be resolved if $\left|\hat{\theta}_k - \theta_k\right| \leq \Delta\theta/2, k = 1, 2$, where $\hat{\theta}_k$ denotes the estimation of the k-th target [13]. The probability of resolution under $M_r = 60, M_t = 20$ and $\text{SNR} = 25\text{dB}$ is plotted in Fig. 1(c). It can be found that the proposed MC based scheme has the same resolution of $\Delta\theta = 0.02^\circ$ as the method with Nyquist sampling. Therefore, a comparable DOA estimation performance is achieved under the proposed scheme as the method that uses Nyquist sampling.

6. CONCLUSIONS

In this paper, we have proposed a MIMO-MC radar approach with transmit beamforming. Each receive antenna performs sub-Nyquist sampling and the full data matrix is recovered at the fusion center via MC. Analysis results have shown that the matrix coherence is independent of the beamformer. The row space coherence of data matrix is optimal if and only if the transmit orthogonal waveforms have constant power during all snapshots. The simulation results show that the proposed scheme could achieve super resolution at a low sub-Nyquist sampling rate.

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