OPTIMUM PHASE-ONLY DISCRETE BROADCAST BEAMFORMING WITH ANTENNA AND USER SELECTION IN INTERFERENCE LIMITED COGNITIVE RADIO NETWORKS

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ABSTRACT

Phase-only beamforming presents certain advantages in radar and communication systems. In cognitive radio, antenna and user selection are the two tools for increasing the quality of service (QoS) for the users. In this paper, discrete single group multicast transmit phase-only beamformer design is presented with antenna subset and user selection. The problem is converted into linear form and solved efficiently by using mixed integer linear programming to find the optimum subset of antennas and secondary users together with optimum beamformer phase coefficients. It is shown that significant power saving is possible compared to fixed antenna systems.

Index Terms— Transmit beamformer, discrete beamformer, mixed integer linear programming, antenna selection

1. INTRODUCTION

In this paper, secondary broadcast network is considered where cognitive base station is equipped with an antenna array. The objective is to transmit a common information to all the secondary users while guaranteeing that the interference on the primary users are below interference temperature in accordance with spectrum underlay perspective of spectrum sharing problem [1]. When the interference limits dictated by the primary network are demanding and the number of primary users is large, admission control or secondary user selection at the cognitive base station becomes a critical task [2], [3]. The main goal in user selection is to maximize the number of secondary users while minimizing the total transmitted power and interference to the primary users.

Transmit antenna selection for the most appropriate antenna subarray is an important problem [4], [5]. Selecting the best antenna set from a larger set of available antennas is an effective approach to reduce hardware cost and complexity. Antenna selection results less transmit power and increases the number of serviced users.

Multicast beamforming problem is usually investigated in continuous case where the beamformer weight vector has continuous amplitude and phase [6], [7]. The continuous problem is nonconvex and NP-hard even for single group multicasting [7], [8]. Hence optimum solution is not guaranteed for the existing approaches. For practical wireless scenarios where the number of users is larger than the antennas, the performance of the well known techniques degrades as the number of users increases [7], [9], [10].

Motivated by the shortcomings of aforementioned approaches, we propose discrete structure for broadcast beamforming. Discrete structure is more suitable for fabrication, decreasing the system complexity and cost as well as increasing the controllability [11], [12]. In practical cases, the exact adjustment of continuous beamformer weight vector may result in energy loss or is not possible since phase shifters and amplifiers might not work continuously [12]. Furthermore, in order to decrease complexity, codebook based beamforming has gained more attention in many standardized systems, such as 3GPP LTE [13]. Recently, beamforming antennas with discrete phase shifters are the key technology of millimeter wave communication systems where it is possible to obtain higher data rate and performance [13].

Although there exist several previous works on user and antenna selection separately in broadcast cognitive scenario [2], [3], [4], [14], there is not much work on joint selection of users and antennas. In some previous papers, optimum discrete phase-only [15] and optimum discrete phase and amplitude [16], [17] transmit beamformer designs are considered in the absence of primary network. This paper extends the work in [17] by including antenna and user selection into the problem in cognitive broadcast scenario. The proposed method finds the optimum secondary user subset as well as the best L out of M antennas together with the optimum phase terms of the transmit beamformer in cognitive radio scenario. The original nonconvex and highly nonlinear problem is converted to a linear form using some nontrivial transformations and integer optimization methods. The final form of the problem is appropriate for mixed integer linear programming which can be solved effectively with the state-of-the art integer programming solvers using branch and cut algorithm. To the best of our knowledge, this paper is the first work which presents an optimum solution to the joint problem.

2. SYSTEM MODEL

It is assumed that there are M antennas in the secondary base station and broadcast signal, $s(t)$, is transmitted to N_s secondary users each having a single antenna. There are N_p primary users. The signal transmitted at the secondary base station is $\mathbf{x}(t) = s(t)\mathbf{w}$ where $\mathbf{w}$ is the $M \times 1$ complex beamformer weight vector. The received signal at the k^{th} secondary user is $y_k(t) = \mathbf{h}_k^H\mathbf{x}(t) + n_k(t), k = 1, \dots N_s$. Here, $\mathbf{h}_k$ is the $M \times 1$ complex downlink channel vector for the k^{th} secondary user, n_k is the additive white noise uncorrelated with the source signal and its variance is σ_k^2. In this case, signal-to-noise ratio (SNR) for the k^{th} secondary user is $SNR_k = \frac{\sigma_s^2 \mathbb{E}\{|\mathbf{w}^H\mathbf{h}_k|^2\}}{\sigma_k^2}$, where σ_s^2 is the source signal variance. $\sigma_s^2 = 1$ is selected without loss of generality throughout the paper. The interference towards the l^{th} primary user is, $T_l = \mathbb{E}\{|\mathbf{w}^H\mathbf{g}_l|^2\}$, where $\mathbf{g}_l$ is the $M \times 1$ complex channel vector for the l^{th} primary user.

It is assumed that only L out of M antennas can transmit the broadcast signal. The goal is to jointly select the best L antennas, and find the corresponding beamforming vector $\mathbf{w}$ such that the transmission power is minimized, subject to receive-SNR constraints per secondary user and interference constraints per primary user. Let $\mathbf{R}_k = \mathbb{E}\{\mathbf{h}_k\mathbf{h}_k^H\}$ and $\mathbf{G}_l = \mathbb{E}\{\mathbf{g}_l\mathbf{g}_l^H\}$ denote the channel covariance matrix of the k^{th} secondary user and l^{th} primary user respectively and they are known at the secondary base station. Considering P_{an} as the per-antenna power and $P_{an_{max}}$ as the maximum antenna power, phase-only continuous problem can be written as follows,

$$\mathcal{P}1: \quad \min_{\mathbf{w}\in\mathbb{C}^M} \ \mathbf{w}^H\mathbf{w} \tag{1.a}$$
$$s.t. \ \ \mathbf{w}^H\mathbf{R}_k\mathbf{w} \geq \gamma_k\sigma_k^2, \quad k=1,...,N_s \tag{1.b}$$
$$\mathbf{w}^H\mathbf{G}_l\mathbf{w} \leq \epsilon_l, \quad l=1,...,N_p \tag{1.c}$$
$$(\mathbf{w}\mathbf{w}^{\mathbf{H}})_{i,i} \in \{0, P_{an}\} \quad i=1,...,M \tag{1.d}$$
$$\mathbf{w}^{\mathbf{H}}\mathbf{w} = LP_{an}, \ P_{an} \leq P_{an_{max}} \tag{1.e}$$

where γ_k and ϵ_l denote the desired SNR for the k^{th} secondary user and the interference threshold for the l^{th} primary user respectively. Solving the above problem requires a combinatorial search over all $\binom{M}{L}$ NP hard problems [4], [7]. The discrete phase version of $\mathcal{P}1$ can be written as,

$$\mathcal{P}2: \quad \min_{\mathbf{w}\in\mathbb{C}^M} \ \mathbf{w}^H\mathbf{w} \tag{2.a}$$
$$s.t. \ \ \mathbf{w}^H\mathbf{R}_k\mathbf{w} \geq \gamma_k\sigma_k^2, \quad k=1,...,N_s \tag{2.b}$$
$$\mathbf{w}^H\mathbf{G}_l\mathbf{w} \leq \epsilon_l, \quad l=1,...,N_p \tag{2.c}$$
$$w_i = \alpha_i\sqrt{P_{an}}e^{j\psi_i} \quad i=1,...,M \tag{2.d}$$
$$\alpha_i \in \{0,1\} \quad i=1,...,M, \quad \sum_{i=1}^{M}\alpha_i = L \tag{2.e}$$
$$\psi_i \in \{0, \Delta\theta, 2\Delta\theta, ..., (2^n-1)\Delta\theta\}, \ \Delta\theta = \frac{360^\circ}{2^n} \tag{2.f}$$
$$P_{an} \leq P_{an_{max}} \tag{2.g}$$

where w_i is the i^{th} element of the beamformer vector $\mathbf{w}$ and ψ_i is the discrete phase with n bits. α_i is the antenna selection coefficient. $\Delta\theta$ is the discrete step size for phase.

In order to convert nonconvex and nonlinear problem $\mathcal{P}2$ to a linear form, an intermediate problem setting is used. Hence in the following part, $\mathcal{P}2$ is shown to be equivalent to $\mathcal{P}3$ which leads to further exploitation.

Theorem 1: $\mathcal{P}2$ is equivalent to the following problem in $\mathcal{P}3$ up to a scale factor in the sense that their optimum solutions differ only by a real scalar.

$$\mathcal{P}3: \quad \max_{\mathbf{w}\in\mathbb{C}^M} \ t \tag{3.a}$$
$$s.t. \ \ \mathbf{w}^H\mathbf{R}_k\mathbf{w} \geq t\gamma_k\sigma_k^2, \quad k=1,...,N_s \tag{3.b}$$
$$\mathbf{w}^H\mathbf{G}_l\mathbf{w} \leq t\epsilon_l, \quad l=1,...,N_p \tag{3.c}$$
$$w_i = \alpha_i\sqrt{P_{an_{max}}}e^{j\psi_i} \quad i=1,...,M \tag{3.d}$$
$$\alpha_i \in \{0,1\} \quad i=1,...,M, \quad \sum_{i=1}^{M}\alpha_i = L \tag{3.e}$$
$$\psi_i \in \{0, \Delta\theta, 2\Delta\theta, ..., (2^n-1)\Delta\theta\}, \ \Delta\theta = \frac{360^\circ}{2^n} \tag{3.f}$$
$$t \geq 1 \tag{3.g}$$

Proof: Assume that $\mathcal{P}2$ is feasible, its optimum solution is $\mathbf{w}_1$ and the associated per-antenna power is P_{an_1}. It is easy to see that at least one of the secondary user SNR constraints in (2.b) should be met with equality. Otherwise $\mathbf{w}_1$ could be scaled down, thereby improving the objective function. Then $\sqrt{P_{an_{max}}/P_{an_1}}\mathbf{w}_1$ satisfies the constraints of $\mathcal{P}3$ with the associated variable $t_1 = P_{an_{max}}/P_{an_1} \geq 1$. Hence $\mathcal{P}3$ is feasible if $\mathcal{P}2$ is feasible. Let $\{\mathbf{w}_2, t_2\}$ be the optimum solution of $\mathcal{P}3$. t_2 can only be greater than or equal to t_1. If t_2 is strictly greater than t_1, then it is possible to satisfy the constraints of $\mathcal{P}2$ using $\frac{\mathbf{w_2}}{\sqrt{t_2}}$ whose per-antenna power is $P_{an_2} = \frac{t_1}{t_2}P_{an_1} < P_{an_1}$, which is a contradiction. Therefore $t_2 = t_1$ and $\mathbf{w}_2 = \sqrt{P_{an_{max}}/P_{an_1}}\mathbf{w}_1$ is the optimum solution. At this point, we have shown that whenever $\mathcal{P}2$ is feasible, the optimum solutions of both problems are the same up to a scale factor. If $\mathcal{P}2$ is not feasible, the constraints of $\mathcal{P}2$ will not be satisfied even with $P_{an} = P_{an_{max}}$. Hence, for the same problem setting (SNR and interference threshold values, γ_k, ϵ_l and noise variance, σ_k^2) no solution can be found for $\mathcal{P}3$, otherwise a feasible solution can be found for $\mathcal{P}2$ by scaling down the solution of $\mathcal{P}3$.

3. USER SELECTION

$\mathcal{P}2$ and $\mathcal{P}3$ are not always feasible due to the interference limitations. In order to improve the feasibility, user selection schemes can be used such that the best user subset with the least transmitted power is chosen [2], [3], [14]. In this paper, a joint optimum user and antenna selection method is presented for the best QoS result.

Theorem 2: Let $\mathbf{w}_{op}$ be the optimum solution of $\mathcal{P}2$ which satisfies the QoS constraints of optimum secondary user subset $\mathcal{K}_\star$. This subset consists as many users as possible where $|\mathcal{K}_\star|$ is the number of serviced secondary users. If $A \geq (\max_k \frac{Tr\{\mathbf{R}_k\}}{\gamma_k\sigma_k^2}LP_{an_{max}})$ and $\beta > (\max_k \frac{Tr\{\mathbf{R}_k\}}{\gamma_k\sigma_k^2}LP_{an_{max}} - 1)$, the optimum solution of the problem $\mathcal{P}4$ is the same as $\mathbf{w}_{op}$ up to a positive scale factor.

$$\mathcal{P}4: \quad \max_{\mathbf{w}\in\mathbb{C}^M, \lambda_k} \ t + \beta\sum_{k=1}^{N_s}\lambda_k \tag{4.a}$$
$$s.t. \ \ \mathbf{w}^H\mathbf{R}_k\mathbf{w} + A(1-\lambda_k)\gamma_k\sigma_k^2 \geq t\gamma_k\sigma_k^2, \quad k=1,...,N_s \tag{4.b}$$
$$\mathbf{w}^H\mathbf{G}_l\mathbf{w} \leq t\epsilon_l, \quad l=1,...,N_p \tag{4.c}$$
$$w_i = \alpha_i\sqrt{P_{an_{max}}}e^{j\psi_i} \quad i=1,...,M \tag{4.d}$$
$$\alpha_i \in \{0,1\} \quad i=1,...,M \tag{4.e}$$
$$\lambda_k \in \{0,1\} \quad k=1,...,N_s \tag{4.f}$$
$$\sum_{k=1}^{N_s}\lambda_k \geq 1, \ \sum_{i=1}^{M}\alpha_i = L, \ t \geq 1 \tag{4.g}$$
$$\psi_i \in \{0, \Delta\theta, 2\Delta\theta, ..., (2^n-1)\Delta\theta\}, \ \Delta\theta = \frac{360^\circ}{2^n} \tag{4.h}$$

Proof: Assume that the optimum solution of $\mathcal{P}2$ with user selection is $\{\mathbf{w}_1, \mathcal{K}_1\}$. The associated per-antenna power is P_{an_1}. $|\mathcal{K}_1|$ shows the number of users in the set $\mathcal{K}_1$. At least one of the SNR constraints of the secondary users in the set K_1 is satisfied with equality. Then $\sqrt{P_{an_{max}}/P_{an_1}}\mathbf{w}_1$ satisfies the constraints of $\mathcal{P}4$ with $\lambda_k = 1$ for $k \in \mathcal{K}_1$. Note that when the user selection coefficient $\lambda_k = 1$, the k^{th} user is selected and its associated SNR constraint in (3.b) is satisfied. Otherwise, for the secondary users which are not in $\mathcal{K}_1$, $\lambda_k = 0$, A is large enough and (4.b) is satisfied independent of $\mathbf{w}^H\mathbf{R}_k\mathbf{w}$ value. If A is larger than the upper bound for t, (4.b) is always satisfied for $\lambda_k = 0$. The upper bound for t can be found from (4.b) assuming that $\lambda_k = 1$, i.e.,

$$t \leq \max_k \frac{Tr\{\mathbf{R}_k\mathbf{w}\mathbf{w}^H\}}{\gamma_k\sigma_k^2} \leq \max_k \frac{Tr\{\mathbf{R}_k\}}{\gamma_k\sigma_k^2}LP_{an_{max}} = t_{max}$$

Hence $A \geq (\max_k \frac{Tr\{\mathbf{R}_k\}}{\gamma_k \sigma_k^2} L P_{an_{max}})$ should be satisfied. Then $\{\sqrt{P_{an_{max}}/P_{an_1}}\mathbf{w}_1, t_1, \mathcal{K}_1\}$ is a feasible solution to $\mathcal{P}4$ where the value of t in (4.a) becomes $t_1 = P_{an_{max}}/P_{an_1} \geq 1$. Hence $\mathcal{P}4$ is feasible if $\mathcal{P}2$ can be solved for at least one secondary user. Note that the feasibility of $\mathcal{P}4$ leads to feasibility of $\mathcal{P}2$ with user selection which can be shown in a similar manner. In the following part, the equivalence between the two problems is shown.

Let $\{\mathbf{w}_2, t_2, \mathcal{K}_2\}$ be the optimum solution of $\mathcal{P}4$. The optimum number of users for $\mathcal{P}2$ and $\mathcal{P}4$ can be compared in three cases, namely $|\mathcal{K}_2| > |\mathcal{K}_1|$, $|\mathcal{K}_2| < |\mathcal{K}_1|$ and $|\mathcal{K}_2| = |\mathcal{K}_1|$ respectively. These three cases are investigated in sequence.

Case 1: Assume that $|\mathcal{K}_2| > |\mathcal{K}_1|$. Then it is possible to find a solution for $\mathcal{P}2$ with the number of secondary users greater than $|\mathcal{K}_1|$ by scaling $\mathbf{w}_2$ with $\sqrt{t_2}$. Hence $|\mathcal{K}_2| > |\mathcal{K}_1|$ is not possible.

Case 2: Assume that $|\mathcal{K}_2| < |\mathcal{K}_1|$. The number of secondary users can be given as $\sum_{k=1}^{N_s} \lambda_k = |\mathcal{K}|$. Since $\{\mathbf{w}_2, t_2, \mathcal{K}_2\}$ is assumed to be the optimum solution of $\mathcal{P}4$, its objective value is greater than that of $\{\sqrt{P_{an_{max}}/P_{an_1}}\mathbf{w}_1, t_1, \mathcal{K}_1\}$, i.e., $t_2 + \beta|\mathcal{K}_2| \geq t_1 + \beta|\mathcal{K}_1|$. Manipulating this inequality results $t_{max} - t_{min} \geq t_2 - t_1 \geq \beta(|\mathcal{K}_1| - |\mathcal{K}_2|)$, where $t_{min} = 1$ from (4.g). Then we obtain, $|\mathcal{K}_1| - |\mathcal{K}_2| \leq \frac{t_{max} - t_{min}}{\beta}$, i.e.,

$$|\mathcal{K}_1| - |\mathcal{K}_2| \leq \frac{\max_k \frac{Tr\{\mathbf{R}_k\}}{\gamma_k \sigma_k^2} L P_{an_{max}} - 1}{\beta} < 1 \qquad (5)$$

where we used the condition for β i.e., $\beta > (\max_k \frac{Tr\{\mathbf{R}_k\}}{\gamma_k \sigma_k^2} L P_{an_{max}} - 1)$. Since $|\mathcal{K}_1| - |\mathcal{K}_2| \geq 1$ ($|\mathcal{K}_1|, |\mathcal{K}_2| \in \mathbb{Z}^+$) from the assumption in Case 2, (5) becomes a contradiction. Hence Case 2 is not possible.

Case 3: Therefore there is only one case left which is $|\mathcal{K}_2| = |\mathcal{K}_1|$. Hence the number secondary users for $\mathcal{P}4$ is the optimum number of users. In this case, t_2 can only be greater than or equal to t_1. If t_2 is strictly greater than t_1, then it is possible to satisfy the QoS constraints of $|\mathcal{K}_2|$ secondary users together with the interference constraints in (2.b) and (2.c). This can be done by scaling $\mathbf{w}_2$, i.e., $\frac{\mathbf{w}_2}{\sqrt{t_2}}$. The per-antenna power then satisfies the following inequality, i.e., $P_{an_2} = \frac{P_{an_{max}}}{t_2} = \frac{t_1}{t_2} P_{an_1} < P_{an_1}$. This is a contradiction. Hence $t_2 = t_1$ and $\mathbf{w}_2 = \sqrt{P_{an_{max}}/P_{an_1}}\mathbf{w}_1$ is the optimum solution. At this point, we have shown that solving $\mathcal{P}2$ with user selection is equivalent to solving $\mathcal{P}4$. Note that $\mathcal{K}_1$ and $\mathcal{K}_2$ sets may be different corresponding to different optimum realizations of the solution. Hence $\{\sqrt{P_{an_{max}}/P_{an_1}}\mathbf{w}_1, t_1, \mathcal{K}_1\}$ is also an optimum solution to $\mathcal{P}4$ since $\mathcal{K}_1$ and $\mathcal{K}_2$ give the same objective value.

In the following section, the nonlinear problem $\mathcal{P}4$ is converted into linear form in order to solve it effectively using mixed integer linear programming.

4. DISCRETE OPTIMIZATION IN LINEAR FORM

Using the same approach in [5], [16] and [17], it is possible to show that $\mathcal{P}4$ can be converted to $\mathcal{P}5$ perfectly. This new form is composed of linear expressions in terms of new optimization variables and suitable for mixed integer linear programming. The details of this conversion can be found in the above references and skipped due to space limitations. Hence the linear form is given as,

$$\mathcal{P}5: \quad \max_{\mathbf{v}_i, \mathbf{u}_{i,p}, a_{i,p}, b_{i,p}, \lambda_k} t + \beta \sum_{k=1}^{N_s} \lambda_k \qquad (6.a)$$

$$s.t. \sum_{i=1}^{M-1} \sum_{p=i+1}^{M} {\mathbf{z}^{\mathbf{s}}}_{k,i,j}^T \cdot \mathbf{u}_{i,p} + \sum_{i=1}^{M} y_{k,i}^s (1 - v_i(1))$$
$$+ A(1 - \lambda_k)\gamma_k \sigma_k^2 \geq t \gamma_k \sigma_k^2 \quad k = 1, ..., N_s \qquad (6.b)$$

$$\sum_{i=1}^{M-1} \sum_{p=i+1}^{M} {\mathbf{z}^{\mathbf{p}}}_{l,i,j}^T \cdot \mathbf{u}_{i,p}$$
$$+ \sum_{i=1}^{M} y_{l,i}^p (1 - v_i(1)) \leq t\epsilon_l \quad l = 1, ..., N_p \qquad (6.c)$$

$$\mathbf{d}^T \cdot \mathbf{u}_{i,p} + b_{i,p} = \mathbf{d}^T \cdot (-\mathbf{v}_i + \mathbf{v}_p) + a_{i,p} 2^n \qquad (6.d)$$

$$a_{i,p},\ \lambda_k,\ v_i(m),\ u_{i,p}(m) \in \{0, 1\} \qquad (6.e)$$

$$b_{i,p} \geq 0,\ b_{i,p} + 2^n(1 - u_{i,p}(1)) \leq 2^n \qquad (6.f)$$

$$-1 \leq v_i(1) + v_p(1) - 2u_{i,p}(1) \leq 0 \qquad (6.g)$$

$$\sum_{i=1}^{M} v_i(1) = M - L,\ \sum_{k=1}^{N_s} \lambda_k \geq 1,\ t \geq 1 \qquad (6.h)$$

$$\sum_{m=1}^{2^n+1} v_i(m) = 1,\ \sum_{m=1}^{2^n+1} u_{i,p}(m) = 1. \qquad (6.i)$$

$$i = 1, 2, ..., M-1,\ p = i+1, ..., M,$$
$$k = 1, ..., N_s,\ m = 1, ..., 2^n + 1$$

where $\mathbf{d} = [\ 0\ 0\ 1\ 2\ ...\ 2^n - 1\ ]^T$, $\mathbf{c} = [\ 0\ cos(0 \cdot \Delta\theta)\ cos(1 \cdot \Delta\theta)\ ...\ cos((2^n - 1) \cdot \Delta\theta)]^T$, $\mathbf{s} = [\ 0\ sin(0 \cdot \Delta\theta)\ sin(1 \cdot \Delta\theta)\ ...\ sin((2^n - 1) \cdot \Delta\theta)]^T$, ${\mathbf{z}^{\mathbf{s}}}_{k,i,j} = 2P_{an_{max}}|R_k(i,p)|$ $(cos(\angle R_k(i,p))\mathbf{c} - sin(\angle R_k(i,p))\mathbf{s})$, ${\mathbf{z}^{\mathbf{p}}}_{l,i,j} = 2P_{an_{max}}$ $|G_l(i,p)|(cos(\angle G_l(i,p))\mathbf{c} - sin(\angle G_l(i,p))\mathbf{s})$, $y_{k,i}^s = P_{an_{max}} R_k(i,i)$ and $y_{l,i}^p = P_{an_{max}} G_l(i,i)$. In $\mathcal{P}5$, t and $b_{i,p}$ are the only continuous variables. The remaining variables are all binary. When $\mathcal{P}5$ is feasible, the global optimum can be found using mixed integer linear programming [18], [19], [20], [21] with branch and cut technique.

Once the solution for $\mathbf{v}_i$'s are found, the phase angles and the antenna selection coefficients of the beamformer vector are obtained as,

$$\psi_i = \mathbf{f}_\psi^T \mathbf{v}_i, \qquad \alpha_i = 1 - v_i(1) \qquad i = 1, ..., M \qquad (7)$$

where $\mathbf{f}_\psi = [\ 0\ 0\ \Delta\theta\ ...\ (2^n - 1)\Delta\theta\ ]^T = \Delta\theta\mathbf{d}$. The selected users can be obtained by considering λ_k, $k = 1, ..., N_s$, where $\lambda_k = 1$ indicates that the user is selected.

5. SIMULATIONS

In the simulations, "Gurobi" [18] which is an efficient mixed integer linear programming solver is used by employing the branch and cut strategy. The evaluation of the proposed method is performed for Rayleigh fading channels. The maximum antenna power is selected as $P_{an_{max}} = \frac{40}{L} W$ in accordance with the maximum downlink power of LTE systems [22].

In the first experiment, $L = 3$ antennas are selected from M antennas. There are $N_s = 12$ secondary and $N_p = 3$ primary users. The interference limit for each primary user is taken as ϵ_l =-6 dB. Fig. 1 shows the average number of selected secondary users for secondary user SNR threshold, γ_k. For antenna selection, $M = 6$ and $M = 9$ antennas are considered. Different number of bits are used for discrete phase angle, namely $n = 3$, $n = 4$ and $n = 5$ respectively. The average of 100 random channel trials at each point is presented in this figure. As the SNR threshold of secondary users increases, the number of secondary users which can be served decreases. There is a significant increase in the number of serviced secondary users for the antenna selection case. Although using more bits for phase improves the solution, the effect of antenna selection is more significant. More users can be served as M is increased relative to L for the same user SNR threshold.

In the second experiment, total transmitted power is compared for different scenarios. For a fair comparison, there is no user selection and the number of serviced secondary users is the same, i.e. $N_s = 4$. There is only one primary user, $N_p = 1$, and the interference threshold is ϵ_1 =-6 dB. Fig. 2 shows the average of the total transmitted power for different secondary user SNR values. Transmitted power decreases significantly with antenna selection. Total transmitted power for antenna selection is lower than that of fixed array even with small number of bits showing the potential of the proposed approach.

In the third experiment, both SNR and interference thresholds are kept constant at γ_k =8 dB and ϵ_l =-6 dB respectively. There are $N_s = 12$ secondary users. The average number of selected secondary users for different number of primary users is shown in Fig. 3. As the number of primary users increases, it becomes more difficult to serve the secondary users. The antenna selection significantly improves the number of secondary users which can be serviced. The improvement in the number of secondary users reaches more than 5 folds for two primary users when antenna selection is performed. This shows the effectiveness of the proposed method as well as the antenna selection idea.

Table 1 shows the computational complexity of the brute force and the proposed method where the average of 10 trials are reported. As it is seen from this table, the proposed optimum method has significantly lower complexity thanks to the efficiency of the mixed integer linear programming with branch and cut technique [18].

6. CONCLUSION

Single group multicast transmit beamformer design with antenna subarray and user selection is considered. The original nonlinear problem is converted to a linear form suitable for mixed integer linear programming. The joint optimum solution is obtained effectively and it is shown that the proposed method performs significantly better compared to the optimum beamformer for the fixed array. Computational complexity is much better than the exhaustive search thanks to the efficiency of the branch and cut algorithm.

Table 1. Computational time of the proposed method (PM) and brute force search (BFS)

	$n = 4$		$n = 5$	
	PM	BFS	PM	BFS
M=6, γ_k =8 dB	3.2 s	1241.3 s	10.4 s	4513 s
M=9, γ_k =8 dB	43.8 s	4505.9 s	102 s	9723.4 s
M=6, γ_k =6 dB	11.9 s	1141 s	18.8 s	4270.5 s
M=9, γ_k =6 dB	54.7 s	2043.2 s	137.9 s	8545.7 s
M=6, γ_k =4 dB	4.9 s	706.6 s	9.5 s	2387.7 s
M=9, γ_k =4 dB	83.7 s	919.34 s	104.76 s	855.9 s

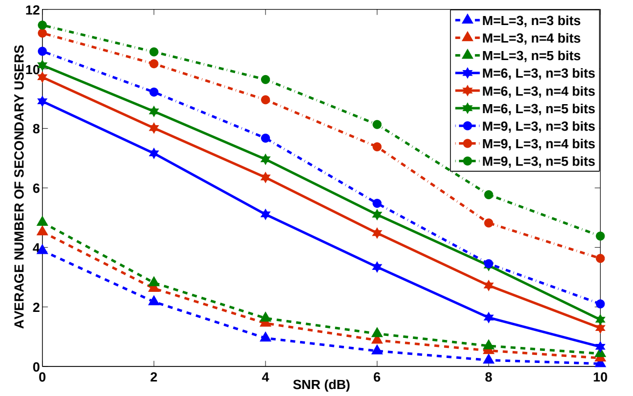

Fig. 1. Average number of serviced secondary users versus secondary user SNR values.

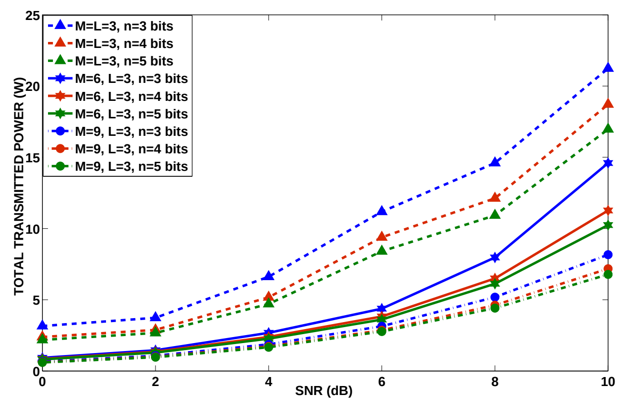

Fig. 2. Total transmitted power versus secondary user SNR values.

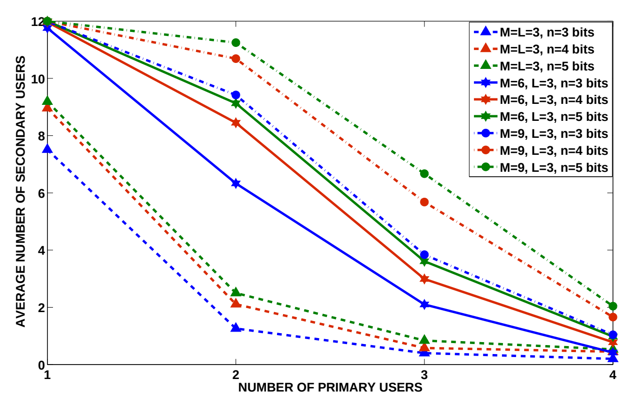

Fig. 3. Average number of secondary users versus the number of primary users.

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