DISTRIBUTED BEAMFORMING FOR COOPERATIVE NETWORKS WITH WIDELY-LINEAR PROCESSING AT THE RELAYS AND THE RECEIVER

Jens Steinwandt, Vimal Radhakrishnan, and Martin Haardt

Communications Research Laboratory, Ilmenau University of Technology P.O. Box 100565, 98684 Ilmenau, Germany, www.tu-ilmenau.de/crl {jens.steinwandt, martin.haardt}@tu-ilmenau.de

ABSTRACT

This paper addresses the distributed beamforming problem, where widely-linear (WL) processing is employed at both the relays and the receiver to take advantage of strictly second-order (SO) noncircular source signals. We consider a single-antenna communication pair in a relay network, which suffers from strong interference. Assuming perfect channel state information (CSI), we design two algorithms based on the maximization of the signal-to-interferenceplus-noise ratio (SINR) under a total relay power constraint. While the first algorithm jointly optimizes the weights at the relays and the receiver using semidefinite relaxation (SDR), the second algorithm performs a separate optimization in closed-form, requiring a substantially lower cost, but yielding almost the same performance. We show through simulations that the respective performance improvements associated with the WL processing at the relays and the receiver accumulate such that significant gains can be achieved compared to linear processing. Also, the complexity of the two algorithms is analyzed.

Index Terms— Widely-linear processing, non-circular sources, distributed beamforming, ad-hoc relay network.

1. INTRODUCTION

In distributed relay networks, the concept of cooperative diversity has been used effectively to improve the coverage, capacity, energyefficiency, and reliability of the transmissions between nodes [1], [2]. In such cooperative schemes, relay nodes assist in the communication by relaying signals through multiple independent paths in the network, which are constructively combined at the destination.

One of the most effective strategies to exploit cooperative diversity capabilities is distributed relay beamforming [3]-[11]. Among various relaying protocols, the amplify-and-forward (AF) protocol [2] is of special interest due to its simplicity. In references [3]-[6], a single communication pair is considered and the beamforming weights are computed based on perfect instantaneous channel state information (CSI) at the receiver. Thus, after the weight computation at the receiver, the weights are fed back to the relays. The studied design criteria [3]-[6] to obtain the beamforming weights minimize the total relay power subject to a target signal-to-noise ratio (SNR) at the receiver, maximize the receiver SNR subject to either individual relay power constraints or a total relay power constraint, or minimize the mean squared error (MSE) at the destination. Techniques that only rely on the statistics of the CSI were examined in [7] and [8], and extensions to multi-antenna receivers as well as multiple communication pairs were developed in [9]-[11].



Fig. 1. A network of K sources, L relays, and one destination.

Recently, the concept of widely-linear (WL) processing, originally applied to the array beamforming problem [12]-[15], has also been applied to distributed beamforming [16], [17]. WL processing takes advantage of the specific structure of second-order (SO) noncircular transmit signals [18]. Important examples of digital modulation schemes that use such signals are BPSK, PAM, O-QPSK, ASK, etc. In [16] and [17], WL processing was only applied at the relays for the case of strictly non-circular signals and weak-sense non-circular data and its conjugate version separately virtually doubles the number of relays, which results in significant performance improvements. However, WL processing has so far not been applied at the receiver or at both the relays and the receiver.

In this paper, we address the distributed beamforming problem, where WL processing is employed at both the relays and the receiver to fully exploit the SO statistics of strictly non-circular (rectilinear) source signals. We consider a single-antenna communication pair that is subject to interference. Assuming perfect CSI, two algorithms are designed based on the maximization of the signal-tointerference-plus-noise ratio (SINR) under a total relay power constraint. In addition to the relay weights, WL processing at the receiver virtually doubles the number of receive antennas and thus, introduces another set of weights to combine the received signals at the destination. While the first algorithm jointly optimizes the weights at the relays and the receiver using semidefinite relaxation (SDR), the second algorithm performs a separate optimization in closed-form, requiring a substantially lower computational cost. We show that the respective performance improvements associated with the WL processing at the relays and the receiver accumulate such that significant gains can be achieved compared to linear processing. Moreover, we compare the complexity of both algorithms and present simulations that illustrate the performance benefits.

2. SYSTEM MODEL AND WIDELY-LINEAR PROCESSING

Consider a distributed network of single-antenna units consisting of one source-destination pair, L relays and K - 1 interfering sources,

This work was supported by the International Graduate School on Mobile Communications (MOBICOM), Ilmenau, Germany.

as depicted in Fig. 1. There is no direct link between the K sources and the destination. Furthermore, we assume that the relays work in half-duplex mode and operate on the same frequency. Moreover, we have flat-fading channels and the network is perfectly synchronized. Each transmission from the sources to the destination is implemented in two consecutive time-slots. In the first time-slot, the sources simultaneously broadcast their signals to the relays and in the second time-slot, the received signals at the relays are processed by the beamforming weights and retransmitted to the destination.

In the first transmission stage, the noisy mixture of source signals received at the relays can be modeled as

$$\boldsymbol{x} = \boldsymbol{F}\boldsymbol{P}^{1/2}\boldsymbol{s} + \boldsymbol{\mu} \in \mathbb{C}^{L \times 1}, \tag{1}$$

where $F = [f_1, \ldots, f_K] \in \mathbb{C}^{L \times K}$ is the channel matrix between the sources and the relays, and $f_i = [f_{i,1}, \ldots, f_{i,L}]^T$, $i = 1, \ldots, K$, contains the channel coefficients from the *i*-th source to the relays. The vector $s = [s_1, \ldots, s_K]^T \in \mathbb{C}^{K \times 1}$ represents the uncorrelated source signals with $\mathbb{E}\{|s_i|^2\} = 1$, $P \in \mathbb{R}^{K \times K}$ is the diagonal matrix with the source powers on its diagonal, and $\mu \in \mathbb{C}^{L \times 1}$ denotes the additive zero-mean circularly symmetric complex Gaussian noise at the relays with variance σ_{μ}^2 . It is assumed that the channel coefficients, the source symbols, and the noise at the relays and the destination are statistically independent.

Due to the assumption of strictly SO non-circular source signals, the complex symbol amplitudes of each source lie on a rotated line in the complex plane. Therefore, the symbol vector s can be written as [19]

$$s = \Psi s_0, \tag{2}$$

where $\mathbf{s}_0 \in \mathbb{R}^{K \times 1}$ is a real-valued symbol vector and $\Psi = \text{diag}\{e^{j\varphi_i}\}_{i=1}^K$ contains complex phase shifts on its diagonal that can be different for each source.

2.1. Widely-Linear Processing at the Relays

Next, we apply WL processing at the relays to take advantage of the strict SO non-circularity of the transmitted signals. This is achieved by processing both the non-circular data and its complex conjugate version at the relays separately, which exploits the additional information contained in the pseudo covariance matrix of the data. To this end, we define the augmented relay vector [16]

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x}^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{F} \boldsymbol{P}^{1/2} \boldsymbol{s} \\ \boldsymbol{F}^* \boldsymbol{P}^{1/2} \boldsymbol{s}^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\mu}^* \end{bmatrix}$$
(3)

$$=\tilde{\boldsymbol{F}}\boldsymbol{P}^{1/2}\boldsymbol{s}+\tilde{\boldsymbol{\mu}}\in\mathbb{C}^{2L\times1},\tag{4}$$

where $\tilde{F} = [\tilde{f}_1, \dots, \tilde{f}_K] = [F^T, F^H \Psi^H \Psi^H]^T \in \mathbb{C}^{2L \times K}$. The extended dimensions of \tilde{x} can be interpreted as a virtual doubling of the number of relays.

In the second stage of the transmission, the retransmitted augmented signal from the relays can be expressed as

$$\tilde{\boldsymbol{r}} = \tilde{\boldsymbol{W}}^H \tilde{\boldsymbol{x}} \in \mathbb{C}^{2L \times 1},$$
(5)

where $\tilde{W} = \text{diag}\{\tilde{w}\}$ and $\tilde{w} \in \mathbb{C}^{2L \times 1}$ contains the 2L virtual beamforming weights to be designed. The physical relays transmit the widely-linear combination [16]

$$\boldsymbol{r} = \boldsymbol{W}_1^H \boldsymbol{x} + \boldsymbol{W}_2^H \boldsymbol{x}^* \in \mathbb{C}^{L \times 1}, \qquad (6)$$

where $W_l = \text{diag}\{w_l\}, l = 1, 2$, with $w_l \in \mathbb{C}^{L \times 1}$ given by $\tilde{w} = [w_1^T, w_2^T]^T$. Next, we define the augmented channel vector $\tilde{g} = [g^T, g^T]^T \in \mathbb{C}^{2L \times 1}$, where $g = [g_1, \ldots, g_L]^T$ is the channel vector between the *L* relays and the destination.

Combining (4) and (5), the received signal at the destination can be written as

$$\tilde{y} = \tilde{\boldsymbol{g}}^T \tilde{\boldsymbol{W}}^H \tilde{\boldsymbol{F}} \boldsymbol{P}^{1/2} \boldsymbol{s} + \tilde{\boldsymbol{g}}^T \tilde{\boldsymbol{W}}^H \tilde{\boldsymbol{\mu}} + n,$$
(7)

where n is the zero-mean noise at the destination with variance σ_n^2 .

2.2. Widely-Linear Processing at the Receiver

Next, we additionally perform WL processing at the receiver using the same concept as in (4). Thus, we apply the stacking operation to (7) and introduce an additional set of weights to linearly combine the data and its complex conjugate version as follows:

$$z = \boldsymbol{v}^{H} \boldsymbol{y}_{\mathrm{a}} = \begin{bmatrix} v_{1} \\ v_{2} \end{bmatrix}^{H} \begin{bmatrix} \tilde{y} \\ \tilde{y}^{*} \end{bmatrix} = v_{1}^{*} \tilde{y} + v_{2}^{*} \tilde{y}^{*}, \qquad (8)$$

where $v \in \mathbb{C}^{2 \times 1}$ is the new weight vector at the receiver. Note that in addition to virtually doubling the number of relays in the previous subsection, the extended dimensions of $y_a \in \mathbb{C}^{2 \times 1}$ correspond to a further virtual doubling of the single antenna at the receiver. Furthermore, y_a can be written as

$$\boldsymbol{y}_{\mathrm{a}} = \begin{bmatrix} \tilde{\boldsymbol{g}}^{T} \tilde{\boldsymbol{W}}^{H} \tilde{\boldsymbol{F}} \boldsymbol{P}^{1/2} \boldsymbol{s} \\ \tilde{\boldsymbol{g}}^{H} \tilde{\boldsymbol{W}}^{T} \tilde{\boldsymbol{F}}^{*} \boldsymbol{P}^{1/2} \boldsymbol{s}^{*} \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{g}}^{T} \tilde{\boldsymbol{W}}^{H} \tilde{\boldsymbol{\mu}} \\ \tilde{\boldsymbol{g}}^{H} \tilde{\boldsymbol{W}}^{T} \tilde{\boldsymbol{\mu}}^{*} \end{bmatrix} + \begin{bmatrix} n \\ n^{*} \end{bmatrix}$$
(9)

$$= \boldsymbol{G}_{\mathrm{a}}^{T} \boldsymbol{W}_{\mathrm{a}}^{H} \boldsymbol{F}_{\mathrm{a}} \boldsymbol{P}^{1/2} \boldsymbol{s} + \boldsymbol{G}_{\mathrm{a}}^{T} \boldsymbol{W}_{\mathrm{a}}^{H} \boldsymbol{\mu}_{\mathrm{a}} + \boldsymbol{n}_{\mathrm{a}}, \qquad (10)$$

where we have

$$\boldsymbol{G}_{\mathrm{a}} = \begin{bmatrix} \tilde{\boldsymbol{g}} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{g}}^* \end{bmatrix} \in \mathbb{C}^{4L \times 2}, \ \boldsymbol{W}_{\mathrm{a}} = \begin{bmatrix} \tilde{\boldsymbol{W}} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{W}}^* \end{bmatrix} \in \mathbb{C}^{4L \times 4L}, \ (11)$$

 $F_{a} = [\tilde{F}^{T}, \tilde{F}^{H} \Psi^{H} \Psi^{H}]^{T} = [f_{a_{1}}, \dots, f_{a_{K}}] \in \mathbb{C}^{4L \times K}$ and $\mu_{a} = [\tilde{\mu}^{T}, \tilde{\mu}^{H}]^{T} \in \mathbb{C}^{4L \times 1}$. The received signal after WL processing at both the relays and the receiver is obtained as

$$z = \underbrace{\sqrt{P_d} v^H G_a^T W_a^H f_{a_d} s_d}_{\text{augm. desired signal}} + \underbrace{v^H G_a^T W_a^H \sum_{k=1, k \neq d}^K \sqrt{P_k} f_{a_k} s_k}_{\text{augm. interference}} + \underbrace{v^H G_a^T W_a^H \mu_a + v^H n_a}_{\text{augm. interference}},$$
(12)

where v and $w_a = \text{diag}\{W_a\} \in \mathbb{C}^{4L \times 1}$ are the beamforming vectors to be designed and the subscript d denotes the desired user.

3. SINR MAXIMIZATION

In this section, we derive the two distributed beamforming algorithms based on the SINR maximization subject to a total relay power constraint. We optimize both the relay weights and the receiver weights. The presented development has been inspired by [9] that uses linear processing and multiple receivers. In contrast to [9], we incorporate WL processing, consider an interference scenario as shown in Fig. 1, avoid the computationally expensive eigendecomposition to solve the resulting generalized eigenvector problem to obtain v, and provide a low-complexity solution in closed-form.

3.1. Joint Optimization of the Weights

In this section, we jointly optimize the two sets of beamforming weights. The optimization problem is stated as

$$\max_{\boldsymbol{w}_{\mathrm{a}},\boldsymbol{v}} \quad \text{SINR}_{\mathrm{WL}} = \frac{P_{\mathrm{s}}}{P_{\mathrm{i}} + P_{\mathrm{n}}}$$
s. t. $P_{\mathrm{r}} \leq P_{\mathrm{max}}.$

$$(13)$$

Here, $P_{\rm s}$, $P_{\rm i}$, and $P_{\rm n}$ represent the power of the desired signal, the interference power, and the noise power at the receiver, respectively. Moreover, $P_{\rm r}$ is the total relay power and $P_{\rm max}$ is the maximum allowable relay transmit power.

Similar to [7]-[11], we next derive the required power expressions. For the total relay power P_r , we have $P_r = \mathbb{E}\{\|\tilde{r}\|^2\} = \tilde{w}^H \tilde{D} \tilde{w}$, where $\tilde{D} \in \mathbb{R}^{2L \times 2L}$ is a diagonal matrix with $\tilde{D} = \tilde{T} \odot I_{2L}$ and $\tilde{T} = \tilde{F} P \tilde{F}^H + \sigma_{\mu}^2 I_{2L}$. Note that \odot denotes the Schur-Hadamard (element-wise) matrix product. The power of the desired signal is computed as $P_s = \mathbb{E}\{|\sqrt{P_d} \ v^H G_a^T W_a^H f_{a_d} s_d|^2\} = P_d \ v^H G_a^T W_a^H f_{a_d} f_{a_d}^H W_a G_a^* v$ and the interference power can be expressed as $P_1 = \mathbb{E}\{|v^H G_a^T W_a^H \sum_{k=1,k\neq d}^{N-1} \sqrt{P_k} f_{k_a} s_k|^2\} = v^H G_a^T W_a^H \tilde{F}_a \bar{P} \bar{F}_a^H W_a G_a^* v$, where \bar{F}_a and \bar{P} only contain the interference channels and the interference powers, respectively. The expression for the noise power at the receiver can be written as $P_n = \mathbb{E}\{|v^H G_a^T W_a^H \mu_a + v^H n_a|^2\} = \sigma_{\mu}^2 \ v^H G_a^T W_a^H W_a G_a^* v + \sigma_n^2 v^H v$.

Then, the optimization problem (13) is given by

$$\max_{\boldsymbol{w}_{a}} \max_{\boldsymbol{v}} \frac{\boldsymbol{v}^{H} \boldsymbol{G}_{a}^{T} \boldsymbol{W}_{a}^{H} \boldsymbol{f}_{a_{d}} \boldsymbol{f}_{a_{d}}^{H} \boldsymbol{W}_{a} \boldsymbol{G}_{a}^{*} \boldsymbol{v}}{\boldsymbol{v}^{H} (\boldsymbol{G}_{a}^{T} \boldsymbol{W}_{a}^{H} \boldsymbol{\tilde{T}}_{a} \boldsymbol{W}_{a} \boldsymbol{G}_{a}^{*} + \sigma_{n}^{2} \boldsymbol{I}_{2}) \boldsymbol{v}}$$
(14)
s. t. $\tilde{\boldsymbol{w}}^{H} \tilde{\boldsymbol{D}} \tilde{\boldsymbol{w}} \leq P_{\max},$

where $\bar{T}_{a} = \bar{F}_{a}\bar{P}\bar{F}_{a}^{H} + \sigma_{\mu}^{2}I_{4L} \in \mathbb{C}^{4L\times 4L}$ and the scaling factor P_{d} can be omitted. Starting with the inner optimization problem, we observe that for any fixed non-zero w_{a} , the optimization with respect to v is a generalized eigenvalue problem with the solution

$$\lambda_{\max}\{(\boldsymbol{G}_{a}^{T}\boldsymbol{W}_{a}^{H}\bar{\boldsymbol{T}}_{a}\boldsymbol{W}_{a}\boldsymbol{G}_{a}^{*}+\sigma_{n}^{2}\boldsymbol{I}_{2})^{-1}\boldsymbol{G}_{a}^{T}\boldsymbol{W}_{a}^{H}\boldsymbol{f}_{a_{d}}\boldsymbol{f}_{a_{d}}^{H}\boldsymbol{W}_{a}\boldsymbol{G}_{a}^{*}\}$$
$$=\boldsymbol{f}_{a_{d}}^{H}\boldsymbol{W}_{a}\boldsymbol{G}_{a}^{*}(\boldsymbol{G}_{a}^{T}\boldsymbol{W}_{a}^{H}\bar{\boldsymbol{T}}_{a}\boldsymbol{W}_{a}\boldsymbol{G}_{a}^{*}+\sigma_{n}^{2}\boldsymbol{I}_{2})^{-1}\boldsymbol{G}_{a}^{T}\boldsymbol{W}_{a}^{H}\boldsymbol{f}_{a_{d}}, (15)$$

where $\lambda_{\max}{A}$ extracts the maximum eigenvalue of a matrix A. Note that in our case, the matrix A is rank-one. Thus, it only has the one non-zero eigenvalue in (15). Hence, we can rewrite (14) as

$$\max_{\boldsymbol{w}_{a}} \quad \boldsymbol{f}_{a_{d}}^{H} \boldsymbol{W}_{a} \boldsymbol{G}_{a}^{*} \left(\boldsymbol{G}_{a}^{T} \boldsymbol{W}_{a}^{H} \bar{\boldsymbol{T}}_{a} \boldsymbol{W}_{a} \boldsymbol{G}_{a}^{*} + \sigma_{n}^{2} \boldsymbol{I}_{2} \right)^{-1} \boldsymbol{G}_{a}^{T} \boldsymbol{W}_{a}^{H} \boldsymbol{f}_{a_{d}}$$

s. t. $\tilde{\boldsymbol{w}}^{H} \tilde{\boldsymbol{D}} \tilde{\boldsymbol{w}} \leq P_{\max}.$ (16)

Applying the matrix inversion lemma [20] to the objective function of (16), we obtain

$$\boldsymbol{f}_{\mathrm{a}_{d}}^{H}\left(\boldsymbol{B}-\boldsymbol{B}\left(\bar{\boldsymbol{T}}_{\mathrm{a}}^{-1}+\boldsymbol{B}\right)^{-1}\boldsymbol{B}\right)\boldsymbol{f}_{\mathrm{a}_{d}},$$
(17)

where $B = \frac{1}{\sigma_n^2} W_a G_a^* G_a^T W_a^H$. Next, we use the fact that $B - B \left(\bar{T}_a^{-1} + B \right)^{-1} B = \bar{T}_a^{-1} - \bar{T}_a^{-1} \left(\bar{T}_a^{-1} + B \right)^{-1} \bar{T}_a^{-1}$ [20] and the epigraph form [21] to reformulate (16) as

 $\max_{w_{2},t} t$

Utilizing the Schur complement [20], problem (18) can be equivalently expressed as

$$\begin{split} \max_{\boldsymbol{w}_{\mathrm{a}},t} & t \\ \text{s. t.} & \left(\begin{array}{c} \bar{\boldsymbol{T}}_{\mathrm{a}}^{-1} + \frac{1}{\sigma_{\mathrm{a}}^{2}} \boldsymbol{W}_{\mathrm{a}} \boldsymbol{G}_{\mathrm{a}}^{*} \boldsymbol{G}_{\mathrm{a}}^{T} \boldsymbol{W}_{\mathrm{a}}^{H} & \bar{\boldsymbol{T}}_{\mathrm{a}}^{-1} \boldsymbol{f}_{\mathrm{a}_{d}} \\ & \boldsymbol{f}_{\mathrm{a}_{d}}^{H} \bar{\boldsymbol{T}}_{\mathrm{a}}^{-1} & \boldsymbol{f}_{\mathrm{a}_{d}}^{H} \bar{\boldsymbol{T}}_{\mathrm{a}}^{-1} \boldsymbol{f}_{\mathrm{a}_{d}} - t \end{array} \right) \succeq \boldsymbol{0} \\ & \tilde{\boldsymbol{w}}^{H} \tilde{\boldsymbol{D}} \tilde{\boldsymbol{w}} \leq P_{\mathrm{max}}. \end{split}$$
(19)

Note that due to the quadratic term in the first constraint, the problem (19) is a bilinear matrix inequality problem (BMI), which is NP-hard. However, problem (19) can be solved approximately using SDR. To this end, we first write the matrix B as

$$\boldsymbol{B} = \frac{1}{\sigma_n^2} \begin{bmatrix} \tilde{\boldsymbol{W}} \tilde{\boldsymbol{g}}^* \tilde{\boldsymbol{g}}^T \tilde{\boldsymbol{W}}^H & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{W}}^* \tilde{\boldsymbol{g}} \tilde{\boldsymbol{g}}^H \tilde{\boldsymbol{W}}^T \end{bmatrix}.$$
 (20)

Then, we perform a change of optimization variables according to $\boldsymbol{u} = \tilde{\boldsymbol{D}}^{1/2} \tilde{\boldsymbol{w}} / \sqrt{P_{\max}}$ such that $\tilde{\boldsymbol{W}} = \sqrt{P_{\max}} \boldsymbol{U} \tilde{\boldsymbol{D}}^{-1/2}$, where $\boldsymbol{U} = \text{diag}\{\boldsymbol{u}\}$. Therefore, we can express \boldsymbol{B} as

$$\boldsymbol{B} = \frac{1}{\sigma_n^2} \begin{bmatrix} \boldsymbol{U} \boldsymbol{C} \boldsymbol{U}^H & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{U}^* \boldsymbol{C}^* \boldsymbol{U}^T \end{bmatrix},$$
(21)

where $C = \frac{P_{\max}}{\sigma_n^2} \tilde{D}^{-1/2} \tilde{g}^* \tilde{g}^T \tilde{D}^{-1/2}$. Next, we define $X = u u^H$ and apply the property $U C U^H = X \odot C$, which holds as U is diagonal. Therefore, after dropping the rank-one constraint, we obtain the convex semidefinite programming (SDP) problem

 $\max t$

 \mathbf{S}

$$\begin{array}{l} \text{t.} \left(\bar{\boldsymbol{T}}_{a}^{-1} + \begin{bmatrix} \boldsymbol{X} \odot \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X}^{*} \odot \boldsymbol{C}^{*} \end{bmatrix} & \bar{\boldsymbol{T}}_{a}^{-1} \boldsymbol{f}_{a_{d}} \\ \boldsymbol{f}_{a_{d}}^{H} \bar{\boldsymbol{T}}_{a}^{-1} & \boldsymbol{f}_{a_{d}}^{H} \bar{\boldsymbol{T}}_{a}^{-1} \boldsymbol{f}_{a_{d}} - t \end{pmatrix} \succeq \boldsymbol{0} \\ \text{Tr}\{\boldsymbol{X}\} \leq 1, \ \boldsymbol{X} \succeq \boldsymbol{0},$$
 (22)

which can be solved efficiently using convex optimization tools [22].

Note that due to the semidefinite relaxation, i.e., dropping the non-convex rank constraint, the optimal value X^* of the problem (22) is not necessarily of rank one in general. It represents an upper bound on the maximum value of the problem (16). If X^* happens to be of rank one, the relaxation is tight, i.e., the optimal values of (22) and (16) are equal. In this case, the desired relay beamforming vector \tilde{w} can be extracted as

$$\tilde{\boldsymbol{w}} = \sqrt{P_{\max}} \, \tilde{\boldsymbol{D}}^{-1/2} \mathcal{P} \big\{ \boldsymbol{X}^* \big\}, \tag{23}$$

where $\mathcal{P}\{\cdot\}$ is the normalized principal eigenvector operator. However, if the rank of X^* is greater than one, we only obtain an approximate solution. In this case, several randomization techniques have been proposed [23]. Interestingly, throughout our extensive simulations, the relaxed SDP problem (22) has always provided a rank-one solution.

Based on (23), we then construct the matrix W_{a} according to (11) and compute the weight vector v as

$$\boldsymbol{v} = \mathcal{P}\left\{ \left(\boldsymbol{G}_{\mathrm{a}}^{T} \boldsymbol{W}_{\mathrm{a}}^{H} \bar{\boldsymbol{T}} \boldsymbol{W}_{\mathrm{a}} \boldsymbol{G}_{\mathrm{a}}^{*} + \sigma_{n}^{2} \boldsymbol{I}_{2} \right)^{-1} \boldsymbol{G}_{\mathrm{a}}^{T} \boldsymbol{W}_{\mathrm{a}}^{H} \boldsymbol{f}_{\mathrm{a}_{d}} \boldsymbol{f}_{\mathrm{a}_{d}}^{H} \boldsymbol{W}_{\mathrm{a}} \boldsymbol{G}_{\mathrm{a}}^{*} \right\}.$$

Note that due to the fact that the matrix $G_{a}^{T}W_{a}^{H}f_{a_{d}}f_{a_{d}}^{H}W_{a}G_{a}^{*}$ is of rank one, the eigendecomposition can be avoided and v can be computed by [16]

$$\boldsymbol{v} = \frac{\bar{\boldsymbol{v}}}{\|\bar{\boldsymbol{v}}\|},\tag{24}$$

where $\bar{\boldsymbol{v}} = (\boldsymbol{G}_{\mathrm{a}}^T \boldsymbol{W}_{\mathrm{a}}^H \bar{\boldsymbol{T}} \boldsymbol{W}_{\mathrm{a}} \boldsymbol{G}_{\mathrm{a}}^* + \sigma_n^2 \boldsymbol{I}_2)^{-1} \boldsymbol{G}_{\mathrm{a}}^T \boldsymbol{W}_{\mathrm{a}}^H \boldsymbol{f}_{\mathrm{a}_d}.$

3.2. Low-Complexity Solution

In this section, we propose a low-complexity solution to problem (14), where the sets of beamforming weights at the relays and the receiver are optimized separately. Despite the suboptimality of this approach, we present a two-step procedure with simple closed-form solutions for \tilde{w} and v, respectively. In the first step, we compute the



Fig. 2. Maximum SINR versus P_{\max} for L = 5, K = 5, SIR = 0 dB, and SNR = 5 dB at the relays and the destination.

beamforming weights at the relays according to [16]. Specifically, we solve the optimization problem [16]

$$\max_{\tilde{\boldsymbol{w}}} \quad \frac{\tilde{\boldsymbol{w}}^{H} \boldsymbol{R} \tilde{\boldsymbol{w}}}{\tilde{\boldsymbol{w}}^{H} (\tilde{\boldsymbol{Q}}_{i} + \tilde{\boldsymbol{Q}}_{n}) \tilde{\boldsymbol{w}} + \sigma_{n}^{2}} \quad \text{s. t.} \quad \tilde{\boldsymbol{w}}^{H} \tilde{\boldsymbol{D}} \tilde{\boldsymbol{w}} \leq P_{\max}, \quad (25)$$

where $\tilde{\boldsymbol{R}} = P_d \tilde{\boldsymbol{h}}_d \tilde{\boldsymbol{h}}_d^H \in \mathbb{C}^{2L \times 2L}$ with $\tilde{\boldsymbol{h}}_d = \tilde{\boldsymbol{g}} \odot \tilde{\boldsymbol{f}}_d$, $\tilde{\boldsymbol{Q}}_i = \sum_{k=1, k \neq d}^K P_k \tilde{\boldsymbol{h}}_k \tilde{\boldsymbol{h}}_k^H \in \mathbb{C}^{2L \times 2L}$ with $\tilde{\boldsymbol{h}}_k = \tilde{\boldsymbol{g}} \odot \tilde{\boldsymbol{f}}_k$, and $\tilde{\boldsymbol{Q}}_n = \sigma_{\mu}^2((\tilde{\boldsymbol{g}}\tilde{\boldsymbol{g}}^H) \odot \boldsymbol{I}_{2L})$.

The solution to (25) is equal to

TT ~

$$\tilde{\boldsymbol{w}} = \sqrt{\frac{P_{\max}}{\bar{\boldsymbol{w}}^H \tilde{\boldsymbol{D}} \bar{\boldsymbol{w}}}} \bar{\boldsymbol{w}},\tag{26}$$

where \bar{w} is computed from $\breve{w} = \left(\tilde{Q}_{i} + \tilde{Q}_{n} + \frac{\sigma_{n}^{2}}{P_{\max}}\tilde{D}\right)^{-1}\tilde{h}_{d}$ by normalizing \breve{w} [16].

In the second step, we construct the matrix $W_{\rm a}$ using (11) and compute v based on (24).

3.3. Complexity Analysis

The computational complexity of the proposed algorithm for the joint optimization is dominated by solving the SDP problem in (22), which has a worst-case complexity of $\mathcal{O}((2L)^5)$, the eigendecomposition in (23) of cost $\mathcal{O}((2L)^3)$, and the matrix inversion in (24) of cost $\mathcal{O}((4L)^3)$. However, the low-complexity algorithm involving the separate optimization only requires the computation of the two matrix inversions in (26) for \tilde{w} and (24) for v of cost $\mathcal{O}((2L)^3)$ and $\mathcal{O}((4L)^3)$, respectively. Hence, the number of necessary mathematical operations for the low-complexity algorithm is considerably lower compared to the algorithm for the joint optimization.

4. SIMULATION RESULTS

In this section, we present simulations that demonstrate the performance of the proposed algorithms ("WL both cvx"/ "WL both low") for WL processing at both the relays and the receiver. For comparison purposes, we include the performance for WL processing only at the relays ("WL rel") [16]. As the modification of the presented development to the case of WL processing only at the receiver is straightforward, we also include these two versions ("WL rec cvx"/ "WL rec low"), respectively, as well as the linear version ("L-DB") [7]. In the simulations, we assume Rayleigh flat-fading channels with unit-variance channel coefficients. The SNR at the relays and



Fig. 3. Maximum SINR versus the number of sources K for L = 5, $P_{\text{max}} = 10 \text{ dB}$, SIR = 0 dB, and SNR = 5 dB at the relays and the destination.

the destination is 5 dB and the desired user transmits with 10 dB, whereas the signal-to-interference ratio (SIR) of the K - 1 interferers is 0 dB. Moreover, we assume that the sources transmit binary phase shift keying (BPSK) symbols. All the curves are obtained by averaging over 1000 Monte Carlo trials.

In Fig. 2, we display the maximum achievable SINR as a function of the maximum total relay transmit power. We have fixed the number of sources to K = 5 and the number of relays to L = 5. The non-circularity phases φ_i contained in Ψ are given by $\varphi = [0, \pi/2, \pi/8, \pi/4, \pi/16]$. It is evident that the two proposed algorithms provide the best performance and "WL both low" is very close to "WL both cvx". Both schemes can attain a performance improvement of up to 6 dB over linear processing in the non-interference case (K = 1) and even more in the interference case (K > 1). Moreover, "WL rec cvx" and "WL rel" perform identical as in both cases the number of channels is virtually doubled. The reason for the performance difference between "WL rec cvx" and "WL rec low" is that "WL rec low" does not take the non-circular interference into account for the computation of w. However, "WL rec low" still outperforms the linear version.

Fig. 3 depicts the SINR as a function of the number of sources K. The number of relays is set to L = 5 and the total relay transmit power is $P_{\rm max} = 10$ dB. The non-circularity phases of the sources are separated by $\pi/4$ starting from 0. It can be seen that the two proposed algorithms again perform close and provide the best performance. Moreover the gap between "WL rec cvx" and "WL rec low" increases as the number of sources grows.

5. CONCLUSION

In this paper, we have presented two distributed beamforming algorithms for WL processing at the relays and the receiver. They exploit the properties of strictly non-circular sources in a network consisting of a single-antenna source-destination pair, multiple relays and multiple interferers. Due to the additional WL processing at the receiver, two sets of beamforming weights have to be optimized. We have designed the weights based on the SINR maximization under a total relay power constraint. While the first algorithm optimizes the weights at the relays and the receiver jointly, the second algorithm performs a separate optimization, requiring a significantly lower cost, but yields almost the same performance. We have analyzed the complexity and shown via simulations that WL processing at both the relays and the receiver provides significant performance improvements as compared to linear processing.

6. REFERENCES

- A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part I. System description," *IEEE Transactions on Communications*, vol. 51, pp. 1927–1938, Nov. 2003.
- [2] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, pp. 3062– 3080, Dec. 2004.
- [3] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *IEEE Transactions on Information Theory*, vol. 55, no. 6, pp. 2499-2517, June 2009.
- [4] L. Zhang, W. Liu, and J. Li, "Low-complexity distributed beamforming for relay networks with real-valued implementation," *IEEE Transactions on Signal Processing*, vol. 61, no. 20, pp. 5039-5048, Oct. 2013.
- [5] J. Steinwandt and M. Haardt, "Reduced-complexity distributed beamforming algorithm for individual relay power constraints," in *Proc. IEEE 5th Int. Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAM-SAP)*, Saint Martin, French Antilles, Dec. 2013.
- [6] N. Khajehnouri and A. H. Sayed, "Distributed MMSE relay strategies for wireless sensor networks," *IEEE Transactions* on Signal Processing, vol. 55, no. 7, pp. 3336–3348, July 2007.
- [7] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z.-Q. Luo, "Distributed beamforming for relay networks based on second-order statistics of the channel state information," *IEEE Transactions on Signal Processing* vol. 56, no. 9, pp. 4306-4316, Sep. 2008.
- [8] J. Li, A. Petropulu, and H. Poor, "Cooperative transmission for relay networks based on second-order statistics of channel state information," *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 1280-1291, Mar. 2011.
- [9] K. L. Law, A. B. Gershman, and S. Shahbazpanahi, "Distributed network beamforming with a multi-antenna receiver," in *Proc. IEEE Int. Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, San Francisco, CA, June 2011.
- [10] S. Fazeli-Dehkordy, S. Shahbazpanahi, and S. Gazor, "Multiple peer-to-peer communications using a network of relays," *IEEE Transactions on Signal Processing*, vol. 57, no. 8, pp. 3053-3062, Aug. 2009.
- [11] H. Chen, A. B. Gershman, and S. Shahbazpanahi, "Distributed peer-to-peer beamforming for multiuser relay networks," in *Proc. IEEE Int. Conf. on Acoust., Speech, Signal Processing (ICASSP)*, Taipei, Taiwan, Apr. 2009.

- [12] T. McWhorter and P. Schreier, "Widely-linear beamforming," in Proc. of the Thirty-Seventh Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, Nov. 2003.
- [13] P. Chevalier, J. P. Delmas, and A. Oukaci, "Optimal widely linear MVDR beamforming for noncircular signals," in *Proc. IEEE Int. Conf. on Acoust., Speech, and Signal Processing* (*ICASSP*), Taipei, Taiwan, Apr. 2009.
- [14] N. Song, J. Steinwandt, L. Wang, R. C. de Lamare, and M. Haardt, "Non-data-aided adaptive beamforming algorithm based on the widely linear auxiliary vector filter," in *Proc. IEEE Int. Conf. on Acoust., Speech, and Signal Processing* (*ICASSP*), Prague, Czech Republic, May 2011.
- [15] J. Steinwandt, R. C. de Lamare, L. Wang, N. Song, and M. Haardt, "Widely linear adaptive beamforming algorithm based on the conjugate gradient method," in *Proc. Int. ITG Workshop on Smart Antennas (WSA)*, Aachen, Germany, Feb. 2011.
- [16] J. Steinwandt and M. Haardt, "Optimal widely-linear distributed beamforming for relay networks," in *Proc. IEEE Int. Conference on Acoustics, Speech, and Signal Processing* (*ICASSP*), Vancouver, Canada, May 2013.
- [17] J. Steinwandt and M. Haardt, "Widely-linear distributed beamforming for weak-sense non-circular sources based on relay power minimization," in *Proc. 10-th International Symposium on Wireless Communications Systems (ISWCS)*, Ilmenau, Germany, Aug. 2013.
- [18] P. J. Schreier and L. L. Scharf, *Statistical Signal Processing of Complex-Valued Data: The Theory of Improper and Non-circular Signals*, Cambridge, U.K.: Cambridge Univ. Press, 2010.
- [19] J. Steinwandt, F. Roemer, M. Haardt, and G. Del Galdo, "R-dimensional ESPRIT-type algorithms for strictly secondorder non-circular sources and their performance analysis," *IEEE Transactions on Signal Processing*, vol. 62, no. 18, pp. 4824–4838, Sep. 15, 2014.
- [20] R. A. Horn and C. R. Johnson, *Matrix Analysis*, New York: Cambridge University Press, 1988.
- [21] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, U.K.: Cambridge University Press, 2004.
- [22] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," available at http://stanford.edu/~boyd/cvx, June 2009.
- [23] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Transactions on Signal Processing*, vol. 54, no. 6, pp. 2239-2251, June 2006.