WEAK INTERFERENCE DIRECTION OF ARRIVAL ESTIMATION IN THE GPS L1 FREQUENCY BAND

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ABSTRACT

Due to its low received power, a GPS signal is vulnerable to both intentional and unintentional interferences. In this paper, the problem of estimating the direction of arrival of a weak GPS interference, which has the same power level as the GPS signals or is even weaker than them, using a GPS antenna array is considered.

To achieve this, a multiple subspace projection algorithm is proposed to cancel GPS signals which are treated as relatively strong interfering sources. Comparisons with the Partitioned Subspace Projection (PSP) method are presented using simulations. Experimental results show that the DOA of an interference with an SNR of -20dB in the GPS L1 band can be accurately estimated[1].

***Index Terms*—** GPS interference, subspace projection, DOA, antenna array

1. INTRODUCTION

The GPS signal is vulnerable to both intentional and unintentional interferences due to its low received power. The need to localize GPS interference sources is becoming more pressing as more systems rely on GPS, while GPS jammers are becoming more widely available. Thus GPS jamming source localization is increasingly important especially for safety critical applications such as airports or machine guidance [1].

Three main methods have been proposed to localize GPS interferences: the first method uses the Carrier to Noise Density (C/No) values on a network of GPS receivers to estimate the ranges of the interference source and thus estimate its position [2-5]; the second method attempts to locate the interference using widely separated antennas by enabling the Time Difference of Arrival (TDOA) to be estimated [6-8]; the third approach estimates the Direction of Arrival (DOA) of the interference by using adaptive antenna array processing techniques [9-11]. This paper extends current antenna array based interference DOA estimation work to be able to accurately estimate the DOA of either wideband or narrowband weak GPS interference sources with signal to noise ratios (SNRs) less than -20dB, thus extending the range at which DOAs can be estimated. At this power level, the GPS signals are usually stronger than the interference signals, and thus can be treated as "interferences" that need to be cancelled before accurate DOA estimation can be carried out.

In the adaptive antenna array processing area, spatial filtering [12] and subspace projection based techniques [13] have been used to reject GPS signals from certain directions, but these methods reduce the array processing gain for the intended signal as one spatial degree of freedom is required to mitigate each interference. Furthermore, if the number of antennas is less than the number of GPS signals, array processing is not capable of rejecting these signals as the least squares equations become underdetermined.

In the GPS area, several methods have been proposed to cancel the unwanted GPS signals from the input data in order to detect a much weaker signal. The classical problem is the near-far problem where a strong GPS signal masks a much weaker one. This is also known as the Coarse /Acquisition (CA) code cross correlation or GPS civilian signal self-interference problem. In [14], a Successive Interference Cancellation (SIC) technique was proposed to subtract strong GPS signals by reconstructing the strong GPS signals using CA code information derived from the tracking loops of a conventional detector. A Partitioned Subspace Projection (PSP) method [15, 16], removes the unwanted GPS signals by projecting the received signal onto the orthogonal subspace of the strong GPS signals. Compared with [14], the PSP method uses a least squares filter to produce better amplitude estimation of the strong signals. The PSP method was further studied in [17, 18] and proved to be independent of the received signal phase which was convenient for non-coherent receivers. To solve the quantization problem when using a low-end GPS receiver with a one or two bit ADC, the adaptive orthogonalization using constraints method [19, 20] and the Delayed Parallel Interference Cancellation (DPIC) method [21, 22] were proposed. The adaptive orthogonalization, using a constraint method, reconstructs the despreading codes making them orthogonal to the strong GPS signals and nearly parallel to

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the weak GPS signals, but it cannot be used to cancel the GPS signals directly. The Delayed Parallel Interference Cancellation (DPIC) method subtracts the cross correlation of the strong GPS signals from the correlation output at the post-correlation stage. The performance of DPIC is slightly worse than PSP because it is designed to have a lower computational burden [22]. However, due to their single rank least mean squares estimation structure, both PSP and DPIC methods can only cancel a line of sight (LOS) GPS signal and thus are unable to cancel multipath components of the unwanted GPS signals or distortions due to a non-ideal frequency response of the GPS front end filters.

In this paper, we extend the subspace projection concept in [15] and [16], to a Multiple Subspace Projection (MSP) method. This MSP method assumes the received GPS L1 signal is received after it is transformed by a Finite Impulse Response (FIR) filter, and thus its signal subspace becomes multi-dimensional while in the PSP method it is assumed to be rank one. Due to its multi-dimensional subspace structure, MSP achieves better cancellation of the received GPS signal if the signal is band-limited, has multipath components, or has fractional delays.

The remainder of the paper is structured as follows: The Multiple Subspace Projection (MSP) method is proposed in Section 2; simulation results are presented in Section 3; in Section 4, the experiment results of the estimation the DOA of a weak GPS interference using this cancellation method are described. Finally conclusions are given in Section 5.

2. GPS SIGNAL CANCELLATION

2.1. Effect of GPS signals on Interference DOA Estimation

Typically the power of the GPS signals is below the total receiver noise power, i.e., SNRs range from -15dB to -30dB, so they usually have no or very limited effects on the DOA estimation of a strong interference. However, if the INR of the interference is at level of -20dB, the GPS signals are now stronger than the interference and will affect the estimation of the DOA of the interference.

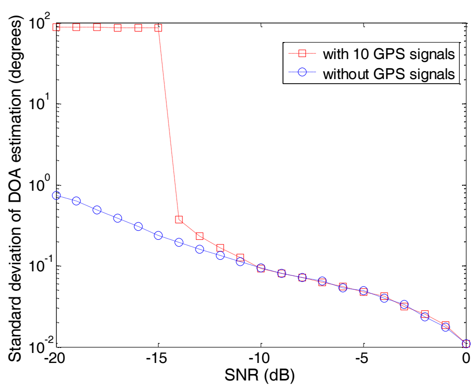

Fig.1 Standard Deviation of the GPS interference DOA estimation using the MUSIC algorithm in the presence of 10 GPS signals (red) and without GPS signals (blue). In this simulation, the antenna array is an 8 element half wavelength uniformly spaced linear array (ULA), the number of snapshots K = 10000, 10 GPS signals are uniformly distributed from 0 to 180 degrees with an SNR of -20dB, each standard deviation is estimated using 200 independent simulations.

Fig.1 shows the standard deviation of the interference DOA estimates using the MUSIC (MUltiple SIgnal Classification algorithm) algorithm with and without GPS signals. In this simulation, the estimation of the DOA of the interference is basically unaffected while its INR is above -10dB. When the INR is between -10dB and -14dB, the interference DOA variance is clearly affected by the GPS signals, while below -14dB the MUSIC algorithm fails completely in estimating the DOA of the interference. Thus the GPS signals need to be cancelled if the DOAs of interferences with an INR of less than -20 dB need to be accurately estimated.

2.2. GPS Signal Cancellation Using Multiple Subspace Projection

The vector form of a single channel down-converted baseband received data $\underline{x}(n)$ with K GPS L1 signals and a single GPS interference is

$$\underline{x}(n) = \sum_{k=1}^{K} \underline{q}_k(n) + \underline{v}(n) + \underline{w}(n) \tag{1}$$

where $\underline{q}_k(n) = [q_k[n],\ q_k[n-1], \dots, q_k[1]]^T$ is the k^{th} GPS L1 signal which is orthogonal to other GPS signals, $\underline{v}(n) = [v[n], v[n-1], \dots, v[1]]^T$ is the interfering signal whose DOA is to be estimated, and $\underline{w}(n) = [w[n], w[n-1], \dots, w[1]]^T$ is Additive White Gaussian Noise (AWGN).

For the k^{th} GPS L1 signal, the code phase, Doppler frequency and the data modulation are obtained through software GPS acquisition and used to construct the signal $\underline{s}_k$ which is the k^{th} transmitted baseband GPS L1 signal with unity gain. The transmitted signal $\underline{s}_k(n)$ is given by

$$s_k[n] = c_k[n - \tau_k] \cdot D_k \cdot e^{j2\pi f_k n} \tag{2}$$

where $c_k[n]$ is the Direct-Sequence Spread Spectrum (DS-SS) signal (the C/A code), τ_k is the code phase, D_k is the data modulation and f_k is the Doppler frequency.

In order to cancel the k^{th} GPS signal $\underline{q}_k(n)$ in the received data $\underline{x}(n)$, the relationship between the unknown signal $\underline{q}_k(n)$ and the known signal $\underline{s}_k(n)$ need to be modeled. In [15] and [16], it is modeled as:

$$\underline{q}_k(n) = A_k e^{j\varphi_k} \underline{s}_k(n) \tag{3}$$

where A_k is the amplitude of the k^{th} GPS signal and φ_k is the carrier phase mismatch between the two signals. This simple but effective model assumes that in the received data

$\underline{x}(n)$, the only parameters of the k^{th} GPS signal, $\underline{q}_k(n)$, that need be estimated are the amplitude and carrier phase difference. However, if the GPS signal has multipath components, fractional delays or is band-limited due to the receiver front end filters, this model will be mismatched.

In order to cancel the GPS signals even if they have multipath components, fractional delays or are band-limited, an L^{th} order Finite Impulse Response (FIR) model to describe the relationship between $\underline{q}_k(n)$ and $\underline{s}_k(n)$ is proposed as shown in Fig.2.

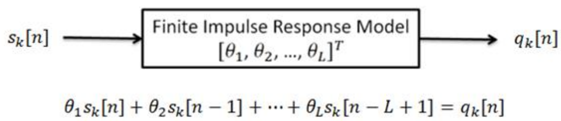

Fig.2 Finite Impulse Response (FIR) model

The vector form of this model can be expressed as

$$\underline{q}_k(n) = \mathbf{S}_k(n)\underline{\theta}_k \tag{4}$$

where $\underline{\theta}_k$ are the weights of the FIR system for the k^{th} GPS signal and $\mathbf{S}_k(n)$ is given by

$$\mathbf{S}_k(n) = [\underline{s}_k(n), \quad \underline{s}_k(n-1), \quad \cdots \quad \underline{s}_k(n-L+1)] \tag{5}$$

where L is the order of the FIR system. In practice, we set L from 15 to 100 taps.

For the k^{th} GPS L1 signal $\underline{q}_k(n) = \mathbf{S}_k(n)\underline{\theta}_k$, the received signal $\underline{x}(n)$ can be rewritten as

$$\underline{x}(n) = \mathbf{S}_k(n)\underline{\theta}_k + \mathbf{S}_o(n)\underline{\theta}_o + \underline{v}(n) + \underline{w}(n) \tag{6}$$

where $\mathbf{S}_o = [\mathbf{S}_1\ \mathbf{S}_2 \ldots \mathbf{S}_{k-1}\ \mathbf{S}_{k+1} \ldots \mathbf{S}_K]$ are the other GPS signals and their respective FIR system weights are $\underline{\theta}_o = [\underline{\theta}_1{}^T\ \underline{\theta}_2{}^T \ldots \underline{\theta}_{k-1}{}^T\ \underline{\theta}_{k+1}{}^T \ldots\ \underline{\theta}_K{}^T]^T$.

In order to cancel the k^{th} GPS signal, the subspace principle is utilized. The subspace orthogonal to that spanned by the k^{th} GPS signal is

$$\mathbf{P}_k = \mathbf{I} - \mathbf{S}_k\left(\mathbf{S}_k{}^H\mathbf{S}_k\right)^{-1}\mathbf{S}_k{}^H \tag{7}$$

The rank of $\mathbf{S}_k{}^H\mathbf{S}_k$ is the order of the FIR system L, so the orthogonal subspace $\mathbf{P}_k$ is a multi-dimensional subspace while it is only rank one in the PSP method [15, 16]. The received data $\underline{x}(n)$ now is projected onto $\mathbf{P}_k$ to cancel the k^{th} GPS signal $\mathbf{S}_k\underline{\theta}_k$

$$\begin{aligned}\mathbf{P}_k\underline{x} &= \left(\mathbf{I} - \mathbf{S}_k\left(\mathbf{S}_k{}^H\mathbf{S}_k\right)^{-1}\mathbf{S}_k{}^H\right)\left(\mathbf{S}_k\underline{\theta}_k + \mathbf{S}_o\underline{\theta}_o + \underline{v} + \underline{w}\right) \\ &= \mathbf{S}_o\underline{\theta}_o + \underline{v} + \underline{w} - \mathbf{S}_k\left(\mathbf{S}_k{}^H\mathbf{S}_k\right)^{-1}\mathbf{S}_k{}^H\left(\mathbf{S}_o\underline{\theta}_o + \underline{v} + \underline{w}\right)\end{aligned} \tag{8}$$

In (8), the received k^{th} GPS signal $\mathbf{S}_k(n)\underline{\theta}_k$ is fully cancelled and $\mathbf{S}_o(n)\underline{\theta}_o + \underline{v}(n) + \underline{w}(n)$ is the desired projection result which only includes the other GPS signals $\mathbf{S}_o(n)\underline{\theta}_o$, the GPS interference $\underline{v}(n)$ and the noise $\underline{w}(n)$. So the projection error is

$$e_k(n) = \mathbf{S}_k\left(\mathbf{S}_k{}^H\mathbf{S}_k\right)^{-1}\mathbf{S}_k{}^H\left(\mathbf{S}_o(n)\underline{\theta}_o + \underline{v}(n) + \underline{w}(n)\right) \tag{9}$$

So,

$$\mathbf{P}_k\underline{x}(n) = \mathbf{S}_o(n)\underline{\theta}_o + \underline{v}(n) + \underline{w}(n) + e_k(n) \tag{10}$$

In (9), due to the CDMA structure of the GPS signals, the GPS signal is uncorrelated with the interference $\underline{v}(n)$ and the white Gaussian noise $\underline{w}(n)$, so $E\{\boldsymbol{S}_k(n)^H\underline{v}(n)\} = 0$ and $E\{\boldsymbol{S}_k(n)^H\underline{w}(n)\} = 0$. Also because of the limited GPS gold code cross-correlation dynamic range, which is about 23.9 dB within 1ms [2], $\mathbf{S}_k{}^H\mathbf{S}_o$ is not strictly but approximately 0 [16]. This means the projection error is very small and can be approximated by 0 ($e_k \approx 0$). Furthermore, as discussed below, the cancellation is iteratively applied on all the GPS signals, so this error will be even smaller after all the GPS signals are subtracted.

So, the projection result is

$$\mathbf{P}_k\underline{x}(n) \approx \mathbf{S}_o\underline{\theta}_o + \underline{v}(n) + \underline{w}(n) \tag{11}$$

where the k^{th} GPS signal $\underline{q}_k(n) = \mathbf{S}_k(n)\underline{\theta}_k$ is cancelled from the received signal $\underline{x}(n)$.

Multiple GPS signals are able to be cancelled by applying the individual projection multiple times, or by combining matrices $\mathbf{S}_k$ and $\mathbf{S}_o$ into $\mathbf{S} = [\mathbf{S}_k\ \mathbf{S}_o]$ and being included in the projection matrix to be cancelled. These GPS signal cancellations are applied on all the antenna array receiver channels whilst the GPS acquisition information can be obtained from only one antenna, or a beam steered at the GPS satellite.

This multiple dimensional subspace structure based on the FIR system model has some important benefits in GPS signal cancellation, as it will accurately subtract the GPS signals even if the GPS signal has multipath components, fractional delays or is band-limited due to the receiver front end filters.

3. SIMULATION RESULTS

The GPS code phase and data modulation are assumed to be accurately estimated and thus not included in the simulations. The sampling rate is 4 MHz, the data length is 20 ms and the FIR model of the MSP method has L = 60 taps.

In simulation 1, there is only one GPS signal (PRN 1) and thermal noise in the received data. The GPS signal (PRN 1) has a multipath component. The multipath

[2] Known as the CA code cross correlation problem, the GPS civilian signal self-interference or the GPS near-far effect.

component is 3 samples delayed and its SNR is 6 dB lower than the direct path signal. The GPS signal with Pseudo-Random Noise (PRN) code sequence 1 is intended to be cancelled. The MSP and PSP cancellation results are shown in Fig.3. Because of the multi-dimensional projection structure, MSP (red) is able to cancel both the LOS and the multipath signal, whilst PSP (green) can only partially cancel them.

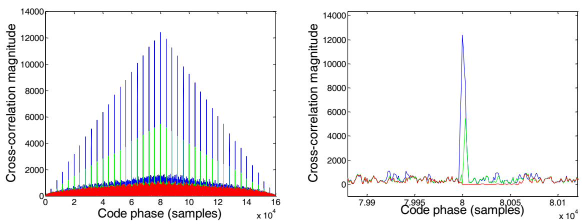

Fig.3 Cross-correlation results comparison between the MSP cancellation (red) and the PSP cancellation (green) with a multipath signal. The blue curve is the cross-correlation result before cancellation. The left graph shows the cross-correlation results for 39 correlation peaks, the right graph shows the cross-correlation results for the highest (centre) peak.

In simulation 2, there is only one GPS signal (PRN 1) and thermal noise in the received data. The GPS signal (PRN 1) has a fractional delay varying from 1 to 2 samples corresponding to ¼ and ½ chip delay. The GPS signal (PRN 1) is intended to be cancelled. As shown in Fig.4, MSP (red) is able to cancel the GPS signal in all the fractional delay conditions, whilst the performance of PSP (green) decays as the fractional delay sample increases.

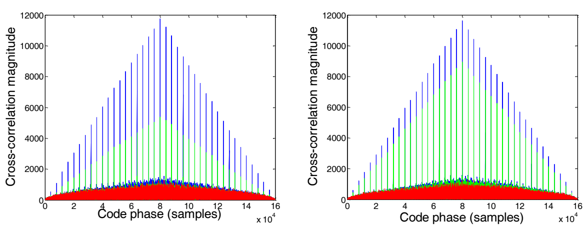

Fig.4 Cross-correlation results comparison between the MSP cancellation (red) and the PSP cancellation (green) with a fraction delay. The blue curve is the cross-correlation result before cancellation. The fractional delays are 1/4 chip (left graph) delay and 1/2 chip delay (right graph).

4. WEAK INTERFERENCE DOA ESTIMATION EXPERIMENT RESULT

Because it is prohibited to transmit a signal in the GPS L1 frequency band, it is hard to find a proper interference source. In the experiment, the interference source was a desktop computer. The interference signal to noise ratio was about -20dB and the distance was about 7 meters from the antenna array, so it was not capable of interfering with the GPS signals during the experiment. Although it was a narrowband interference, the DOA was estimated using a wideband signal assumption. The antenna array was a 7 element uniform circular array with an additional 1 element in the centre. The radius of the circular array was 10 cm.

The interference DOA was estimated by the MUSIC algorithm where the two eigenvectors corresponding to the largest two eigenvalues determined the signal subspace. The DOA estimation results with and without GPS subtraction are shown in Fig.5. The covariance matrix was pre-whitened to mitigate the coloured noise in the system. Before GPS cancellation, the estimated DOA of the interference is interfered by the GPS signals. After GPS signal cancellation the estimated azimuth angle of the interference was 179.4^0 compared with a true value of 180^0. The clean MUSIC spectrum suggested a good estimation result.

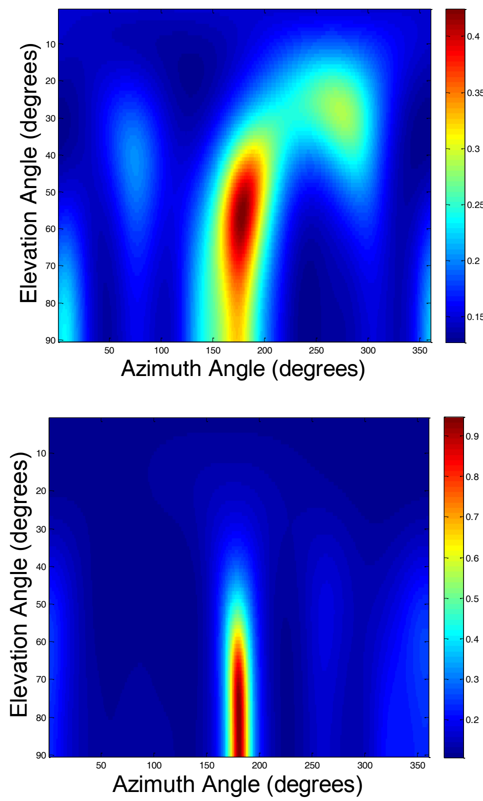

Fig.5 DOA estimations using MUSIC with GPS subtraction (lower) and without GPS subtraction (upper).

5. CONCLUSIONS

In this paper, a Multiple Subspace Projection method is proposed to cancel the GPS signals in the received data of a GPS antenna array to enable accurate DOA estimation of weak GPS interferences. This method is capable of subtracting the GPS signals even if the GPS signal has multipath components, fractional delays or is band-limited due to the receiver front end filters. The experiment result shows the DOA of an interference with SNR of -20dB in the GPS L1 band was accurately estimated.

In the future, this work can be improved by applying the Kalman filter or its extensions to track the estimated FIR linear system, which will help and improve the GPS signal cancellation performance in large data blocks, where real time application is necessary.

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