LOST-FIND: A SPECTRAL-SPACE-TIME DIRECT BLIND GEOLOCALIZATION ALGORITHM

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ABSTRACT

In the literature of direct blind geolocalization algorithms, two algorithms are mainly concurrent: DPD and LOST. The first one appears to be very sensitive to the spectral contents and the second, although presenting wide scope of scenarii with better performance than DPD, does not exploit the TDoA. This last algorithm could therefore be again improved. The purpose of this paper is to propose a new algorithm LOST-FIND which exploits (for non monochromatic sources) the structured TDoA between stations thanks to a spectral estimation of sources. The proposed algorithm is an iterative extension of LOST algorithm initially designed for monochromatic sources. Simulations confirm the expected improvements versus DPD and LOST.

Index Terms— AoA estimation, TDoA, direct geolocalization, performance

1. INTRODUCTION

The context of this work is the geolocation of multiple radiating sources with multiple separated arrays (also called stations). Traditional technics [1] rely on a two steps strategy where measurements such as Angle of Arrival (AoA), Time Differential of Arrival (TDoA), Frequency Differential of Arrival (FDoA), *etc.* are obtained from each array in a first step and combined in a second step to estimate the sources position. The sources impingings on each station are assumed to be narrowband and far-field. For instance, the AoAs of sources are estimated by each station independently in the first step and, in a second step, the locations of source are computed from the AoAs (*e.g.*, by triangulation) [1].

However, such 2-steps methods present drawbacks [3] and are generally less efficient than the direct algorithms (1-step methods) [4]-[8]. The recently direct geolocation algorithms [7], [8] use the array of arrays (global array) to directly estimate the sources locations. Unfortunately, the sources are generally wideband with respect to that global array [12]. These algorithms transform this broadband problem into narrowband problems in order to efficiently apply algorithms such as MUSIC [2]. For that, DPD (Direct Position Determination) [7] decomposes the received signal into K narrowbands subsignals and LOST (LOcalization by Space-Time) [8] uses a space-time approach. Even if the DPD exploits the TDoA of sources between stations, its principal drawback consists in the high spectrum sensitivity in the sense that an absence of signal or even a low SNR on a FFT bin (subband) leads to outliers on the position estimation. On the other hand, the LOST algorithm has not this drawback as it has been designed to be spectrum adaptive and, consequently, it is not sensitive to the source spectrum. Moreover, in the case of monochromatic sources, it is optimal. In presence of non monochromatic sources it would be interesting to exploit the TDoA between stations.

The purpose of this paper is to propose a new algorithm, named LOST-FIND (LOcalization by Space Time with Frequency Identification in Narrowband Decomposition), which is spectrum adaptive and explicitly exploits the TDoA structure contained in the model. For that, we start with the space-time approach of LOST and exploit the modeling of the space-time observation in order to use in one hand the TDoA and in a second hand a source selectivity into the frequency domain.

2. SIGNAL MODELING AND PROBLEM FORMULATION

2.1. Signal Modeling and hypothesis

The global geolocation system is composed of L perfectly synchronized remote stations with M_l sensors for $1 \leq l \leq L$. In presence of Q sources, the associated observation vectors, $\mathbf{x}_l(t)$, whose components $x_m^l(t)$ ($1 \leq m \leq M_l$) are the signals complex envelopes at the output of the antennas stemmed from the Line of Sight (LoS) assumption, are thus:

$$\mathbf{x}_l(t) = \sum_{q=1}^{Q} \rho_{l,q} \mathbf{a}_l(\theta_l(\mathbf{p}_q)) s_q(t - \tau_l(\mathbf{p}_q)) + \mathbf{n}_l(t) \tag{1}$$

where $s_q(t)$ is the complex envelope of the q-th source, $\mathbf{n}_l(t)$ is a spatially white noise vector and $\mathbf{a}_l(\theta_l(\mathbf{p}))$ is the steering vector, noted $\mathbf{a}_l(\mathbf{p})$ in the remainder of the paper. In addition, $\rho_{l,q}$, $\theta_l(\mathbf{p}_q)$ and $\tau_l(\mathbf{p}_q)$ are the complex attenuation, the AoA and Time of Arrival (ToA) of the q-th source arriving on the l-th station respectively. Let us note $\Delta\tau_{ij}(\mathbf{p}) = \tau_i(\mathbf{p}) - \tau_j(\mathbf{p})$ the TDoAs. The carrier (resp. sampling) frequency of $x_m^l(t)$ is f_0 (resp. F_e). For sake of simplicity the signal of the q-th source is:

$$s_q(t) = \sum_{k=1}^{K_q} \alpha_{qk} e^{i2\pi f_{qk} t} \text{ and } |f_{qk}| \leq \frac{F_e}{2} \quad (2)$$

where f_{qk} and α_{qk} are the residual frequency and attenuation of the k-th component of the q-th source. The sources are decorrelated between each other. Then, the associated bandwidth is $B_q = \max_k(f_{qk}) - \min_k(f_{qk})$.

The direct geolocation algorithms use the global array and the associated observation vector is then:

$$\mathbf{x}(t) = \left[\mathbf{x}_1^T(t), ..., \mathbf{x}_L^T(t)\right]^T \quad (3)$$

where $(\cdot)^T$ denotes the transpose operator. For a given source location $\mathbf{p}_q$, the global observation is generally broadband as $|s_q(t - \tau_1(\mathbf{p}_q))|, ..., |s_q(t - \tau_L(\mathbf{p}_q))|$ are different. That is why a conventional narrowband algorithm [2] cannot be used to estimate $\mathbf{p}_q$ from $\mathbf{x}(t)$. More precisely, the narrowband hypothesis is verified on the global array if and only if:

$$\max_{q,i,j} |\tau_i(\mathbf{p}_q) - \tau_j(\mathbf{p}_q)| \times B_q \ll 1 \quad (4)$$

2.2. Space-Time observation

A space-time observation takes into account that the signal $\mathbf{x}(t)$ is broadband in general. Let us define:

$$\mathbf{y}(t) = \left[\mathbf{x}^T(t), \mathbf{x}^T(t - T_e), ..., \mathbf{x}^T(t - (K-1)T_e)\right]^T \quad (5)$$

where T_e is the sampling period such that $T_e = \frac{1}{F_e}$. According to Eq.(2), the signal $s_q(t - \tau_l(\mathbf{p}_q) - (j-1)T_e)$ is:

$$s_q(t - \tau_l(\mathbf{p}_q) - (j-1)T_e) = \sum_{k=1}^{K_q} s_{q,k}(t)\, c^{j-1}(f_{qk}) z_l(\mathbf{p}_q, f_{qk}) \quad (6)$$

with $c(f) = e^{-i2\pi f T_e}$, $z_l(\mathbf{p}, f) = e^{-i2\pi f \tau_l(\mathbf{p})}$ and $s_{q,k}(t) = \alpha_{qk} e^{i2\pi f_{qk} t}$. The space-time observation of Eq.(5) becomes:

$$\mathbf{y}(t) = \sum_{q=1}^{Q} \sum_{k=1}^{K_q} \mathbf{v}(\mathbf{p}_q, f_{qk}, \boldsymbol{\rho}_q) s_{q,k}(t) + \mathbf{n}(t) \quad (7)$$

where $\boldsymbol{\rho}_q = [\rho_{1,q}, ..., \rho_{L,q}]^T$ and

$$\begin{cases} \mathbf{v}(\mathbf{p}_q, f_{qk}, \boldsymbol{\rho}_q) = \mathbf{c}(f_{qk}) \otimes \mathbf{u}(\mathbf{p}_q, f_{qk}, \boldsymbol{\rho}_q) \\ \mathbf{u}(\mathbf{p}_q, f_{qk}, \boldsymbol{\rho}_q) = \mathbf{U}(\mathbf{p}_q)\boldsymbol{\Lambda}(\mathbf{p}_q, f_{qk})\boldsymbol{\rho}_q \end{cases} \quad (8)$$

with

$$\mathbf{U}(\mathbf{p}) = \begin{bmatrix} \mathbf{a}_1(\mathbf{p}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{a}_L(\mathbf{p}) \end{bmatrix}, \mathbf{c}(f) = \begin{bmatrix} c^0(f) \\ \vdots \\ c^{K-1}(f) \end{bmatrix} \quad (9)$$

$$\boldsymbol{\Lambda}(\mathbf{p}, f) = \begin{bmatrix} z_1(\mathbf{p}, f) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & z_L(\mathbf{p}, f) \end{bmatrix} \quad (10)$$

Thanks to Eq.(7), one could remark that the space-time observation is composed of a finite number of steering vectors $\mathbf{v}(\mathbf{p}_q, f_{qk}, \boldsymbol{\rho}_q)$ depending on the frequency f_{qk} as in a context of narrowband hypothesis.

2.3. Problem formulation

Let us move on the problem formulation which will show that the LOST algorithm does not exploit the TDoA.

LOST exploits the structure of the space-time observation covariance matrix $\mathbf{R}_y = \mathbb{E}\left[\mathbf{y}(t)\mathbf{y}^H(t)\right]$ with $\mathbb{E}[\cdot]$ is the mathematical expectation. Thus, the signal subspace of rank $R = \sum_{q=1}^{Q} K_q$ is spanned by the vectors $\mathbf{v}(\mathbf{p}_q, f_{qk}, \boldsymbol{\rho}_q)$. The parameters $(\mathbf{p}_q, f_{qk}, \boldsymbol{\rho}_q)$ for $1 \leq q \leq Q$ and $1 \leq k \leq K_q$ can then be estimated with a MUSIC approach by searching the zeros of:

$$J_{MUSIC}(\mathbf{p}, f, \boldsymbol{\rho}) = \frac{\mathbf{v}^H(\mathbf{p}, f, \boldsymbol{\rho})\boldsymbol{\Pi}^\perp \mathbf{v}(\mathbf{p}, f, \boldsymbol{\rho})}{\mathbf{v}^H(\mathbf{p}, f, \boldsymbol{\rho})\mathbf{v}(\mathbf{p}, f, \boldsymbol{\rho})} \quad (11)$$

where $\boldsymbol{\Pi}^\perp$ is the projector onto the noise subspace from $\mathbf{R}_y$. However, the optimization of the cost function $J_{MUSIC}(\mathbf{p}, f, \boldsymbol{\rho})$ can be simplified by reducing the number of interest parameters to the location vector $\mathbf{p}$. Indeed, according to Eqs.(8-10), the steering vector can be rewritten as:

$$\mathbf{v}(\mathbf{p}, f, \boldsymbol{\rho}) = \mathbf{W}(\mathbf{p})\boldsymbol{\beta}(f, \mathbf{p}, \boldsymbol{\rho}) \quad (12)$$

$$\text{where} \quad \begin{cases} \mathbf{W}(\mathbf{p}) = (\mathbf{I}_K \otimes \mathbf{U}(\mathbf{p})) \\ \boldsymbol{\beta}(f, \mathbf{p}, \boldsymbol{\rho}) = \mathbf{c}(f) \otimes (\boldsymbol{\Lambda}(\mathbf{p}, f)\boldsymbol{\rho}) \end{cases} \quad (13)$$

where $\mathbf{I}_K$ is the identity matrix of dimension K. In LOST, the steering vector $\mathbf{v}(\mathbf{p}, f, \boldsymbol{\rho})$ in Eq.(11) is thus replaced by $\mathbf{W}(\mathbf{p})\boldsymbol{\beta}(f, \mathbf{p}, \boldsymbol{\rho})$ and, for each $\mathbf{p}$, the criterion $J_{MUSIC}(\mathbf{p}, f, \boldsymbol{\rho})$ is minimized with respect to $\boldsymbol{\beta}(f, \mathbf{p}, \boldsymbol{\rho})$. Thanks to the Rayleight's quotient [9], [10], we define the LOST criterion:

$$J_{LOST}(\mathbf{p}) = \lambda_{\min}\left\{\mathbf{Q}_2^{-1}(\mathbf{p})\mathbf{Q}_1(\mathbf{p})\right\} \quad (14)$$

$$\text{where} \quad \begin{cases} \mathbf{Q}_1(\mathbf{p}) = \mathbf{W}^H(\mathbf{p})\boldsymbol{\Pi}^\perp \mathbf{W}(\mathbf{p}) \\ \mathbf{Q}_2(\mathbf{p}) = \mathbf{W}^H(\mathbf{p})\mathbf{W}(\mathbf{p}) \end{cases} \quad (15)$$

where $\lambda_{\min}\{\mathbf{A}\}$ is the minimum eigenvalue of $\mathbf{A}$. In LOST, the sources location $\mathbf{p}_q$ for $1 \leq q \leq Q$ can then be estimated by searching the Q zeros of $J_{LOST}(\mathbf{p})$. The advantage is that the LOST criterion only depends on $\mathbf{p}$. However, LOST is based on the separation of $\mathbf{W}(\mathbf{p})$ and the vector $\boldsymbol{\beta}(f, \mathbf{p}, \boldsymbol{\rho})$ which depends on the sources TDoAs according to Eqs.(6), (10) and (13). That is why the LOST algorithm cannot exploit the stations TDoAs. In addition, as the q-th source is associated to K_q frequencies f_{qk} (*cf.* Eq.(2)), a source of location $\mathbf{p}_q$ is associated to K_q steering vectors $\mathbf{v}(\mathbf{p}_q, f_{qk}, \boldsymbol{\rho}_q)$ and only one matrix $\mathbf{W}(\mathbf{p}_q)$. This last remark shows that the frequency diversity of the sources is not optimally exploited in LOST (*cf.* Eq.(14)). The purpose of this paper is to exploit the structure of the vectors $\boldsymbol{\beta}(f_{qk}, \mathbf{p}_q, \boldsymbol{\rho}_q)$ for $1 \leq k \leq K_q$ to take into account the TDoA and the spectrum diversity of sources for the estimation of the location $\mathbf{p}_q$.

3. NEW SPACE-TIME GEOLOCATION ESTIMATION

The aim of this approach is to propose a spectrum adaptive algorithm which explicitly and simultaneously uses the link between the TDoA, AoA and the sources coordinates of the spatio-temporal process. This strategy will be incorporated in a relaxed optimization algorithm.

3.1. Estimation of the source frequencies from its location

We first desire to estimate the frequencies set $\mathcal{F}_q$ of the q-th source from its known location $\mathbf{p}_q$. According to Eqs.(11), (14) and (15), the eigenvector $\mathbf{v}_{\min}$ associated to the minimum eigenvalue of $\mathbf{Q}_2^{-1}(\mathbf{p}_q)\mathbf{Q}_1(\mathbf{p}_q)$ verifies:

$$J_{MUSIC}(\mathbf{p}_q, f \in \mathcal{F}_q, \boldsymbol{\rho}_q) = \frac{\mathbf{v}_{\min}^H \mathbf{Q}_1(\mathbf{p}_q)\mathbf{v}_{\min}}{\mathbf{v}_{\min}^H \mathbf{Q}_2(\mathbf{p}_q)\mathbf{v}_{\min}} \tag{16}$$

where $\mathcal{F}_q = \{f_{qk} : 1 \leq k \leq K_q\}$.

It is known that, assuming that $\mathbf{p}_q$ is the true source location and if $\boldsymbol{\rho}_q$ is the true complex source attenuation, $J_{MUSIC}(\mathbf{p}_q, f \in \mathcal{F}_q, \boldsymbol{\rho}_q)$ is null. As a consequence, $\mathbf{Q}_2^{-1}(\mathbf{p}_q)\mathbf{Q}_1(\mathbf{p}_q)$ has K_q null eigenvalues with the associated eigenvectors:

$$\mathbf{v}_{\min}(j) = \sum_{k=1}^{K_q} \xi_{kj} \boldsymbol{\beta}(f_{qk}, \mathbf{p}_q, \boldsymbol{\rho}_q) \quad \text{for } 1 \leq j \leq K_q \tag{17}$$

with ξ_{kj} is one coefficient of a change of basis matrix.

However, f_{qk} and $\boldsymbol{\rho}_q$ are unknown. Thus, a cost function independent of $\boldsymbol{\rho}$ is preferred. Hence, we write the vector $\boldsymbol{\beta}(f, \mathbf{p}_q, \boldsymbol{\rho})$ as, using Eqs.(9) and (13):

$$\boldsymbol{\beta}(f, \mathbf{p}_q, \boldsymbol{\rho}) = \boldsymbol{\Lambda}_K(\mathbf{p}_q, f)\boldsymbol{\rho} \tag{18}$$
$$\boldsymbol{\Lambda}_K(\mathbf{p}_q, f) = \mathbf{c}(f) \otimes \boldsymbol{\Lambda}(\mathbf{p}_q, f) \tag{19}$$

Then, the frequencies f_{qk} can be estimated by a MUSIC approach. Indeed, the K_q vectors $\boldsymbol{\beta}(f, \mathbf{p}_q, \boldsymbol{\rho})$ are orthogonal to the following projector onto the noise subspace:

$$\boldsymbol{\Pi}_{\mathbf{v}}^{\perp} = \mathbf{I}_{KL} - \sum_{j=1}^{K_q} \mathbf{v}_{\min}(j)\mathbf{v}_{\min}^H(j) \tag{20}$$

Consequently, from [9], [10] and Eq.(19), the f_{qk} for $1 \leq k \leq K_q$ can be estimated by searching the zeros of the following criterion:

$$J_{\mathcal{F}_q}(f) = \lambda_{\min}\left\{\mathbf{A}_2^{-1}(f)\mathbf{A}_1(f)\right\} \tag{21}$$

$$\text{where} \quad \begin{cases} \mathbf{A}_1(f) = \boldsymbol{\Lambda}_K^H(\mathbf{p}_q, f)\boldsymbol{\Pi}_{\mathbf{v}}^{\perp}\boldsymbol{\Lambda}_K(\mathbf{p}_q, f) \\ \mathbf{A}_2(f) = \boldsymbol{\Lambda}_K^H(\mathbf{p}_q, f)\boldsymbol{\Lambda}_K(\mathbf{p}_q, f) \end{cases} \tag{22}$$

From the knowledge of the estimated f_{qk}'s and the location vector $\mathbf{p}_q$, the vector $\boldsymbol{\rho}_q$ can finally be estimated from the vectors $\mathbf{v}_{\min}(j)$ of Eq.(17). Indeed, the vector $\mathbf{v}_{\min}(j)$ can be expressed as:

$$\mathbf{v}_{\min}(j) = \mathbf{V}_q\left(\boldsymbol{\xi}_{\min}(j) \otimes \boldsymbol{\rho}_q\right) \tag{23}$$

where $\mathbf{V}_q = \left[\boldsymbol{\Lambda}_K(\mathbf{p}_q, f_{q1}), ..., \boldsymbol{\Lambda}_K(\mathbf{p}_q, f_{qK_q})\right]$ and $\boldsymbol{\xi}_{\min}(j) = [\xi_{1j}, ..., \xi_{K_q j}]^T$. We define $\boldsymbol{\eta}_j = \boldsymbol{\xi}_{\min}(j) \otimes \boldsymbol{\rho}_q$ and is deduced from the least squares:

$$\boldsymbol{\eta}_j = \left(\mathbf{V}_q^H\mathbf{V}_q\right)^{-1}\mathbf{V}_q^H\mathbf{v}_{\min}(j) \tag{24}$$

The $\boldsymbol{\eta}_j$ components being reshaped into a $K_q \times L$ matrix $\boldsymbol{\Omega}_j = \boldsymbol{\xi}_{\min}(j)\boldsymbol{\rho}_q^T$, one could deduce the vector $\boldsymbol{\rho}_q$ from:

$$\boldsymbol{\Omega} = \sum_{j=1}^{K_q} \boldsymbol{\Omega}_j^T\boldsymbol{\Omega}_j^* = \boldsymbol{\rho}_q\boldsymbol{\rho}_q^H\left(\sum_{j=1}^{K_q} \boldsymbol{\xi}_{\min}^H(j)\boldsymbol{\xi}_{\min}(j)\right) \tag{25}$$

where $(\cdot)^*$ is the conjugate operator. Indeed, $\boldsymbol{\rho}_q$ is its eigenvector of $\boldsymbol{\Omega}$ associated to the maximum eigenvalue.

3.2. Direct geolocalization from the frequencies set

After the estimation of the frequencies set of the q-th source, one could directly geolocalise it using this information.

The q-th source subspace is spanned by the vectors $\mathbf{v}(\mathbf{p}, f_{qk}, \boldsymbol{\rho}_q)$ for $(1 \leq k \leq K_q)$. Using Eqs.(12) and (13):

$$\mathbf{v}(\mathbf{p}, f_{qk}, \boldsymbol{\rho}_q) = \mathbf{B}(\mathbf{p}, f_{qk})\boldsymbol{\rho}_q \tag{26}$$
$$\mathbf{B}(\mathbf{p}, f_{qk}) = \mathbf{W}(\mathbf{p})\left(\mathbf{c}(f_{qk}) \otimes \boldsymbol{\Lambda}(\mathbf{p}, f_{qk})\right) \tag{27}$$

The location $\mathbf{p}_q$ of the source is estimated with a Weighting Subspace Fitting (WSF) approach [11] by searching the zeros of the following criterion:

$$J_{FIND,q}(\mathbf{p}) = \operatorname{trace}\left(\boldsymbol{\Pi}^{\perp}\boldsymbol{\Pi}_q(\mathbf{p})\right) \tag{28}$$

$$\boldsymbol{\Pi}_q(\mathbf{p}) = \mathbf{E}_q(\mathbf{p})\left(\mathbf{E}_q^H(\mathbf{p})\mathbf{E}_q(\mathbf{p})\right)^{-1}\mathbf{E}_q^H(\mathbf{p}) \tag{29}$$

with $\mathbf{E}_q(\mathbf{p}) = \left[\mathbf{v}(\mathbf{p}, f_{q1}, \boldsymbol{\rho}_q), ..., \mathbf{v}(\mathbf{p}, f_{qK_q}, \boldsymbol{\rho}_q)\right]$.

From Eqs.(26) and (27), it can be shown that $\mathbf{v}^H(\mathbf{p}, f_1, \boldsymbol{\rho}_q) \times\mathbf{v}(\mathbf{p}, f_2, \boldsymbol{\rho}_q) = 0$ for $f_2 - f_1 = \frac{kF_e}{K}$, $k \in \mathbb{Z} \setminus \{0\}$, and $||\mathbf{v}(\mathbf{p}, f, \boldsymbol{\rho}_q)||_2^2 = KM$. Hence, in presence of frequencies such that $f_{qi} - f_{qj} = \frac{kF_e}{K}$, the LOST-FIND criterion becomes:

$$J_{FIND1,q}(\mathbf{p}) = \sum_{k=1}^{K} \frac{\mathbf{v}^H(\mathbf{p}, f_{qk}, \boldsymbol{\rho}_q)\boldsymbol{\Pi}^{\perp}\mathbf{v}(\mathbf{p}, f_{qk}, \boldsymbol{\rho}_q)}{KM} \tag{30}$$

If $\boldsymbol{\rho}_q$ is unknown or has not been estimated, we inject Eq.(26) in Eq.(30) and the vector $\boldsymbol{\rho}_q$ can be jointly estimated to $\mathbf{p}_q$. Indeed, the location $\mathbf{p}_q$ is deduced from:

$$J_{FIND2,q}(\mathbf{p}) = \min_{\boldsymbol{\rho} \in \mathbb{C}^L} J_{FIND1,q}(\mathbf{p}, \boldsymbol{\rho}) = \lambda_{\min}\left\{\mathbf{Q}(\mathbf{p})\right\} \tag{31}$$

$$\mathbf{Q}(\mathbf{p}) = \sum_{k=1}^{K} \frac{\mathbf{B}^H(\mathbf{p}, f_{qk})\boldsymbol{\Pi}^{\perp}\mathbf{B}(\mathbf{p}, f_{qk})}{KM} \tag{32}$$

When the assumption $f_{qi} - f_{qj} = \frac{kF_e}{K}$ is not verified, $J_{FIND1,q}(\mathbf{p}) \neq J_{FIND,q}(\mathbf{p})$ but $J_{FIND1,q}(\mathbf{p}_q) = J_{FIND,q}(\mathbf{p}_q) = 0$. In this case, the source location can be estimated with $J_{FIND1,q}(\mathbf{p})$. In addition, this estimator is not the optimal one (in the sense that it does not exploit all the available information) which is $J_{FIND,q}(\mathbf{p})$.

3.3. LOST-FIND algorithm

The steps of the LOST-FIND algorithm are then:

- **Step-1**: First estimation of $\mathbf{p}_q$ for $(1 \leq q \leq Q)$ with LOST algorithm for each q from Eq.(14),
- **Step-2**: Frequencies estimation $\mathcal{F}_q = \{f_{qk} : 1 \leq k \leq K_q\}$ from searching the zeros of $J_{\mathcal{F}_q}(f)$ in Eq.(21),
- **Step-3**: From $\mathcal{F}_q$, $\mathbf{p}_q$ and $\boldsymbol{\Omega}$ (Eq.(25)), estimation of the attenuation vector $\boldsymbol{\rho}_q$: eigenvector associated to the maximum eigenvalue of $\boldsymbol{\Omega}$,
- **Step-4**: From the knowledge of $\mathcal{F}_q$ and $\boldsymbol{\rho}_q$, new estimation of $\mathbf{p}_q$ with the criterion $J_{FIND,q}(\mathbf{p})$ (Eq.(28)) or $J_{FIND1,q}(\mathbf{p})$ (Eq.(30)),
- **Step-5** (optional): Go to Step-2.

4. SIMULATIONS

In this part we compare the LOST-FIND algorithm to the LOST, the DPD, the classical triangulation (AoA/AoA) and the localization in 2-step combining the AoA of one station and the TDoA with 1 sensor of each station (AoA/TDoA). We will consider two arrays. In a Cartesian coordinate system, we place the first array at (-400m,0), and the second at (+400m,0). The arrays are composed of six sensors where five are in a circular formation around a sixth in the center. The arrays radius is 0.8m and we consider $K = 4$ temporal shifts for the space-time process (LOST and LOST-FIND) and $K = 4$ decompositions of the stations bandwidth for the DPD.

We first consider the single source case, where the source is located at (0,+5m). Such a scenario is very severe for algorithms which exploits only the AoA, where their performances are strongly deteriorated. This source is composed of 4 subcarrier frequencies fairly distributed in each subband of the DPD decomposition. Indeed, to correctly operate, the DPD needs a signal presence in all the subbands. In Fig.1, we plot the RMSE of the estimated source position as a function of the bandwidth of the arrays with SNR= 10dB. We observe that, for the two algorithms which do not exploit the TDoA (*e.g.* LOST and AoA/AoA), the arrays bandwidth has no impact on the performance. Furthermore, for the methods which exploit the TDoA, we note that the higher the bandwidth is, the smaller the RMSE are. In addition, the performance is similar between the DPD and the AoA/TDoA because, with a single source, when the signal is narrowband after the DPD decomposition, it was shown that the performance is similar [12]. Finally, exploiting the space-time structure, the LOST-FIND gives a better RMSE. In the remainder of this part we fix the arrays bandwidth at $B = 500$kHz.

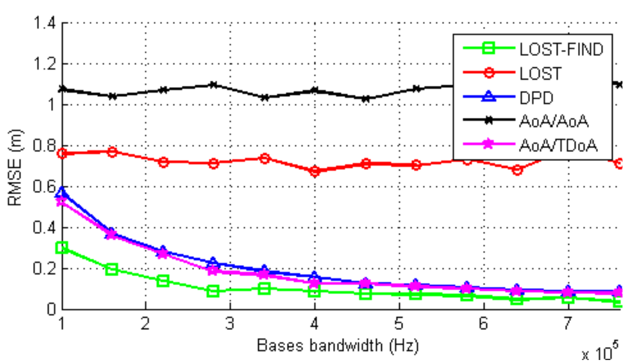

Fig. 1. Visualization of the RMSE as a function of the bandwidth of the arrays

In Fig.2 we keep the same source position, but considering now a source composed of 3 subcarrier frequencies, and we plot the RMSE of the source position as a function of the SNR. We observe that the algorithms exploiting the TDoA have a better performance. The DPD has poorer performance than in Fig.1 because the source signal is not present in each decomposition subband. Moreover, we constate that the LOST-FIND is the algorithm having the better performance.

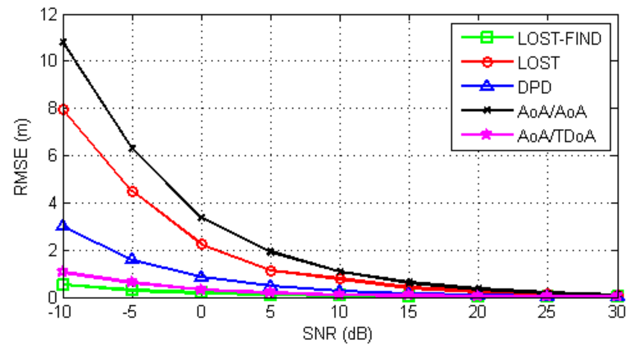

Fig. 2. One source at position (0,+5m)

In Fig.3 we consider the two sources case. We place the sources locations in a more favorable context for the AoA estimation. We place the first source at (0,+200m) and the second at (+5m,+200m) and we have 3 (resp. 2) subcarrier frequencies for the first (resp. second) source. We first observe that the AoA/TDoA gives bad results as, the two sources being really too close, the temporal resolution of the criterion does not allow to separate the two sources. Although, due to the total exploitation of the system information (AoA of the two arrays and their TDoA simultaneously), the DPD is not optimal in this context, it gives better performance than the AoA/AoA methods. Thanks to the identification of the frequencies of each source, the LOST-FIND algorithm permits to reject the source we do not want to locate (the second source). Consequently, it allows to have better performance than the other algorithms. Finally, we apply the optional iteration given by the step-5 of LOST-FIND and we remark that the performance is better.

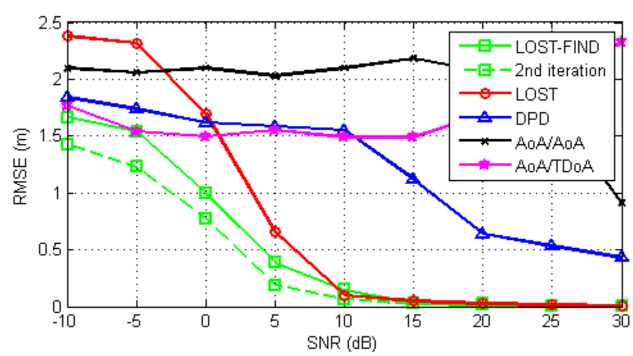

Fig. 3. Two sources of positions (0,+200m), (+5m,+200m), estimation of the first source

5. CONCLUSIONS

A new algorithm LOST-FIND was proposed for blind geolocalization. This algorithm is spectrum adaptive and simultaneously exploits the TDoA and AoA. Indeed, the spatio-temporal structure of the model is more correctly handled than the recently introduced LOST algorithm. Moreover, this property gives a good robustness when DPD and/or LOST fail. The proposed algorithm outperforms the existing ones as shown in the simulations.

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