LARGE REGION ACOUSTIC SOURCE MAPPING USING MOVABLE ARRAYS

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ABSTRACT

Mapping environmental noise with high resolution on a large scale (such as a city) is prohibitively expensive with current approaches, which use a large, dense array spanning the entire region of interest, or sequential noise measurements at thousands of locations on a dense grid. We propose instead a new acoustic measurement scheme using a small movable array (for example, mounted on a vehicle driving along the streets of a city) to rapidly acquire measurements at many different locations. A multiple-point sparse constrained deconvolution approach for the mapping of acoustic sources (MPSC-DAMAS) and a multiple-point covariance matrix fitting (MP-CMF) approach are developed to accurately estimate the locations and powers of stationary noise sources across the region of interest. Computer simulations of large region acoustic mapping demonstrate that superior resolution and much lower power estimation errors are achieved by the proposed approaches compared to the state-of-the-art SC-DAMAS approach and CMF approach.

Index Terms— microphone arrays, source localization, acoustic source mapping

1. INTRODUCTION

Mapping of acoustic sources using microphone arrays have been widely used for acoustic source localization and power estimation in aeroacoustic measurements [1], vehicle noise mapping and vibration detection since the mid 1990s [2]. Microphone arrays are used to virtually steer into the desired scanning points and estimate the sound pressure level (SPL) at those points to identify the dominant noise sources. The state-of-the-art power estimation approaches are the delay-and-sum (DAS) beamformer, the deconvolution approach for the mapping of acoustic sources (DAMAS) [3], the sparse constrained deconvolution (SC-DAMAS) [4], and the covariance matrix fitting (CMF) [4, 5]. The DAS beamformer is the simplest approach, but it suffers from high sidelobes and spatial aliasing effects. The DAMAS approach achieves super resolutions, but the convergence constraint of the Gauss-Seidel method [4] in the DAMAS approach cannot be often satisfied. The SC-DAMAS and CMF approaches successfully solve the convergence problem of the DAMAS approach, but they are constrained to the physical regions within the aperture of microphone arrays.

A recent large-scale study in Europe has discovered significant adverse impact of environmental noise on health and longetivity. Locating environmental noise sources and measuring their level on a city- or even nation-scale is essential to address this problem, yet deploying dense acoustic arrays on this scale would be prohibitively expensive. To rapidly and cost-effectively address this need, we propose to mount a small microphone array on a vehicle which can conveniently drive around on public streets and acquire data at many locations across the region of interest, creating a kind of non-coherent virtual array of much larger aperture. We propose a multiple-point SC-DAMAS (MPSC-DAMAS) approach and a multiple-point CMF (MP-CMF) approach for accurately mapping the location and intensity of stationary acoustic noise sources across the region from such data. Compared to the existing SC-DAMAS and CMF approaches studied in [4], our proposed approaches have modest increases in the computational complexity, but are found to provide superior performance.

2. PROBLEM FORMULATION

Consider a wave field divided into a dense grid of I "scanning" locations at which the noise source power is to be estimated. The number of monopole acoustic sources located in the field are assumed to be sparse and less than I. A movable microphone array of M microphones is used to record data at a total of K recording points. Notice that the current acoustic mapping approaches use a fixed microphone array and have only one recording point. We present the acoustic mapping problem based on a multiple-point data measurement scheme. The Cartesian coordinates of the I scanning points $\mathbf{p}_i = [p_{ix}, p_{iy}, p_{iz}]^T$ and the microphone positions at

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kth recording points $\mathbf{p}_m^{(k)} = [p_{mx}^{(k)}, p_{my}^{(k)}, p_{mz}^{(k)}]^T$ are known. The total snapshots of each microphone at each recording point are divided into N segments, where each segment consists of length-L snapshots. Applying an L-point fast Fourier transform (FFT), the array output vector of an M-element microphone array in the presence of additive noise at the kth recording point can be represented as,

$$\mathbf{z}_n^{(k)}(f_l) = \mathbf{A}^{(k)}(\mathbf{p}, f_l)\mathbf{s}_n(f_l) + \mathbf{v}_n^{(k)}(f_l), n = 1, 2, ..., N \quad (1)$$

where $f_l \in [f_{min}, f_{max}]$ denotes the lth frequency bin; $\mathbf{A}^{(k)}(\mathbf{p}, f_l)$ is the $M \times I$ steering matrix over all the scanning points at the kth recording point and it is defined as $\mathbf{A}^{(k)}(\mathbf{p}, f_l) = [\mathbf{a}^{(k)}(\mathbf{p}_1, f_l), ..., \mathbf{a}^{(k)}(\mathbf{p}_I, f_l)]$. The source signal vector of the whole scanning region is given by $\mathbf{s}_n(f_l) = [s_{n,1}(f_l), ..., s_{n,I}(f_l)]$, $n = 1, 2, ..., N$. It is assumed that the source signal vector $\mathbf{s}_n(f_l)$ is zero mean and uncorrelated with the additive noise vector $\mathbf{v}_n^{(k)}(f_l)$. The steering vector for scanning point i and recording point k is

$$\mathbf{a}^{(k)}(\mathbf{p}_i, f_l) = \Big[\frac{1}{r_{i,1}^{(k)}} e^{-j2\pi f_l r_{i,1}^{(k)}/cf_L}, ..., \frac{1}{r_{i,M}^{(k)}} e^{-j2\pi f_l r_{i,M}^{(k)}/cf_L}\Big]^T \quad (2)$$

where $r_{i,m}^{(k)}$ represents the propagating distance from the ith scanning point to the mth microphone at the recording point k. The problem is to estimate the power levels of the scanning points using the observed vectors $\mathbf{z}_n^{(k)}(f_l), k = 1, 2, ..., K$.

The existing approaches assume that the scanning region is as large as the physical array size and the observation is made at one recording point where $K = 1$. The power estimation of the DAS beamformer for the scanning point $\mathbf{p}_i$ and the frequency bin f_l is given as

$$y^{(1)}(\mathbf{p}_i, f_l) = \frac{1}{M^2}\tilde{\mathbf{a}}^{(1)}(\mathbf{p}_i, f_l)^H \mathbf{R}^{(1)}(f_l)\tilde{\mathbf{a}}^{(1)}(\mathbf{p}_i, f_l), \quad (3)$$

where

$$\tilde{\mathbf{a}}^{(1)}(\mathbf{p}_i, f_l) = \Big[r_{i,1}^{(1)} e^{-j2\pi f_l r_{i,1}^{(1)}/cf_L}, ..., r_{i,1}^{(1)} e^{-j2\pi f_l r_{i,M}^{(1)}/cf_L}\Big]^T, \quad (4)$$

and the covariance matrix $\mathbf{R}^{(1)}(f_l)$ at the recording point 1 is modeled by

$$\begin{aligned}\mathbf{R}^{(1)}(f_l) &= E[\mathbf{z}_n^{(1)}(f_l)\mathbf{z}_n^{(1)}(f_l)^H] \\ &= \mathbf{A}(\mathbf{p}, f_l)\mathbf{X}(f_l)\mathbf{A}(\mathbf{p}, f_l)^H + \sigma^2(f_l)\mathbf{I} \quad (5)\end{aligned}$$

where the recording point index (1) is omitted in $\mathbf{A}(\mathbf{p}, f_l)$ and $\sigma^2(f_l)$ for the sake of representation; the matrix $\mathbf{X}(f_l)$ is the covariance matrix of the sources. Using (5) and assuming that the sources are stationary and mutually uncorrelated, and that the noise is absent, the DAMAS approach [3] formulates the following linear system of equations

$$\mathbf{y}^{(1)}(f_l) = \mathbf{C}^{(1)}(f_l)\mathbf{x}(f_l) \quad (6)$$

where $\mathbf{x}(f_l) = [x_1(f_l), ..., x_I(f_l)]^T$ and $x_i(f_l)$ represents the averaging power at the frequency f_l and the scanning point i, with $i = 1, 2, ..., I$; $\mathbf{y}^{(1)}(f_l) = [y_1^{(1)}(f_l), ..., y_I^{(1)}(f_l)]^T$ is the DAS output power vector; and the $I \times I$ matrix $\mathbf{C}^{(1)}$ has the elements

$$c_{i,j}^{(1)}(f_l) = \frac{1}{M^2}|\tilde{\mathbf{a}}^{(1)}(\mathbf{p}_i, f_l)^H \mathbf{a}^{(1)}(\mathbf{p}_j, f_l)|^2, i, j = 1, 2, ..., I \quad (7)$$

Estimating the power vector $\mathbf{x}(f_l)$ in (6) is an inverse problem. If the square matrix $\mathbf{C}^{(1)}(f_l)$ is full rank and invertible, the problem can be directly solved. However, the matrix $\mathbf{C}^{(1)}(f_l)$ often has a very low rank and is not invertible for acoustic mapping problems. The DAMAS approach solves the inverse problem in (6) using the Gauss-Seidel method. However, the DAMAS approach requires the matrix $\mathbf{C}^{(1)}(f_l)$ be diagonally dominant and $c_{i,i}^{(1)}(f_l) = 1$. This constraint is not always true for large region acoustic mapping where the scanning points have large distance ratios. Therefore, the DAMAS approach cannot guarantee to converge for solving the inverse problem in (6).

3. THE PROPOSED MPSC-DAMAS AND MP-CMF

The SC-DAMAS approach [4] solves the inverse problem (6) by formulating the following sparsity constrained problem:

$$\begin{cases} \min_{\mathbf{x}} \mathcal{J}(\mathbf{x}) = ||\mathbf{y}^{(1)} - \mathbf{C}^{(1)}\mathbf{x}||^2 \\ \text{s.t.} \|\mathbf{x}\|_1 \le \beta, x_i \ge 0, i = 1, 2, ..., I, \end{cases} \quad (8)$$

where β is the upper bound of the total source power and every element of $\mathbf{x}$ is enforced to be nonnegative. For the sake of representations, the frequency bin index f_l is omitted in (8) and in the following derivations. By the prior knowledge of the sparsity of $\mathbf{x}$, the SC-DAMAS approach works effectively for small region acoustic mappings [4]. When the region of interest increases, the SC-DAMAS approach loses the capability of correctly identifying the dominant sources without increasing the array aperture accordingly. To solve this problem, we propose to use the multiple-point measurements (1) for $k = 1, 2, ..., K$. In this case, we stack up all DAS output power vectors $\mathbf{y}^{(k)}$, $k = 1, 2, ..., K$ in (6), and yield the following expanded linear system of equations

$$\begin{bmatrix} \mathbf{y}^{(1)} \\ \vdots \\ \mathbf{y}^{(K)} \end{bmatrix} = \begin{bmatrix} \mathbf{C}^{(1)} \\ \vdots \\ \mathbf{C}^{(K)} \end{bmatrix} \mathbf{x} \quad (9)$$

Denoting $\mathbf{y} = [\mathbf{y}^{(1)}, ..., \mathbf{y}^{(K)}]^T$ and $\mathbf{C} = [\mathbf{C}^{(1)}, ..., \mathbf{C}^{(K)}]^T$, a multiple-point SC-DAMAS (MPSC-DAMAS) approach is then proposed as follows:

$$\begin{cases} \min_{\mathbf{x}} \mathcal{J}(\mathbf{x}) = ||\mathbf{y} - \mathbf{C}\mathbf{x}||^2 \\ \text{s.t.} \|\mathbf{x}\|_1 \le \beta, x_i \ge 0, i = 1, 2, ..., I, \end{cases} \quad (10)$$

where the initialization of β can be made using the eigendecomposition method similarly to the SC-DAMAS approach.

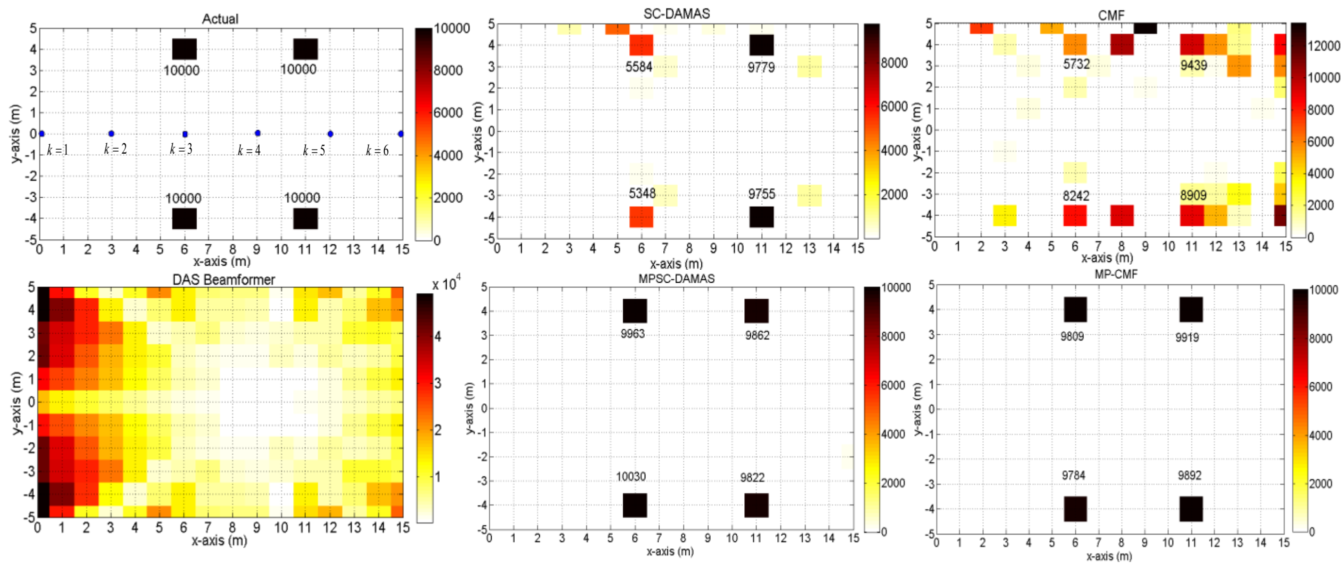

Fig. 1. Maps of the actual sources, DAS beamformer, SC-DAMAS, CMF, MPSC-DAMAS, and MP-CMF ($f = 1kHz$, $\sigma^2 = 100$).

Compared to the SC-DAMAS approach, the MPSC-DAMAS approach increases the degree of freedom of the matrix $\mathbf{C}$ using the array measurements from different points. Therefore, the MPSC-DAMAS approach is expected to estimate the same number of unknowns in $\mathbf{x}$ more accurately. A multiple-point SC-RDAMAS approach can also be worked out based on the SC-RDAMAS approach presented in [6] for more robust estimation with strong additive noises. It will not be further discussed here due to the page limit.

Now let's discuss the CMF approach for estimating the source powers using the observation data given in (1). Assuming a single recording point is used, the CMF approach [4] directly works on the sample covariance matrix and the steering matrix. Notice from (5) that the covariance matrix can be decomposed to a linear representation of the source covariance matrix. The covariance matrix can be estimated from the sample covariance matrix, which is an averaged estimate of the covariance matrix $\mathbf{R}^{(1)}(f_l)$ using the N segments:

$$\hat{\mathbf{R}}^{(1)} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{z}_n^{(1)}(\mathbf{z}_n^{(1)})^H. \tag{11}$$

For mutually uncorrelated sources, the matrix $\mathbf{X}$ is a diagonal matrix with I unknowns $x_i, i = 1, 2, ..., I$. Using the sparsity of $\{x_i\}_i^I$, the CMF approach estimates the noise power σ^2 and the diagonal elements $\{x_i\}_i^I$ of $\mathbf{X}$ as follows:

$$\left\{ \begin{array}{l} \min_{\{x_i\}_i^I, \sigma^2} ||\hat{\mathbf{R}}^{(1)} - \mathbf{A}^{(1)}(\mathbf{p})\mathbf{X}\mathbf{A}^{(1)}(\mathbf{p})^H - (\sigma^{(1)})^2\mathbf{I}||_F^2 \\ \text{s.t. } x_i \geq 0, i = 1, 2, ..., I, \sum_{i=1}^{I} x_i \leq \beta, (\sigma^{(1)})^2 \geq 0, \end{array} \right. \tag{12}$$

We found that the performance of the CMF approach degrades when the region of interest increases. To overcome this problem, we propose a multiple-point CMF (MP-CMF) approach using the multiple-point measurements (1) as follows:

$$\left\{ \begin{array}{l} \min_{\{x_i\}_i^I, \sigma^2} ||\hat{\mathbf{R}} - \mathbf{A}(\mathbf{p})\mathbf{X}\mathbf{A}(\mathbf{p})^H - \sigma^2\mathbf{I}||_F^2 \\ \text{s.t. } x_i \geq 0, i = 1, 2, ..., I, \sum_{i=1}^{I} x_i \leq \beta, \sigma^2 \geq 0, \end{array} \right. \tag{13}$$

where we compute the matrices $\hat{\mathbf{R}} = 1/K\sum_{k=1}^{K} \hat{\mathbf{R}}^{(k)}$ and $\mathbf{A}(\mathbf{p})\mathbf{X}\mathbf{A}(\mathbf{p})^H = 1/K\sum_{k=1}^{K} \mathbf{A}^{(k)}(\mathbf{p})\mathbf{X}\mathbf{A}^{(k)}(\mathbf{p})^H$ as the averages over the K recoding points; and $\sigma^2 = 1/K\sum_{k=1}^{K}(\sigma^{(k)})^2$ is the averaged noise power. The initialization of β and σ^2 are made similarly to the CMF approach.

All the above sparse constraint formulations are quadratic convex optimization problems and can be solved via readily available interior point methods with the free *Self-Dual Minimization* software package [7]. The MPSC-DAMAS approach adds modest computational complexity due to the increase of matrix dimensions and the MP-CMF approach adds negligible computational complexity for the average computations.

4. EXPERIMENTAL RESULTS

In this section, the performance of the proposed MPSC-DAMAS and MP-CMF approaches was evaluated and compared with the existing DAS beamformer, SC-DAMAS and CMF approaches [4]. In the first simulation, we used an 8-channel circular array with a diameter of $1m$. The scanning region of interest was a $15m \times 10m$ plane and the scanning points were set on a $1m \times 1m$ grid. The received signals were generated according to Eq. (1) where the microphone array were placed on the scanning plane. Four

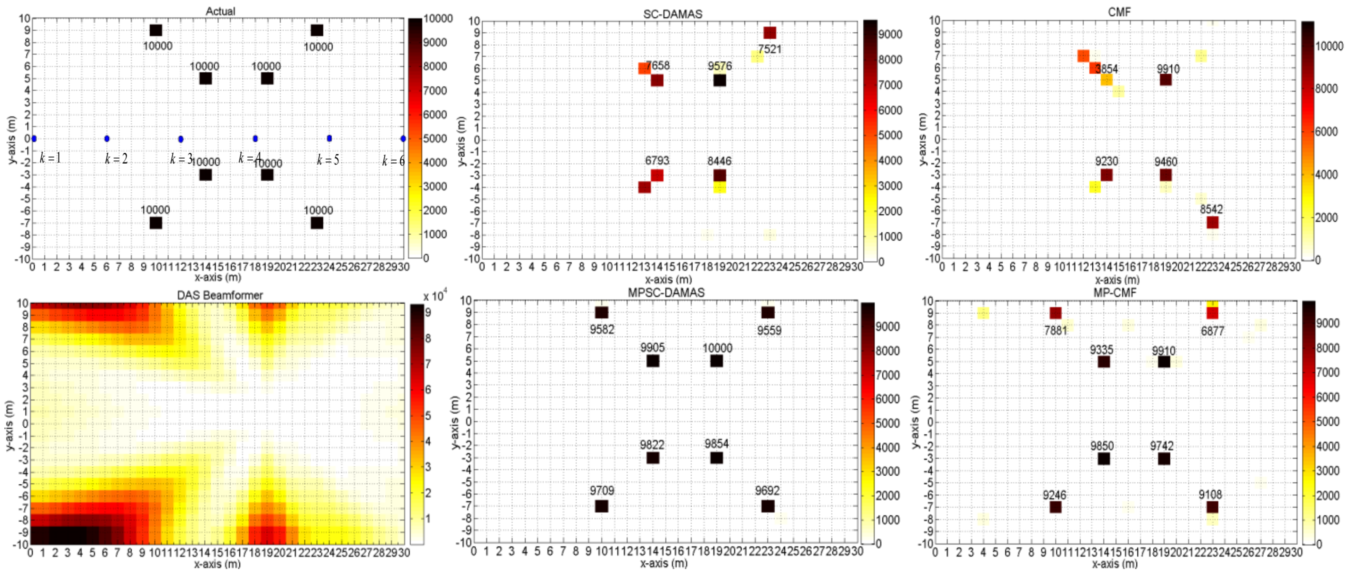

Fig. 2. Maps of the actual sources, DAS beamformer, SC-DAMAS, CMF, MPSC-DAMAS, and MP-CMF ($f = 500Hz$, $\sigma^2 = 100$).

acoustic sources and six recording points were considered as shown in Fig 1. The sources and additive noise were synthetic complex Gaussian noise signals with zero mean and powers of $40dB$ and $20dB$, respectively. The frequency of interest was set to $1kHz$. A total of 10000 snapshots were used at each recording point. The 4th recording point was used in the DAS beamformer, SC-DAMAS and CMF for the better performance. All recording points were used for MPSC-DAMAS and MP-CMF. The resulting acoustic maps were shown in Fig 1, where the x-axis and y-axis represent the 2D scanning plane and the power levels are represented in a hot color bar with linear values. It is observed that the DAS beamformer gives the worse resolution and power estimation. The dominant sources cannot be identified from the results of the DAS beamformer and the CMF approach. Compared to SC-DAMAS and CMF, MPSC-DAMAS and MP-CMF produce much lower estimation errors and better resolutions, and the dominant sources can be clearly identified from the maps. Among all the compared approaches, MPSC-DAMAS produces the best performance for the acoustic mapping.

In the second simulation, the large aperture microphone directional array (LAMDA) studied in [5] was used. The scanning region of interest was increased to a $30m \times 20m$ plane. The number of target sources was increased to eight and six recording points were selected as illustrated in Fig 2. The source frequency of interest was reduced to $500Hz$. Without loss of generality, all the other settings are same to the first simulation. Fig 2 shows the acoustic mapping results of all the approaches. A similar observations was made as Fig 1. MPSC-DAMAS and MP-CMF clearly identify the 8 dominant sources, while both SC-DAMAS and CMF missed 3 sources and produced several false sources. MP-CMF has slightly increased power estimation errors and MPSC-DAMAS has the best performance on the power estimation.

The processing time for both simulations using Matlab implementations showed MPSC-DAMAS runs 2 times slower than SC-DAMAS, and runs 3 times slower than CMF and MP-CMF. Our further tests showed that increasing the number of recording points generally improves the performance, but also increases the computational and measurement cost. Optimal settings of the recording points for a given scanning region needs to be studied in our future work.

5. CONCLUSIONS

A multiple-point measurement scheme using small movable arrays for large region acoustic mapping has been shown in principle and simulation to be feasible. The proposed MPSC-DAMAS approach and MP-CMF approach both produce accurate source location and source power estimates. Simulation results of large region acoustic mapping showed that the MPSC-DAMAS and MP-CMF approaches greatly outperform the state-of-the-art DAS beamformer, SC-DAMAS and CMF approaches.

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