PERIODIC RF TRANSMITTER GEOLOCATION USING A MOBILE RECEIVER

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ABSTRACT

In this paper we propose a method to localize a periodic RF transmitter using a single mobile receiver. The receiver measures the time-of-arrival (TOA) of the periodic messages at different locations along its trajectory. By comparing the TOA of successive messages at different points along its trajectory, the receiver can eventually estimate the transmitter location. The challenge lies in separating the time offset due to receiver movement from that caused by local oscillator (LO) drift. We propose an extended Kalman filter framework that estimates the LO drift and the transmitter location simultaneously, using the TOA measurements and the receiver location as inputs. The proposed algorithm is implemented and tested on a software-defined radio testbed, and experimental results demonstrate that the proposed method is able to simultaneously locate the transmitter and estimate the LO drift with good accuracy.

Index Terms— Transmitter localization, EKF, periodic signal, local oscillator time offset, software defined radio

1. INTRODUCTION

Localization of RF transmitters is an interesting yet challenging problem with applications both in commercial and military fields, such as vehicular wireless networks, cellular network, robotics and navigation systems [1–4]. In traditional RF localization, the transmitter is localized by measuring and estimating some parameters of the RF signal, such as received signal strength (RSS), angle-of-arrival (AOA), time-of-arrival (TOA) or time-difference-of-arrival (TDOA) [5–9]. When transmitters are non-cooperative, TDOA-based methods that usually have better accuracy over RSS and AOA based approaches are often used. In such methods, the difference in propagation time between the transmitter and a pair of receivers is used to localize the transmitter. The TDOA measured by each pair of receivers describes a hyperbola of possible transmitter locations, requiring at least three receivers to fully resolve the transmitter location. Moreover, in TDOA methods, stringent time synchronization is required between the different receivers to obtain accurate TDOA measurements. It should be noted that, in many wireless standards, transmitters are required to send periodic messages. The 3G standard, for example, requires base station to send a periodic synchronization signal for mobile terminals to synchronize to the base station. Such periodic synchronization signals can be used for localizing a transmitter [10, 11].

In this paper, we consider the problem of localizing a periodic RF transmitter using a single mobile receiver. The receiver measures the TOA of the periodic message as it moves along its trajectory. The difference in TOA between different locations, compensated for the transmitter period, is called the *virtual* TDOA (VTDOA), and can be used for transmitter localization, similarly to traditional TDOA. One major challenge here is to separate the time offsets caused by the local oscillator (LO) drift from the time offset due to the receiver movement. The LO time offset exist between a transmitter and a receiver occurs due to slight temperature differences and manufacturing tolerances of LOs [12]. The concept of VTDOA (for a transmitter sending a known signal) was recently proposed in [4], and the influence of LO drift was investigated. If the time offset due to LO drift is ignored, the moving receiver should move at a high speeds (300 m/s) to keep the localization error reasonably low. For non-flying and terrestrial vehicles, the time offsets due both to LO drift and the receiver movement will be in the order of tens of nanoseconds, and simply ignoring the LO drift will lead to large errors in the time measurements. In this paper, we develop an adaptive filtering framework to estimate both the LO drift and transmitter location simultaneously, using the TOA of the transmitter's periodic messages. The mechanism of the proposed filtering framework implicitly uses VTDOA to localize the transmitter, even though the VTDOA is not explicitly estimated.

The main contributions of this paper can be summarized as follows:

1. We propose a method to localize a periodic transmitter using a mobile receiver. The receiver uses an extended Kalman filter (EKF) to estimate the LO offset, skew, and transmitter location and speed simultaneously, using TOA of the periodic messages and the location of the receiver as inputs.
2. Simulation results are presented to demonstrate that the proposed method performs well, both for static and moving transmitters.
3. The proposed method is implemented on a software-defined radio (SDR) testbed, and outdoor experimental results are presented (for static transmitters) to evaluate the performance of our technique in real environments.

The rest of this paper is organized as follows. The concept of time offsets and challenges for localization are detailed in Section 2. The EKF used for LO tracking and transmitter localization is presented in Section 3, and simulation results are discussed. The details of implementation and experimental results are presented in Section 4. Finally, Section 5 concludes the paper and discusses some future directions.

2. TIME OFFSETS AND LOCALIZATION CHALLENGES

2.1. Time offsets

Consider a transmitter sending a signal with periodicity T_0. This can be a repetitive beacon sent from a transmitter in a search and rescue operation, or a synchronization signal transmitted by a base station in a cellular network. If the first message is transmitted by the transmitter at time t_0, the receiver will receive the k-th message at time

$$t_k = t_0 + (k-1)T_0 + \Delta t_k + \Delta\tau_k$$

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where Δt_k corresponds to the propagation time between the transmitter and the receiver, and $\Delta\tau_k$ is the time offset due to LO drift between t_0 and the time of measurement k. We define a *cycle* as the time period over which one message is transmitted by the transmitter (and received by the receiver), and the local time of the receiver LO at cycle k is defined as

$$\tau_k \triangleq t_0 + (k-1)T_0 + \Delta\tau_k.$$

If the LO offset is zero, the VTDOA between two successive points can be estimated by evaluating $t_k - t_{k-1} - T_0$. However, due to the presence of the LO offset $\Delta\tau_k$, the challenge lies in estimating the propagation time Δt_k using the measurements t_k. To overcome this problem, we will consider the time offset due both to LO drift and to receiver movement, and estimate both Δt_k and $\Delta\tau_k$ simultaneously using an adaptive filtering framework.

2.2. LO dynamics

Let us start by considering the dynamics of LOs. The local time of a particular LO can be captured by considering the following two-state state-space model

$$\begin{bmatrix} \tau_k \\ \beta_k \end{bmatrix} = \mathbf{F}_{\text{LO}} \begin{bmatrix} \tau_{k-1} \\ \beta_{k-1} \end{bmatrix} + \mathbf{n}_k\left(\mathbf{Q}_{\text{LO}}\right) \tag{1}$$

where $\mathbf{F}_{\text{LO}} = \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix}$, τ_k is the LO local time at the cycle k, and β_k is the LO skew at time t_k. The term $\mathbf{n}_k\left(\mathbf{Q}_{\text{LO}}\right)$ is a zero-mean Gaussian noise vector with covariance matrix $\mathbf{Q}_{\text{LO}}$ defined by

$$\mathbf{Q}_{\text{LO}} = q_1^2 \begin{bmatrix} T_0 & 0 \\ 0 & 0 \end{bmatrix} + q_2^2 \begin{bmatrix} \frac{T_0^3}{3} & \frac{T_0^2}{2} \\ \frac{T_0^2}{2} & T_0 \end{bmatrix} \tag{2}$$

where q_1^2 and q_2^2 are the process noise parameters corresponding to white frequency noise and random walk frequency noise, respectively [13]. For high-quality oven-controlled crystal oscillators (OCXO) used in our testbed, the values are $q_1^2 = 5.25 \times 10^{-24}$ and $q_2^2 = 1.77 \times 10^{-21}$. These can be deduced by fitting the theoretical phase noise curve with phase noise values found in the LO datasheet [14].

3. PERIODIC TRANSMITTER LOCALIZATION METHOD

We develop an EKF to estimate the local LO time, LO skew and transmitter's location and speed at each cycle. The details of the state-space transition model and measurement model considered are presented in the following.

3.1. Extended Kalman Filter Formulation

The state to be estimated is defined as

$$\mathbf{x}_k = \left[\tau_k, \beta_k, x_{T,k}, \dot{x}_{T,k}, y_{T,k}, \dot{y}_{T,k}\right]^T$$

where $x_{T,k}$ and $y_{T,k}$ are the x- and y-coordinates of the transmitter, respectively, and $\dot{x}_{T,k}$ and $\dot{y}_{T,k}$ are the x- and y-coordinate speed of the transmitter, respectively.[1] We consider a state-space model, defined as

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{w}_k(\mathbf{Q}) \tag{3}$$

[1] Although in this paper we consider only 2D localization, our analysis can be easily extended to the 3D case.

where $\mathbf{w}_k(\mathbf{Q})$ is a zero-mean Gaussian noise vector with covariance $\mathbf{Q}$, and $\mathbf{F}$ is the state-space transition matrix defined as

$$\mathbf{F} = \mathbf{I}_3 \otimes \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix}$$

where $\mathbf{I}_3$ is a 3×3 identity matrix and $\otimes$ is the Kronecker product. The covariance matrix of the process noise $\mathbf{Q}$ can be defined as

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{\text{LO}} & \mathbf{0}_{4\times4} \\ \mathbf{0}_{2\times2} & \mathbf{Q}_{\text{Tx}} \end{bmatrix}$$

in which $\mathbf{Q}_{\text{LO}}$ is the 2×2 matrix defined in (2) and $\mathbf{Q}_{\text{Tx}}$ is the covariance matrix related to transmitter motion defined as

$$\mathbf{Q}_{\text{Tx}} = \begin{bmatrix} \sigma_{a,x}^2 & 0 \\ 0 & \sigma_{a,y}^2 \end{bmatrix} \otimes \begin{bmatrix} \frac{T_0^4}{4} & \frac{T_0^3}{2} \\ \frac{T_0^3}{2} & T_0^2 \end{bmatrix}$$

where $\sigma_{a,x}^2$ and $\sigma_{a,y}^2$ are the variances of random acceleration in the x and y directions, respectively.

The measured TOA at the receiver $\hat{t}_k$ can be expressed as

$$\hat{t}_k = h(\mathbf{x}_k) + v_k(R) \tag{4}$$

where $v_k(R)$ is a zero-mean Gaussian measurement noise with covariance R, and $h(\mathbf{x}_k)$ is a nonlinear function defined as

$$\begin{aligned} h(\mathbf{x}_k) &= t_k = \tau_k + \Delta t_k \\ &= \tau_k + \frac{\sqrt{(x_{R,k} - x_{T,k})^2 + (y_{R,k} - y_{T,k})^2}}{c_0} \end{aligned} \tag{5}$$

where c_0 is the speed of light. Equation (5) assumes that the propagation time between transmitter and receiver is dominated by the line-of-sight path. The state-space equation (3) and nonlinear measurement equation (4) define an extended Kalman filter (EKF), which can be used to track the state vector $\mathbf{x}_k$. The EKF algorithm consists of two steps: a prediction step and an update step. The prediction step provides a prediction of the current state, conditioned on the previous state $\mathbf{x}_{k|k-1}$. The update step corrects the predicted state $\mathbf{x}_{k|k-1}$ using the measurement $\hat{t}_k$ to obtain the filtered state $\mathbf{x}_{k|k}$. The EKF also provides an estimate of the state error covariance matrix $\mathbf{P}_{k|k}$. Details of EKF prediction and update step can be found in [15, 16]. Note that, compared to a conventional Kalman filter, the EKF introduces some error by linearizing (5). However, as T_0 is typically small, the movement of the transmitted and receiver are small during that timeframe, in which case the error introduced by the linearization process remains small. Unfortunately, unlike the linear Kalman filter, no convergence guarantees can be given for an EKF.

When running simulations or experiments with the proposed EKF, we observe that the LO states (τ_k and β_k) behave fairly independently of the transmitter location states ($x_{T,k}$, $y_{T,k}$, $\dot{x}_{T,k}$ and $\dot{y}_{T,k}$). The filter will first try to estimate the LO states without correcting the transmitter location state estimates. Once the LO states are close to their real values, the filter will start correcting the transmitter location state estimates. Therefore, in order to provide some insight into the steady-state behavior of the EKF, we consider the simplified two-state system composed of the local LO time and LO skew only, i.e. $\mathbf{x}_{\text{LO},k} = [\tau_k, \beta_k]^T$. The state-space transition equation of this simplified system is given by (1). If we do not consider the effect of transmitter and receiver movement in our system, the TOA measurement of our two-state Kalman filter can be expressed as

$$\hat{t}_{\text{LO},k} = \mathbf{H}\mathbf{x}_{\text{LO},k} + v_k(R) \tag{6}$$

where

$$\mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The state-space equation (1) and measurement equation (6) can be used to define a linear Kalman filter. It can easily be verified that this Kalman filter is fully observable and controllable [16]. In that case, the state error covariance matrix $\mathbf{P}_{\mathrm{LO},k|k}$ of the simplified Kalman filter is guaranteed to converge towards the steady-state error covariance matrix, which can be obtained by solving the discrete-time algebraic Ricatti equation (DARE):

$$\mathbf{P}_{\mathrm{LO},k+1|k+1} = \mathbf{F}_{\mathrm{LO}}\mathbf{P}_{\mathrm{LO},k|k}\mathbf{F}_{\mathrm{LO}}^T - \mathbf{F}_{\mathrm{LO}}\mathbf{P}_{\mathrm{LO},k|k}\mathbf{H}^T \left(\mathbf{H}\mathbf{P}_{\mathrm{LO},k|k}\mathbf{H}^T + \mathbf{R}\right)^{-1} \mathbf{H}\mathbf{P}_{\mathrm{LO},k|k}\mathbf{F}_{\mathrm{LO}}^T + \mathbf{Q}_{\mathrm{LO}}$$

Note that the solution for this recursive problem can easily be found by using Matlab's DARE solver.

Figure 1 compares the solution of the DARE with the elements of the state error covariance matrix of the simplified linear Kalman filter. As expected, the elements of the state error covariance matrix converge to the solution of the DARE. We also show the elements of the error covariance matrix (corresponding to the LO states) of the EKF. It can be seen that these also converge to the solution of the DARE (albeit a bit slower than for the linear Kalman filter), suggesting that the LO states do indeed behave independently from the transmitter location states. Note that the Kalman filter is simulated by extracting the LO states of the simulation presented in Section 3.2, and the EKF simulation correspond to the first simulation in Section 3.2. Given the EKF measurement equation (4)-(5), the error

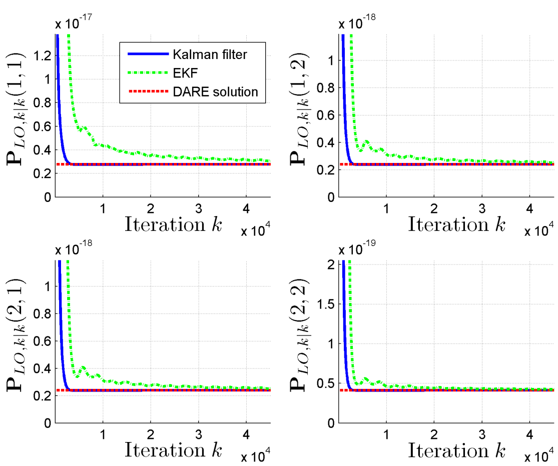

Fig. 1. Elements of the state covariance matrix related to the LO states for the Kalman filter, EKF and DARE solution.

on the local LO time will propagate on the transmitter-to-received distance r defined as

$$r = \sqrt{(x_{R,k} - x_{T,k})^2 + (y_{R,k} - y_{T,k})^2}$$

with a factor of c_0, which will determine the transmitter location error. Since the steady-state error covariance on the local LO time can be approximated from the DARE solution, we can get a reasonable prediction on the transmitter location error covariance. For the local LO time error covariance of $(1.66 \text{ ns})^2$ shown in Figure 1, the corresponding localization error covariance is $(0.50 \text{ m})^2$, which matches well with the simulation results presented later in Figure 2.

3.2. Simulations and Discussion

We present some simulation results to evaluate the performances of our proposed localization method. In the following, we will consider both static and mobile transmitter cases. It should be noted that the movement of transmitter and receiver are simulated, but that the LO drift used in the simulation is a real LO drift that was measured with our setup described in Section 4. The transmitter and receiver were connected with a cable, and the TOAs of the periodic messages were estimated. Since the cable length is fixed, the measured TOA values contain only the effects of LO drift (and no effect of transmitter or receiver movement). These measured TOA values are used to model the LO drift in our simulations.

In a first simulation, we assume a static transmitter at location $(0, 0)$ transmitting a periodic signal with period $T_0 = 10$ ms. The receiver moves around the transmitter with a speed of 1.5 m/s. The receiver trajectory is a circle with its origin at $(0, 0)$ and radius 10 m. The measurement noise covariance is equal to $R = 40$ ns. The results are shown in Figure 2. The red solid curve represents the receiver trajectory, the blue dashed curve represents the estimated transmitter location and the black square represents the real transmitter location. The results in this figure are related to two different initial guesses $(x_0, y_0) = (7, 7)$ and $(x_0, y_0) = (15, 15)$ for the transmitter location, and are marked by crosses in the figure . The estimation errors (difference between real value and the estimated one) in $\sqrt{x_{T,k}^2 + y_{T,k}^2}$ are also plotted as a function of the cycle number. It can be seen that the EKF is able to successfully localize the transmitter in both simulations.

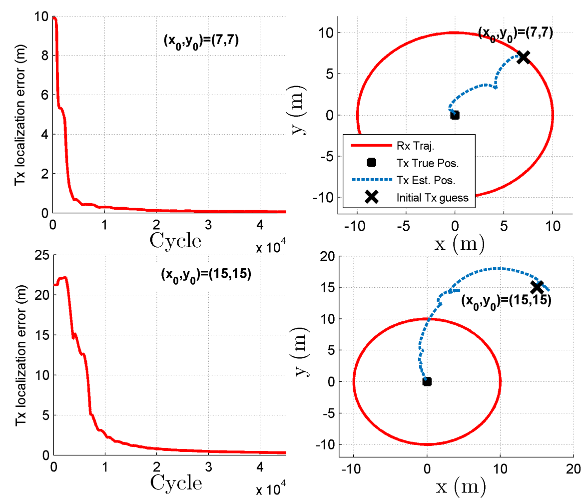

Fig. 2. Position estimation of a static transmitter placed at $(0, 0)$, from two different initial guess $(x_0, y_0) = (7, 7)$ and $(15, 15)$.

In the following simulation, we consider a moving transmitter sending a periodic message with $T_0 = 10$ ms. The transmitter starts at location $(0, 0)$ and we consider four values for the transmitter's speed, from $V_T = 0.05$ m/s to 1 m/s. The receiver's trajectory is a circle with radius 10 m and with origin $(0, 0)$. We suppose that the receiver knows the initial location of the transmitter, but has no information about the transmitter's motion. The simulation results are shown in Figure 3. It can be seen that the receiver is able to estimate and track the transmitter's motion. For higher transmitter speeds, the accuracy of the estimation is a little decreased. This is mostly due to the fact that the movement of the receiver is small compared to the movement of the transmitter, resulting in a poor geometry for localizing the transmitter.

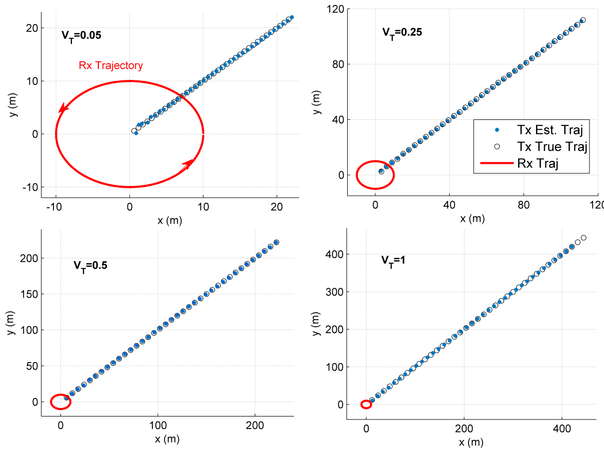

Fig. 3. Position estimation of a moving transmitter starting at $(0, 0)$ with different speeds $V_T = 0.05,\ 0.25,\ 0.5$, and 1 m/s.

The trajectory of the receiver will in great part determine the accuracy of the final transmitter location estimate. The problem of receiver trajectory optimization knowing the transmitter's position has been considered in [17]. The case of simultaneous trajectory optimization and localization using VTDOA measurements has not been addressed in the literature, and will be considered in our future work.

4. IMPLEMENTATION AND EXPERIMENTAL RESULTS

Our proposed EKF algorithm is implemented on a SDR testbed for real-time operation. The SDRs used are the USRP-N210, equipped with WBX daughterboards [18], for both the transmitter and the receiver. All the processing is done in real-time on host laptops with GNU Radio software, that facilitates real-time baseband processing [19]. The transmitter sends a pre-generated QPSK-modulated message at a 1 MHz symbol rate. The messages are 1 ms long, and are transmitted at a rate of 10 Hz. The USRP carrier frequency is set to 855 MHz. The receiver's block diagram is shown in Figure 4. The USRP sampling rate is set to 10 MHz. The receiver processes the baseband samples in real-time: after a low-pass filter, the receiver correlates its received signal with the known transmitted message to obtain a TOA measurement. To increase the resolution beyond 10 MHz, the receiver applies a quadratic interpolation between the three highest points of the correlation function [20], which increases the TOA accuracy to typical values below 5 ns. The receiver is also equipped with a high-precision differential GPS (DGPS) which measures the receiver's position with high accuracy. The receiver's position is measured at a rate of 10 Hz, and converted to x-y coordinates. The TOA and receiver x-y coordinates are sent to the EKF at a rate of 10 Hz, which estimates the LO drift and transmitter location. The filtered estimate is then recorded at the receiver side.

We conducted several experiments to evaluate the performance of our proposed localization method. The transmitter was placed in the center of the measurement area, and the receiver was moved in a roughly circular trajectory around the transmitter. The results are shown in Figure 5, for different initial guesses of the transmitter location. Figure 5 illustrates the trajectory of the receiver (in red),

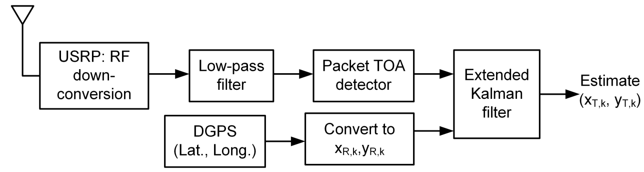

Fig. 4. Block diagram of the receiver.

the real position of the target (in black) and estimated position of the target (in blue) for different (x_0, y_0) in the EKF algorithm. It is observed in Figure 5 that the algorithm converges toward the true transmitter position for all cases in the experiment.

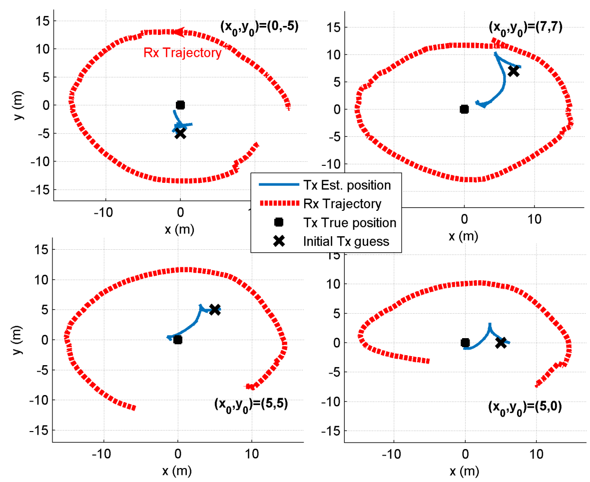

Fig. 5. Position estimation of a static transmitter placed at $(0, 0)$ with different initial guess $(x_0, y_0) = (0, -5), (7, 7), (5, 5)$ and $(5, 0)$.

5. CONCLUSION

In this paper we investigated the problem of localizing a periodic transmitter using a mobile receiver. The receiver measures the TOA of the periodic messages as it moves along its trajectory, and uses these TOA measurements to localize the transmitter. An extended Kalman filter was proposed to simultaneously estimate the transmitter location and the LO offset between transmitter and receiver. Simulation results suggest that the proposed method is able to localize both static and moving transmitters. The proposed method was implemented on a SDR testbed and its performance has been evaluated in several experiments. The results show that the presented method can estimate the LO time offset accurately and localize the transmitter successfully. One interesting problem for future research is the optimization of the receiver's trajectory in order to estimate the transmitter location. Another possible future direction is the problem of self-localization of mobile receivers using a single or multiple known periodic transmitters.

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