OPTIMUM DISCRETE DISTRIBUTED BEAMFORMING FOR SINGLE GROUP MULTICASTING RELAY NETWORKS WITH RELAY SELECTION

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ABSTRACT

In this paper, broadcast beamforming with relay selection is considered in wireless relay networks where there is no direct link between the source and the receivers. A source node transmits common information to many users employing multiple relays which use amplify-and-forward relay protocol. The channel state information is assumed to be available at a single relay to compute the relay weights and distribute them. Discrete relay weights are used due to several advantages including decreased overhead for the feedback channel. Multiple relay selection is employed in order to decrease network complexity and improve bandwidth efficiency. Nonlinear joint optimization problem is converted to a liner form and optimum solution is found by using mixed integer linear programming.

***Index Terms*—** Distributed beamforming, relay selection, discrete beamforming, mixed integer linear programming

1. INTRODUCTION

In this paper, we consider cooperative relaying where single antenna relays work as a distributed beamformer to take advantage of spatial diversity. Single group multicasting scenario is investigated where a source node transmits common information to multiple users through a relay network. Both transmitter and receivers have single antenna. Amplify-and-forward relay protocol is used for simplicity. Each relay multiplies the received signal by a complex weight such that quality of service (QoS) constraints are satisfied for the receivers while minimum total relay power is used.

In [1], distributed beamforming is presented for multi-group multicasting relay network. In [1], [2] and [3], it is assumed that the relays can adjust their powers arbitrarily and use continuous phase and amplitude for their complex weights. However, it is not practical to consider a continuous range for phase and amplitude due to limited feedback [4]. Furthermore, discrete beamformer structure has several advantages [5], [6]. In this paper, we propose discrete beamforming where the phase and amplitude terms are chosen from finite discrete sets to increase the network lifetime as well as to obtain optimum solution. Furthermore, broadcast relay beamforming is extended to include relay selection. In other words, the best relays of the network are selected by minimizing total power while QoS constraints are satisfied at the receivers. Relay selection is important to decrease network complexity, overhead and improve bandwidth efficiency [4], [7], [8]. Single relay selection is considered frequently in the literature [9], [10] but it may not be suitable for the multiuser case. While relay selection is used in different scenarios, it is not employed for distributed multicast beamforming. In this paper, relay selection for single group multicasting is considered by using discrete beamforming structure. Joint optimization problem is solved optimally by converting the problem into a linear form.

2. SYSTEM MODEL

Consider a single group multicasting (broadcasting) wireless relay network where a source node transmits a broadcast signal to N destination nodes through M relays. All nodes in this network are equipped with a single antenna. It is assumed that there is no direct link between the source and destination nodes due to path losses. We consider the two-hop data transmission. In the first phase of the transmission, the transmitter node broadcasts its signal to the relays and in the second phase, all relays simultaneously transmit to the destination nodes. Amplify-and-forward relay protocol is used where the relay forwards an amplified and phase-adjusted version of its received signal to the destination nodes. The received relay signal is $\mathbf{r} = \mathbf{f}s + \mathbf{n}^r$, where s is the information symbol transmitted by the source node, $\mathbf{f} = [\ f_1\ f_2\ ...\ f_M\ ]^T$, f_i is the complex channel gain between the source node and the i^{th} relay, $\mathbf{n}^r = [\ n_1^r\ n_2^r\ ...\ n_M^r\ ]^T$ is the relay noise vector and $\mathbf{r} = [\ r_1\ r_2\ ...r_M\ ]^T$, r_i is the received signal at the i^{th} relay. The i^{th} relay multiplies its received signal, r_i, by a complex weight $w_i^\star$ and transmits the resulting signal, $t_i = w_i^\star r_i$, to the destination nodes. The transmitted signal from the relays is given as, $\mathbf{t} = \mathbf{W}^H\mathbf{r}$ where $\mathbf{t} = [\ t_1\ t_2\ ...\ t_M\ ]^T$ and $\mathbf{W}$ is a diagonal matrix whose elements are complex conjugates of the complex weights, i.e., $\mathbf{W} = diag\{w_1, w_2,\ ..., w_M\}$. The received signal at the k^{th} receiver is, $y_k = \mathbf{g}_k^T\mathbf{t} + n_k^d = \mathbf{g}_k^T(\mathbf{W}^H\mathbf{f}s + \mathbf{W}^H\mathbf{n}^r) + n_k^d$ where $\mathbf{g}_k = [\ g_{k,1}\ g_{k,2}\ ...\ g_{k,M}\ ]^T$ and $g_{k,i}$ denotes the complex channel gain between i^{th} relay and k^{th} destination. n_k^d is the noise at the k^{th} receiver. Defining $\mathbf{G}_k = diag\{g_{k,1}, g_{k,2}, ..., g_{k,M}\}$ and $\mathbf{w} = [\ w_1\ w_2\ ...\ w_M\ ]^T$, the received signal can be written as $y_k = \mathbf{w}^H\mathbf{G}_k\mathbf{f}s + \mathbf{w}^H\mathbf{G}_k\mathbf{n}^r + n_k^d$. It is assumed that the information symbol s, the relay and the receiver noises are mutually uncorrelated in accordance with [1], [2] and [3]. Furthermore, the instantaneous channels are assumed to be known at the relays [1], [3]. In this paper, relay beamformer is designed by using QoS approach [1], [2]. Hence, it is desired to minimize the total power transmitted from the relays by ensuring that signal-to-noise ratio (SNR) constraints at the receivers are satisfied. The SNR of the k^{th} receiver is given as,

$$SNR_k = \frac{\mathbb{E}\{|\mathbf{w}^H\mathbf{G}_k\mathbf{f}s|^2\}}{\mathbb{E}\{|\mathbf{w}^H\mathbf{G}_k\mathbf{n}^r + n_k^d|^2\}} \tag{1}$$

where $\mathbb{E}\{|\mathbf{w}^H\mathbf{G}_k\mathbf{f}s|^2\} = P_s\mathbf{w}^H\mathbf{G}_k\mathbf{f}\mathbf{f}^H\mathbf{G}_k^H\mathbf{w}$ and $\mathbb{E}\{|\mathbf{w}^H\mathbf{G}_k\mathbf{n}^r + n_k^d|^2\} = \sigma_r^2\mathbf{w}^H\mathbf{G}_k\mathbf{G}_k^H\mathbf{w} + \sigma_{d,k}^2$ assuming that the channels are known in accordance with [1]. Here, the relay noise is assumed to be spatially white without loss of generality. P_s, σ_r^2 and $\sigma_{d,k}^2$ denote the source power, the variance of the relay noise and the noise of the k^{th} receiver, respectively. The total transmitted relay power can be written as,

$$P_T = \sum_{i=1}^{M}\mathbb{E}\{|t_i|^2\} = \sum_{i=1}^{M}|w_i|^2\mathbb{E}\{|r_i|^2\} = \mathbf{w}^H\mathbf{D}\mathbf{w} \tag{2}$$

where $\mathbf{D}$ is the diagonal matrix whose elements are $\mathbb{E}\{|r_i|^2\}$, i.e., $\mathbf{D} = diag\{\mathbb{E}\{|r_1|^2\}, ..., \mathbb{E}\{|r_M|^2\}\}$. $\mathbb{E}\{|r_i|^2\} = P_s|f_i|^2 + \sigma_r^2$. The optimization problem to minimize the total transmitted relay power subject to user SNR constraints is,

$$\min_{\mathbf{w}\in\mathbb{C}^M} \mathbf{w}^H\mathbf{D}\mathbf{w} \tag{3.a}$$

$$s.t.\ \mathbf{w}^H\mathbf{T}_k\mathbf{w} \geq \gamma_k\sigma_{d,k}^2, \quad k = 1, 2, ..., N \tag{3.b}$$

$$|w_i|^2 D_{i,i} \leq p_i, \quad i = 1, 2, ..., M \tag{3.c}$$

where $\mathbf{T}_k = P_s\mathbf{G}_k\mathbf{f}\mathbf{f}^H\mathbf{G}_k^H - \gamma_k\sigma_r^2\mathbf{G}_k\mathbf{G}_k^H$ and γ_k is the desired SNR value at the k^{th} receiver. $D_{i,i}$ shows the i^{th} diagonal element of $\mathbf{D}$. (3.c) is used to keep the individual relay power below a threshold, i.e., p_i. The fact that the relays may not want to use too much power due to their limited battery lifetime motivates us to include the individual power constraints in (3.c) to the original QoS relay beamforming problem [1]. The problem in (3) is not convex and there are efficient algorithms to find suboptimal solutions to this problem [1], [2], [3]. All these previous works cannot guarantee optimum solution and produce continuous beamformer weights. However, continuous power adjustment may not be practical [4]. In this paper, a discrete version of the problem in (3) is proposed by embedding the relay selection. In this case, at most L out of M relays are selected to assist transmission. The discrete QoS relay beamforming problem with relay selection can be written as,

$$\min_{\psi_i,\alpha_i} \mathbf{w}^H\mathbf{D}\mathbf{w} \tag{4.a}$$

$$s.t.\ \ \mathbf{w}^H\mathbf{T}_k\mathbf{w} \geq \gamma_k\sigma_{d,k}^2, \quad k = 1, ..., N \tag{4.b}$$

$$\psi_i \in \{0, \Delta\theta, 2\Delta\theta, ..., (2^n-1)\Delta\theta\},\ \Delta\theta = \frac{2\pi}{2^n} \tag{4.c}$$

$$\alpha_i \in \{\ 0, \Delta_i, 2\Delta_i, ..., (2^m-1)\Delta_i\},\ \Delta_i = \frac{\sqrt{p_i}}{2^m-1} \tag{4.d}$$

$$w_i = \frac{\alpha_i}{\sqrt{D_{i,i}}}e^{j\psi_i} \quad i = 1, ..., M, \tag{4.e}$$

$$\sum_{i=1}^{M} 1_r(\alpha_i) \leq L \tag{4.f}$$

where n and m are the number of bits to represent the discrete phase and amplitude respectively. $\Delta\theta$ and Δ_i's are the discrete step sizes for phase and amplitude respectively. $1_r(\alpha_i)$ is the indicator function determining the relay selection for the i^{th} relay, i.e., $1_r(\alpha_i) = 1$ if $\alpha_i > 0$, otherwise $1_r(\alpha_i) = 0$. Note that (3.c) is satisfied automatically by (4.e). Using the fact that $\mathbf{T}_k$ is a Hermitian symmetric matrix and defining $\beta_{i,p} = -\psi_i + \psi_p$ and $\mu_{i,p} = \alpha_i\alpha_p$, $i = 1, 2, ..., M-1$, $p = i+1, ..., M$, the optimization problem can be written as,

$$\min_{\psi_i,\alpha_i,\beta_{i,p},\mu_{i,p}} \sum_{i=1}^{M}\alpha_i^2 \tag{5.a}$$

$$\begin{aligned} s.t. \sum_{i=1}^{M-1}\sum_{p=i+1}^{M} 2\frac{\mu_{i,p}}{\sqrt{D_{i,i}D_{p,p}}}|T_{k_{i,p}}|(&cos(\angle T_{k_{i,p}})cos\beta_{i,p} \\ &-sin(\angle T_{k_{i,p}}sin\beta_{i,p}) \\ +\sum_{i=1}^{M}\frac{\alpha_i^2}{D_{i,i}}T_{k_{i,i}} \geq \gamma_k\sigma_{d,k}^2,& \quad k = 1, ..., N \end{aligned} \tag{5.b}$$

$$\beta_{i,p} = -\psi_i + \psi_p \tag{5.c}$$

$$\mu_{i,p} = \alpha_i\alpha_p, \tag{5.d}$$

$$i = 1, 2, ..., M-1,\ p = i+1, ..., M$$

(4.c), (4.d), (4.f)

where $T_{k_{i,p}}$ denotes the i^{th} row, p^{th} column element of $\mathbf{T}_k$ matrix. The problem in (5) is not convex and difficult to solve. In the following section, additional tools are used to map the same problem into a linear form in terms of the optimization variables in order to find an optimum solution.

3. DISCRETE OPTIMIZATION IN LINEAR FORM

The discrete optimization problem given in (5) is composed of nonlinear expressions of optimization variables. In this part, (5.a-d) and (4.f) are converted into linear expressions of optimization variables. Note that the formulation in this part is significantly different than [5], since QoS problem with relay selection is considered.

Let the first and second parts of the left hand side of the inequality in (5.b) be represented as A and B respectively, i.e., $A = \sum_{i=1}^{M-1}\sum_{p=i+1}^{M} A_{i,p}$ and $B = \sum_{i=1}^{M} B_i$ where $A_{i,p} = 2\frac{\mu_{i,p}}{\sqrt{D_{i,i}D_{p,p}}}|T_{k_{i,p}}|(cos(\angle T_{k_{i,p}})cos\beta_{i,p} - sin(\angle T_{k_{i,p}})sin\beta_{i,p})$ and $B_i = \frac{\alpha_i^2}{D_{i,i}}T_{k_{i,i}}$. The fact that $\mu_{i,p}$ and $\beta_{i,p}$ constitute finite discrete sets can be used to write A in terms of some indicator vectors whose function is to choose the appropriate values from the predefined discrete sets. Let $\mathbf{c}$ and $\mathbf{s}$ be composed of all possible $cos\beta_{i,p}$ and $sin\beta_{i,p}$ terms, i.e., $\mathbf{c} = [\ 0\ cos(0\cdot\Delta\theta)\ cos(1\cdot\Delta\theta)\ ...\ cos((2^n-1)\cdot\Delta\theta)]^T$ and $\mathbf{s} = [\ 0\ sin(0\cdot\Delta\theta)\ sin(1\cdot\Delta\theta)\ ...\ sin((2^n-1)\cdot\Delta\theta)]^T$. "0" corresponds to a term related to the relay selection which nullifies the corresponding element in the beamformer weight vector $\mathbf{w}$. In order to access each term in A, indicator vectors $\mathbf{u}_{i,p}$'s are defined, whose elements are all zero except one which is a positive integer. These types of vectors are known as special ordered sets of type 1 (SOS1) [11]. The index value of the nonzero element indicates the selected cosine or sine values from $\mathbf{c}$ and $\mathbf{s}$ respectively and the integer element amplitude at this index is $\frac{\mu_{i,p}}{\Delta_i\Delta_p}$, i.e., $\mathbf{c}^T\mathbf{u}_{i,p} = \frac{\mu_{i,p}}{\Delta_i\Delta_p}cos(\beta_{i,p})$, $\mathbf{s}^T\mathbf{u}_{i,p} = \frac{\mu_{i,p}}{\Delta_i\Delta_p}sin(\beta_{i,p})$. The above is true when the i^{th} and p^{th} relays are selected and $\mu_{i,p}$ is nonzero in accordance with (5.d). When i^{th} and p^{th} relays are not jointly selected, $\mu_{i,p} = 0$ (5.d) and the first element of $\mathbf{u}_{i,p}$ is one in order to have $A_{i,p} = 0$. The first element of $\mathbf{u}_{i,p}$ vector, u_{i,p_1} is a binary variable which is either 0 or 1. A in (5.b) can be expressed as a linear expression of $\mathbf{u}_{i,p}$'s as,

$$\begin{aligned} A = \sum_{i=1}^{M-1}\sum_{p=i+1}^{M} 2\frac{\Delta_i\Delta_p}{\sqrt{D_{i,i}D_{p,p}}}|T_{k_{i,p}}|(&cos(\angle T_{k_{i,p}})\mathbf{c}^T \\ &-sin(\angle T_{k_{i,p}})\mathbf{s}^T)\cdot\mathbf{u}_{i,p} \end{aligned} \tag{6}$$

Note that u_{i,p_1} and $1_r(\mu_{i,p})$ are the complements of each other as binary variables described as in Table 1.There is a dependency between $\mathbf{u}_{i,p}$ vectors due to (5.c) and (5.d). The relationship between $\mathbf{u}_{i,p}$ vectors can be established over SOS1 vectors, $\mathbf{v}_i$. $(2^n+1)\times 1$ vector, $\mathbf{v}_i$, carries the phase and amplitude information of the individual elements of the beamformer vector. The first element of $\mathbf{u}_{i,p}$ and $\mathbf{v}_i$, namely u_{i,p_1} and v_{i_1} are the relay selection parameters. v_{i_1} is the complement of $1_r(\alpha_i)$ as given in Table 1. If the i^{th} relay is selected, $v_{i_1} = 0$, the index value of the nonzero element stands for $\frac{\psi_i}{\Delta\theta}$ and the element amplitude is $\frac{\alpha_i}{\Delta_i}$.

Table 1. The binary relationship between variables

$1_r(\alpha_i)$	$1_r(\alpha_p)$	$1_r(\mu_{i,p})$	v_{i_1}	v_{p_1}	u_{i,p_1}	$b_{i,p}, c_{i,p}$
0	0	0	1	1	1	≥ 0
0	1	0	1	0	1	≥ 0
1	0	0	0	1	1	≥ 0
1	1	1	0	0	0	0

$\mathbf{u}_{i,p}$ and $\mathbf{v}_i$ are SOS1 vectors carrying phase and amplitude information. Phase and amplitude information can be separated by defining SOS1 operators $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ respectively. $\mathcal{T}_1$ generates the phase coded "binary" SOS1 vectors $\mathbf{u}'_{i,p}$ and $\mathbf{v}'_i$ from $\mathbf{u}_{i,p}$ and $\mathbf{v}_i$ respectively. $\mathcal{T}_2$ and $\mathcal{T}_3$ generate $\mathbf{v}''_i$ and $\mathbf{u}''_{i,p}$ binary SOS1 vectors respectively. These two vectors have only the coded amplitude information and are described in the following parts.

The function of $\mathcal{T}_1\{.\}$ operator is the normalization, i.e., $\mathbf{v}'_i = \mathcal{T}_1\{\mathbf{v}_i\} = \mathcal{T}_1\{[\ 0\ ...\ \frac{\alpha_i}{\Delta_i}\ ...\ 0\ ]^T\} = \frac{\mathbf{v}_i \Delta_i}{\alpha_i} = [\ 0\ ...\ 1\ ...\ 0\ ]^T$ and therefore it generates the phase coded vector $\mathbf{v}'_i$. $\mathcal{T}_1$ operates similarly on $\mathbf{u}_{i,p}$.

The equation in (5.c) can now be written in terms of new variables $\mathbf{v}'_i$ and $\mathbf{u}'_{i,p}$ respectively. In order to do this, (5.c) is first normalized with $\Delta\theta = \frac{2\pi}{2^n}$. Let $\mathbf{d} = [\ 0\ 0\ 1\ 2\ ...\ (2^n-1)\ ]^T$ be known vector of integers. After normalization, (5.c) can be written as,

$$\mathbf{d}^T \cdot \mathbf{u}'_{i,p} = \mathbf{d}^T \cdot (-\mathbf{v}'_i + \mathbf{v}'_p) + 2^n a_{i,p} \tag{7}$$

$\beta_{i,p}$ in (5.c) can take values in $(-2\pi, 2\pi)$. In order to decrease the length of $\mathbf{u}_{i,p}$ and $\mathbf{v}_i$ vectors from 2^{n+1} to (2^n+1), $\beta_{i,p}$ is considered in $[0, 2\pi)$ range. In this case, the left hand side of the equation (7) is nonnegative. To overcome phase ambiguity problem when $(-\psi_i + \psi_p)$ is negative, binary variable $a_{i,p}$ is used in (7) in order to have a valid equation for all cases. When there is a 2^n difference between $\mathbf{d}^T \cdot (-\mathbf{v}'_\mathbf{i} + \mathbf{v}'_\mathbf{p})$ and $\mathbf{d}^T \cdot \mathbf{u}'_{i,p}$ (corresponding to 2π phase difference between $-\psi_i + \psi_p$ and $\beta_{i,p}$), $a_{i,p}$ becomes 1 to satisfy the equality in (7). Note that (7) is defined when i^{th} and p^{th} relays are used simultaneously. For the other cases of relay existences, equation (7) can still be made valid if an additional variable $b_{i,p} \geq 0$ is added to the left hand side of equation (7). In this case, the right hand side of (7) is always nonnegative and $\mathbf{d}^T \cdot \mathbf{u}'_{i,p} = 0$ requiring a nonnegative $b_{i,p}$ to satisfy the equality, i.e.,

$$\mathbf{d}^T \cdot \mathbf{u}'_{i,p} + b_{i,p} = \mathbf{d}^T \cdot (-\mathbf{v}'_i + \mathbf{v}'_p) + a_{i,p} 2^n \tag{8}$$

$b_{i,p}$ is a nonnegative slack variable and it should be zero when both i^{th} and p^{th} relays are selected as indicated in Table 1. The relation between u_{i,p_1} and $b_{i,p}$ in Table 1 can be established using Big-M inequality [12], i.e., $b_{i,p} + 2^n(1 - u_{i,p_1}) \leq 2^n$. The relation between the relay selection parameters, v_{i_1}, v_{p_1} and u_{i,p_1} in Table 1 can also be expressed as a double sided inequality, i.e., $-1 \leq v_{i_1} + v_{p_1} - 2u_{i,p_1} \leq 0$. (8) is the linear form of the expression in (5.c) in terms of new variables $\mathbf{u}'_{\mathbf{i},\mathbf{p}}$, $\mathbf{v}'_\mathbf{i}$, $a_{i,p}$ and $b_{i,p}$ respectively. In the following part, (5.d), B, (5.a) and (4.f) are converted into the desired form. In this context, multiplication of two variables is converted to addition through a mapping operation. More specifically, this nonlinear expression of optimization variables is converted to an addition in terms of new optimization variables.

Let α_i and α_p be two amplitude values in multiplication, i.e, $\alpha_i \cdot \alpha_p = \mu_{i,p}$, where $\frac{\alpha_i}{\Delta_i}$, $\frac{\alpha_p}{\Delta_p}$ and $\frac{\mu_{i,p}}{\Delta_i \Delta_p}$ belong to nonnegative integers, i.e., $\frac{\alpha_i}{\Delta_i}, \frac{\alpha_p}{\Delta_p}, \frac{\mu_{i,p}}{\Delta_i \Delta_p} \in \{0\} \cup \mathbb{Z}^+$. $\frac{\alpha_i}{\Delta_i}$ and $\frac{\alpha_p}{\Delta_p}$ belong to a finite discrete set, i.e., $\frac{\alpha_i}{\Delta_i}, \frac{\alpha_p}{\Delta_p} \in S$ where $S = \{0, 1, 2, 3, ..., (2^m - 1)\}$ for m bits as can be seen in (4.d). There are $2^m + \binom{2^m}{2}$ multiplication couples since multiplication is commutative. When the multiplication of $\frac{\alpha_i}{\Delta_i}$ and $\frac{\alpha_p}{\Delta_p}$ in S is considered, the discrete set of couples can be given as, $P = \{0 \cdot 0,\ ...\ 0 \cdot (2^m-1),\ 1 \cdot 1,\ ...\ , 1 \cdot (2^m - 1),\ 2 \cdot 2,\ 2 \cdot 3,\ ...\ , 2 \cdot (2^m-1),\ ...\ , (2^m-1) \cdot (2^m-1)\}$. Let $\mathbf{q}$ be a vector whose elements are unique and ordered values of the multiplication results corresponding to couples in P, i.e., $\mathbf{q} = [\ 0\ 1\ 2\ 3\ ...\ (2^m-1) \cdot (2^m-1)\ ]^T$. The values in the discrete set S should be coded by some g_i in order to generate a linear expression and one-to-one mapping between multiplication and addition. This can be easily done by using logarithm operation since $\alpha_i \cdot \alpha_p = \mu_{i,p}$ can be equivalently written as $log\alpha_i + log\alpha_p = log\mu_{i,p}$. Let $\mathbf{g}$ be a vector whose elements are the logarithm of the elements in S except 0, i.e., $\mathbf{g} = [\ 0\ g_1\ g_2\ ...\ g_{2^m-1}\ ]^T$ where $g_i = logi$ and "0" corresponds to a term related to the relay selection. In a similar manner, the logarithm of the values in $\mathbf{q}$ except 0 are used to obtain $\mathbf{h}$ as $\mathbf{h} = [\ 0\ h_1\ h_2\ ...]^T$ where $h_i = logq_{i+1}$. In order to select or access each element of $\mathbf{g}$ and $\mathbf{h}$, we need to define binary SOS1 vectors $\mathbf{v}''_i$ and $\mathbf{u}''_{i,p}$ from $\mathbf{v}_i$ and $\mathbf{u}_{i,p}$ respectively. This is possible by defining $\mathcal{T}_2\{.\}$ and $\mathcal{T}_3\{.\}$ operators. These two operators map a SOS1 vector into a binary SOS1 vector. Note that these operators generate SOS1 vectors which carry only the amplitude information.

$\mathcal{T}_2\{\}$ operates on $\mathbf{v}_i$ and $\mathcal{T}_3\{\}$ operates on $\mathbf{u}_{i,p}$ only. $\mathcal{T}_2\{\mathbf{v}_i\}$ sums the elements of $\mathbf{v}_i$ except the first element and results a binary vector of $2^m \times 1$ size with a nonzero element index being equal to the summed value. If $v_{i_1} = 1$ then $v''_{i_1} = 1$ and $\mathbf{g}^T \cdot \mathbf{v}''_i = 0$. If $u_{i,p_1} = 0$, $\mathcal{T}3\{\mathbf{u}_{i,p}\}$ takes the nonzero element value in $\mathbf{u}_{i,p}$ and finds its index value in $\mathbf{q}$. It generates a binary vector whose only nonzero value is at this index. Otherwise, $u''_{i,p_1} = 1$ and $\mathbf{h}^T \cdot \mathbf{u}''_{i,p} = 0$. Now consider the multiplication, $\frac{\mu_{i,p}}{\Delta_i \Delta_p} = \frac{\alpha_i}{\Delta_i} \cdot \frac{\alpha_p}{\Delta_p}$ which is equivalent to $log\frac{\mu_{i,p}}{\Delta_i \Delta_p} = log\frac{\alpha_i}{\Delta_i} + log\frac{\alpha_p}{\Delta_p}$. This equation can be written in terms of known vectors, $\mathbf{g}$, $\mathbf{h}$, and binary SOS1 vectors $\mathbf{v}''_i$ and $\mathbf{u}''_{i,p}$ as,

$$\mathbf{h}^T \cdot \mathbf{u}''_{i,p} = \mathbf{g}^T \cdot (\mathbf{v}''_i + \mathbf{v}''_p) \tag{9}$$

Note that (9) is defined when i^{th} and p^{th} relays are used simultaneously. For the other cases of relay selection, equation (9) can still be made valid if an additional variable $c_{i,p} \geq 0$ is added to the left hand side of equation (9). In this case, the right hand side of (9) is always nonnegative and $\mathbf{h}^T \cdot \mathbf{u}''_{i,p} = 0$ requiring a nonnegative $c_{i,p}$ to satisfy the equality, i.e.,

$$\mathbf{h}^T \cdot \mathbf{u}''_{i,p} + c_{i,p} = \mathbf{g}^T \cdot (\mathbf{v}''_i + \mathbf{v}''_p) \tag{10}$$

Similar to $b_{i,p}$, $c_{i,p}$ should satisfy $c_{i,p} + 2^m(1 - u_{i,p_1}) \leq 2^m$ in accordance with Table 1. As a result (5.d) is converted into a linear additive expression in (10) in terms of optimization variables, $\mathbf{v}''_i$, $\mathbf{u}''_{i,p}$ and $c_{i,p}$. B is the sum of weighted squared amplitudes of the beamformer elements. Define a vector $\mathbf{e}$ which is composed of the squared values in S, i.e., $\mathbf{e} = [\ 0^2\ 1^2\ 2^2\ ...\ (2^m-1)^2\ ]^T$. Using $\mathbf{e}$, B can be written as,

$$B = \sum_{i=1}^{M} \Delta_i^2 \frac{T_{k_{i,i}}}{D_{i,i}} \mathbf{e}^T \cdot \mathbf{v}''_i \tag{11}$$

Similarly (5.a) can be expressed as,

$$\sum_{i=1}^{M} \Delta_i^2 \mathbf{e}^T \cdot \mathbf{v}''_i \tag{12}$$

(4.f) can also be written using binary variables v_{i_1} as,

$$\sum_{i=1}^{M} (1 - v_{i_1}) \leq L \tag{13}$$

Now the expressions in (5.a-d) and (4.f) are converted to linear expressions in terms of $\mathbf{v}_i$, $\mathbf{u}_{i,p}$, $\mathbf{v}'_i$, $\mathbf{u}'_{i,p}$, $\mathbf{v}''_i$, $\mathbf{u}''_{i,p}$, $a_{i,p}$, $b_{i,p}$ and $c_{i,p}$ in (12), (6), (11), (8), (10) and (13) respectively. The final optimization problem is written as,

$$\min_{\mathbf{v}_i, \mathbf{u}_{i,p}, \mathbf{v}'_i, \mathbf{u}'_{i,p}, \mathbf{v}''_i, \mathbf{u}''_{i,p}, a_{i,p}, b_{i,p}, c_{i,p}} \sum_{i=1}^{M} \Delta_i^2 \mathbf{e}^T \cdot \mathbf{v}''_i \tag{14.a}$$

$$\begin{aligned} s.t. \sum_{i=1}^{M-1} \sum_{p=i+1}^{M} 2 \frac{\Delta_i \Delta_p}{\sqrt{D_{i,i} D_{p,p}}} |T_{k_{i,p}}| (cos(\angle T_{k_{i,p}}) \mathbf{c}^T \\ - sin(\angle T_{k_{i,p}}) \mathbf{s}^T) \cdot \mathbf{u}_{i,p} \\ + \sum_{i=1}^{M} \Delta_i^2 \frac{T_{k_{i,i}}}{D_{i,i}} \mathbf{e}^T \cdot \mathbf{v}''_i \geq \gamma_k \sigma_{d,k}^2 \quad k = 1, ..., N \end{aligned} \tag{14.b}$$

$$\mathbf{d}^T \cdot \mathbf{u}'_{i,p} + b_{i,p} = \mathbf{d}^T \cdot (-\mathbf{v}'_i + \mathbf{v}'_p) + a_{i,p}2^n \quad (14.c)$$

$$\mathbf{h}^T \cdot \mathbf{u}''_{i,p} + c_{i,p} = \mathbf{g}^T \cdot (\mathbf{v}''_i + \mathbf{v}''_p) \quad (14.d)$$

$$\sum_{i=1}^{M} (1 - v_{i_1}) \leq L \quad (14.e)$$

$$a_{i,p}, v_{i_1}, u_{i,p_1}, v'_{i_t}, u'_{i,p_t}, v''_{i_t}, u''_{i,p_t} \in \{0,1\}, \quad t = 1, ..., (2^n + 1) \quad (14.f)$$

$$v_{i_t}, u_{i,p_t} \in \{0\} \cup \mathbb{Z}^+, \ \ t = 2, ..., (2^n + 1) \quad (14.g)$$

$$\mathbf{v}_i, \mathbf{u}_{i,p}, \mathbf{v}_i + \mathbf{v}'_i, \ \mathbf{u}_{i,p} + \mathbf{u}'_{i,p} \in \text{ SOS1} \quad (14.h)$$

$$\mathbf{v}'_i, \ \mathbf{u}'_{i,p}, \ \mathbf{v}''_i, \ \mathbf{u}''_{i,p} \in \text{ SOS1}_b \quad (14.i)$$

$$[\, 0\ 1\ 1\ ...\ 1 \,]\mathbf{v}_i = \mathbf{p}^T \mathbf{v}''_i \quad (14.j)$$

$$[\, 0\ 1\ 1\ ...\ 1 \,]\mathbf{u}_{i,p} = \mathbf{q}^T \mathbf{u}''_{i,p} \quad (14.k)$$

$$b_{i,p} \geq 0, \ b_{i,p} + 2^n(1 - u_{i,p_1}) \leq 2^n \quad (14.l)$$

$$c_{i,p} \geq 0, \ c_{i,p} + 2^m(1 - u_{i,p_1}) \leq 2^m \quad (14.m)$$

$$-1 \leq v_{i_1} + v_{p_1} - 2u_{i,p_1} \leq 0 \quad (14.n)$$

All of the variables in these expressions except $b_{i,p}$ and $c_{i,p}$ are integer variables. Furthermore $\mathbf{v}'_i$, $\mathbf{v}''_i$, $\mathbf{u}'_{i,p}$ and $\mathbf{u}''_{i,p}$ are binary SOS1 (SOS1_b) vectors and $a_{i,p}$'s are binary variables. The expressions in (14.h-k) are used to implement $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ operators [5], [6] where $\mathbf{p} = [\ 0\ 1\ ...\ (2^m - 1)\ ]^T$. Therefore the problem can be solved using mixed integer linear programming with branch and cut procedure which is known to return the global optimum [13], [14], [15], [16]. Once the solution for $\mathbf{v}_i$'s are found, the phase angles and the amplitudes of the beamformer vector are obtained as, $\psi_i = \mathbf{f}_\psi^T \mathbf{v}'_i$, $\alpha_i = \frac{\Delta_i}{\sqrt{D_{i,i}}} \mathbf{f}_\alpha^T \mathbf{v}_i$, $i = 1, ..., M$ where $\mathbf{f}_\psi = [\ 0\ 0\ \Delta\theta\ ...\ (2^n - 1)\Delta\theta\ ]^T$ and $\mathbf{f}_\alpha = [\ 0\ 1\ 1\ ...\ 1\ ]^T$ respectively.

4. SIMULATION RESULTS

The evaluation of the proposed method is performed for flat-fading Rayleigh channels with unit variance. The source power, relay and receiver noise variances are selected as, $P_s = 10$ W, $\sigma_r^2 = 0.1$ and $\sigma_{d,k}^2 = 0.1$ respectively. We assume equal SNR thresholds at the receivers. Maximum relay power for each relay is the same and selected as $p_i = 2$ W. There are $N = 12$ receivers.

In Fig. 1, total power transmitted from the relays is plotted for different number of bits for phase (n) and amplitude (m). In this experiment, there is no relay selection and $M = 4$ relays are used. The average of 100 random channel trials at each point is presented. It is shown that very good user SNR levels are achieved by using reasonably low total relay power even when there are less relays compared to the number of users. As the number of bits increases, transmitted power decreases and the effect of amplitude becomes more significant.

In the second experiment, total power transmitted from the relays is considered for the relay selection as well as fixed relay case. Different number of relays are selected from $M = 6$ relays, i.e. $L = 2, 3, 4$. Fig. 2 shows the total power transmitted from the relays for γ_k ranging from 0 to 10 dB. $n = 3$ and $m = 3$ bits are used for phase and amplitude respectively. In this figure, it is shown that the best L out of M relay selection decreases the total relay power significantly. In fact, more than 2 dB improvement in power is possible for $L = 2$ and $L = 3$ compared to $M = L$ fixed relays.

Table 2 shows the computational complexity of the brute force and the proposed method where the average of 100 trials are reported. As it is seen from this table, the proposed optimum method has significantly lower complexity thanks to the efficiency of the mixed integer linear programming with branch and cut technique.

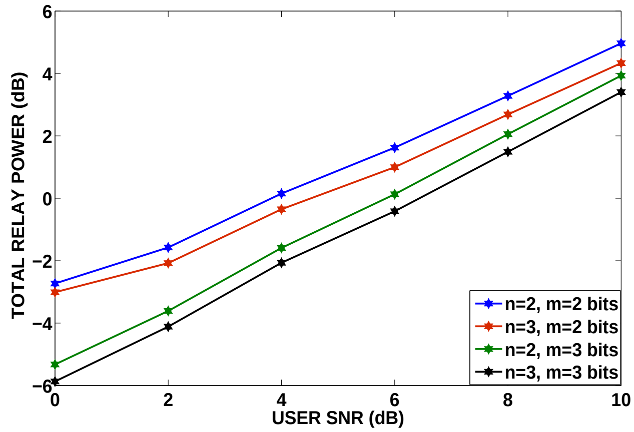

Fig. 1. Total relay power versus user SNR values for different number of bits for phase and amplitude.

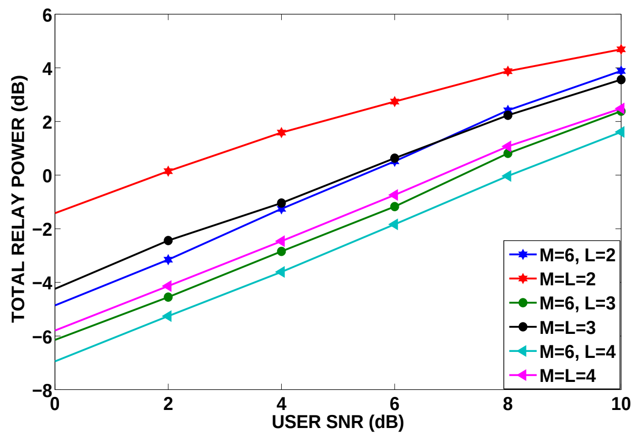

Fig. 2. Total relay power versus user SNR values for different relay selection and fixed cases.

Table 2. Computational time of the proposed method (PM) and brute force search (BFS)

$M = 6, m = 3$		$\gamma = 0$ dB	$\gamma = 4$ dB	$\gamma = 8$ dB
$n = 2, L = 3$	PM	1.45 s	4.84 s	7.35 s
	BFS	11 s	10 s	10 s
$n = 2, L = 4$	PM	4.20 s	15.41 s	46.25 s
	BFS	280 s	275 s	273 s
$n = 3, L = 3$	PM	3.33 s	8.76 s	12.96 s
	BFS	44 s	44 s	44 s
$n = 3, L = 4$	PM	21.6 s	51.03 s	138.24 s
	BFS	2215 s	2231 s	2198 s

5. CONCLUSION

Single group multicast relay beamformer design with multiple relay selection is considered for amplify-and-forward relaying protocol. The phase and amplitude coefficients of the relay weights are chosen from discrete sets. The joint nonlinear problem is converted to a linear form suitable for mixed integer linear programming. The optimum solution is obtained effectively and it is shown that the proposed method performs significantly better compared to the optimum beamformer for the fixed relay. Computational complexity is much better than the exhaustive search thanks to the efficiency of the branch and cut algorithm.

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