ROBUST WIDELY LINEAR BEAMFORMER BASED ON A PROJECTION CONSTRAINT

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ABSTRACT

For noncircular signals, optimal widely linear (WL) minimum variance distortionless response (MVDR) beamformer has a powerful performance by exploiting the noncircularity of the received signals. Though, the noncircularity rate can be estimated by the steering vector (SV) of the signal of interest (SOI), the performance degrades as there exist errors in the SOI's SV. This paper introduces a new robust WL beamformer. In the proposed approach, the assumed extended steering vector (ESV) of the SOI is used to construct an interference-plus-noise subspace projection matrix, and the new ESV is estimated by maximizing the WL beamformer output power under a constraint that prevents the ESV from converging to the interference. The proposed algorithm only needs imprecise knowledge of the antenna array geometry and the SOI's angular sector. Simulations verify the effectiveness of the proposed algorithm.

Index Terms— Array signal processing, widely linear, projection matrix, robust widely linear beamformer

1. INTRODUCTION

In array signal processing, beamforming is a widely used technology in radar, sonar and wireless communications [1]. Conventional beamforming techniques, such as linearly constrained minimum variance (LCMV) and minimum variance distortionless response (MVDR), aim at a linear and time invariant (TI) complex filter for stationary observations, whose complex envelopes have been proved to be necessarily second-order (SO) circular [2]. However, as signals are second-order noncircular and nonstationary in radio communication, such as amplitude phase-shift keying (ASK), binary phase-shift keying (BPSK), minimum shift keying (MSK) and unbalanced quaternary phase shift keying (UQPSK) signals [3], the conventional linear and TI approaches like MVDR beamformer turn out to be suboptimal and the optimal complex filters become the widely linear (WL) [4], [5], [6].

The WL MVDR beamformer is firstly introduced by Chevalier [7] and shows better performance than conventional beamformers. Then, by exploiting the noncircularity of the signal of interest (SOI), the optimal WL MVDR beamformer proposed in [8] further improves the performance, but the SOI's steering vector (SV) and noncircularity coefficient are the priori knowledge. In many applications, the noncircularity coefficient is unavailable, thus limit the practical implement of the beamformer. Subsequently, Xu et al. [9] estimate the noncircularity coefficient of the SOI, which makes the optimal WL MVDR beamformer available in practical applications. However, the estimator requires the SOI's exact SV to achieve optimal performance. A robust WL beamformer is presented in [10], the noncircularity coefficient uncertainty and the SOI's SV uncertainty are set to against errors. However, it is sensitive to large mismatch of noncircularity coefficient. Recently, a robust WL beamformer based on spatial spectrum of noncircularity coefficient is proposed in [11]. The method reconstructs the extended interference-plus-noise covariance matrix to get a corrected extended steering vector (ESV). It relies on the accurate antenna array geometry, so the method is effective on look direction error, signal spatial signature mismatch but not robust to channel gain and phase error, sensor location error.

In this paper, we propose a new robust WL beamformer. As the aforementioned WL beamformer needs accurate information to reconstruct covariance matrix, we find a more robust WL beamformer with less priori information needed. Motivated by conventional robust beamformer [12], which constructs an interference-plus-noise subspace projection matrix, we develop this projection matrix in area of the WL beamformer. Then considering the ESV is sensitive to mismatch, we set the largest projection value in the SOI's angular sector to relax the projection constraint. Finally, the ESV is obtained by solving a quadratically constrained quadratic programming (QCQP) problem. The priori information we need are the imprecise knowledge of the antenna array geometry and the SOI's angular sector.

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2. OPTIMAL WIDELY LINEAR MVDR

2.1 signal model

We consider an array of N antennas to receive narrowband signals and the array output $\mathbf{x}(t)$ is an $N \times 1$ complex vector. $\mathbf{x}(t)$ can be modeled as

$$\mathbf{x}(t) = s(t)\mathbf{a} + \mathbf{v}(t) \tag{1}$$

where $s(t)$ is the SOI's complex envelope, assumed SO noncircular, $\mathbf{a}$ is the SOI's SV and $\mathbf{v}(t)$ is the total interference-plus-noise vector, respectively. The noise is assumed to be circularly symmetric Gaussian white process.

The SO statistics of the noncircular observation $\mathbf{x}(t)$ are defined by

$$\mathbf{R}_x \triangleq \left\langle E\left[\mathbf{x}(t)\mathbf{x}(t)^H\right]\right\rangle = \pi_s \mathbf{a}\mathbf{a}^H + \mathbf{R} \tag{2}$$

$$\mathbf{C}_x \triangleq \left\langle E\left[\mathbf{x}(t)\mathbf{x}(t)^T\right]\right\rangle = \pi_s \gamma_s \mathbf{a}\mathbf{a}^T + \mathbf{C} \tag{3}$$

where $\langle \cdot \rangle$ denotes the time-averaging operation, with respect to the time index t, $\pi_s = \left\langle E\left[|s(t)|^2\right]\right\rangle$ is the time-averaged power of the SOI, $\gamma_s = \left\langle E\left[s(t)^2\right]\right\rangle / \pi_s$ is the noncircularity coefficient of the SOI, with $\gamma_s = |\gamma_s| e^{j\phi_s}$, $0 \le |\gamma_s| \le 1$, where $|\gamma_s|$ and ϕ_s denote the noncircularity rate and phase, $\mathbf{R}$ and $\mathbf{C}$ are the correlation matrices of the total interference-plus-noise, which are defined as $\mathbf{R} = \left\langle E\left[\mathbf{v}(t)\mathbf{v}(t)^H\right]\right\rangle$ and $\mathbf{C} = \left\langle E\left[\mathbf{v}(t)\mathbf{v}(t)^T\right]\right\rangle$.

2.2 optimal WL MVDR beamformer

To exploit the noncircularity of $\mathbf{x}(t)$, the WL MVDR beamformer utilizes the extended observation vector as $\tilde{\mathbf{x}}(t) \triangleq \left[\mathbf{x}(t)^T, \mathbf{x}(t)^H\right]^T$, using the signal model (1), we get

$$\tilde{\mathbf{x}}(t) = s(t)\tilde{\mathbf{a}}_1 + s(t)^* \tilde{\mathbf{a}}_2 + \tilde{\mathbf{v}}(t) \tag{4}$$

where $\tilde{\mathbf{a}}_1 \triangleq \left[\mathbf{a}^T, \mathbf{0}_N^T\right]^T$, $\tilde{\mathbf{a}}_2 \triangleq \left[\mathbf{0}_N^T, \mathbf{a}^H\right]^T$, and $\tilde{\mathbf{v}}(t) \triangleq \left[\mathbf{v}(t)^T, \mathbf{v}(t)^H\right]^T$. The SO statistics of $\tilde{\mathbf{x}}(t)$ and $\tilde{\mathbf{v}}(t)$ are

$$\mathbf{R}_{\tilde{x}} \triangleq \left\langle E\left[\tilde{\mathbf{x}}(t)\tilde{\mathbf{x}}(t)^H\right]\right\rangle = \begin{bmatrix} \mathbf{R}_x & \mathbf{C}_x \\ \mathbf{C}_x^* & \mathbf{R}_x^* \end{bmatrix} \tag{5}$$

$$\mathbf{R}_{\tilde{v}} \triangleq \left\langle E\left[\tilde{\mathbf{v}}(t)\tilde{\mathbf{v}}(t)^H\right]\right\rangle = \begin{bmatrix} \mathbf{R} & \mathbf{C} \\ \mathbf{C}^* & \mathbf{R}^* \end{bmatrix} \tag{6}$$

Further exploiting the noncircularity of the SOI, the optimal WL MVDR beamformer in [8] gives the following orthogonal decomposition of $s(t)^*$

$$s(t)^* = \gamma_s^* s(t) + \left[\pi_s\left(1 - |\gamma_s|^2\right)\right]^{1/2} s'(t) \tag{7}$$

with $\left\langle E\left[s(t)s'(t)^*\right]\right\rangle = 0$ and $\left\langle E\left[|s'(t)|^2\right]\right\rangle = 1$. Then the extended observation vector can be written as

$$\begin{aligned}\tilde{\mathbf{x}}(t) &= s(t)\underbrace{\left(\tilde{\mathbf{a}}_1 + \gamma_s^* \tilde{\mathbf{a}}_2\right)}_{\tilde{\mathbf{a}}_\gamma} + \underbrace{s'(t)\left[\pi_s\left(1 - |\gamma_s|^2\right)\right]^{1/2} \tilde{\mathbf{a}}_2 + \tilde{\mathbf{v}}(t)}_{\tilde{\mathbf{v}}_\gamma(t)} \\ &\triangleq s(t)\tilde{\mathbf{a}}_\gamma + \tilde{\mathbf{v}}_\gamma(t)\end{aligned} \tag{8}$$

where $\tilde{\mathbf{a}}_\gamma = \tilde{\mathbf{a}}_1 + \gamma_s^* \tilde{\mathbf{a}}_2 = \left[\mathbf{a}^T, \gamma_s^* \mathbf{a}^H\right]^T$ is the equivalent ESV of the SOI, which now depends on γ_s, and $\tilde{\mathbf{v}}_\gamma(t)$ is the global noise vector for the extended observation vector $\tilde{\mathbf{x}}(t)$. The optimal WL beamformer is then designed as

$$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}^H \mathbf{R}_{\tilde{v}_\gamma} \tilde{\mathbf{w}} \quad \text{subject to} \quad \tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_\gamma = 1 \tag{9}$$

where $\mathbf{R}_{\tilde{v}_\gamma} = \left\langle E\left[\tilde{\mathbf{v}}_\gamma(t)\tilde{\mathbf{v}}_\gamma(t)^H\right]\right\rangle$, the optimal solution of (9) is

$$\tilde{\mathbf{w}}_{\text{MVDR}} = [\tilde{\mathbf{a}}_\gamma^H \mathbf{R}_{\tilde{v}_\gamma}^{-1} \tilde{\mathbf{a}}_\gamma]^{-1} \mathbf{R}_{\tilde{v}_\gamma}^{-1} \tilde{\mathbf{a}}_\gamma \tag{10}$$

The output signal-to-interference-plus-noise ratios (SINR) of a WL filter $\tilde{\mathbf{w}}$ is defined by

$$\text{SINR}[\tilde{\mathbf{w}}] = \frac{\pi_s \left|\tilde{\mathbf{w}}^H \tilde{\mathbf{a}}_\gamma\right|^2}{\tilde{\mathbf{w}}^H \mathbf{R}_{\tilde{v}_\gamma} \tilde{\mathbf{w}}} \tag{11}$$

In practical application, the exact $\mathbf{R}_{\tilde{v}_\gamma}$ and $\mathbf{a}$ are unavailable, and replaced by the extended sample covariance matrix $\hat{\mathbf{R}}_{\tilde{x}} = 1/K \sum_{k=1}^{K} \tilde{\mathbf{x}}(k)\tilde{\mathbf{x}}(k)^H$, where K is the number of snapshots, and the presumed SV $\bar{\mathbf{a}}$. Moreover, when the noncircular coefficient γ_s is known as a priori information, the WL weight vector (10) becomes to

$$\tilde{\mathbf{w}}_{\text{MVDR}} = [\bar{\tilde{\mathbf{a}}}_\gamma^H \hat{\mathbf{R}}_{\tilde{x}}^{-1} \bar{\tilde{\mathbf{a}}}_\gamma]^{-1} \hat{\mathbf{R}}_{\tilde{x}}^{-1} \bar{\tilde{\mathbf{a}}}_\gamma \tag{12}$$

where $\bar{\tilde{\mathbf{a}}}_\gamma = \left[\bar{\mathbf{a}}^T, \gamma_s^* \bar{\mathbf{a}}^H\right]^T$. In [9], The estimate of γ_s is given as

$$\hat{\gamma}_s = -\frac{\mathbf{a}^H \mathbf{E} \mathbf{a}^*}{\mathbf{a}^H \mathbf{D} \mathbf{a}} \cdot \frac{\mathbf{a}^H \mathbf{a}}{\mathbf{a}^H (\mathbf{I}_N - \hat{\eta} \mathbf{R}_x^{-1}) \mathbf{a}} \tag{13}$$

where $\mathbf{D} \triangleq (\mathbf{R}_x - \mathbf{C}_x \mathbf{R}_x^{*-1} \mathbf{C}_x^*)^{-1}$, $\mathbf{E} \triangleq -\mathbf{D}\mathbf{C}_x \mathbf{R}_x^{*-1}$ and $\hat{\eta}$ is the minimum eigenvalue of $\mathbf{R}_{\tilde{x}}$. Substituting (13) in (12), we can implement the optimal WL MVDR beamformer practically. Note that an inaccurate SV would result in a bad estimate of γ_s, thus degrading the WL beamformer's performance.

3. PROPOSED ALGORITHM

In this section, we introduce a new robust WL beamformer. First, we construct the interference-plus-noise subspace projection matrix $\tilde{\mathbf{P}}_{i+n}$. The extended sample covariance matrix $\hat{\mathbf{R}}_{\tilde{x}}$ can be eigendecomposed as $\hat{\mathbf{R}}_{\tilde{x}} = \sum_{i=1}^{2N} \lambda_i \mathbf{e}_i \mathbf{e}_i^H$, where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{2N} = \sigma^2$ are the eigenvalues of $\hat{\mathbf{R}}_{\tilde{x}}$ in descending order, and $\mathbf{e}_i$ for $i = 1,2,\cdots,2N$ are the corresponding eigenvectors, σ^2 denotes the noise power. When the mismatch between the assumed ESV $\bar{\tilde{\mathbf{a}}}_{\hat{\gamma}} = \left[\bar{\mathbf{a}}^T, \hat{\gamma}_s^* \bar{\mathbf{a}}^H\right]^T$ and the actual one $\tilde{\mathbf{a}}_\gamma = \left[\mathbf{a}^T, \gamma_s^* \mathbf{a}^H\right]^T$ is small, as in [12, 13], the eigenvectors corresponding to the small projections of $\bar{\tilde{\mathbf{a}}}_{\hat{\gamma}}$ on the eigenvector $\mathbf{e}_i$ can be used to structure the $\bar{\mathbf{U}}_{i+n}$ which spans the new estimated interference-plus-noise subspace.

The projections $p(i) = \left|\mathbf{e}_i^H \bar{\tilde{\mathbf{a}}}_{\hat{\gamma}}\right|^2$ $(i = 1,2,\cdots,2N)$ are arranged in descending order, $p(1) \geq p(2) \geq \cdots \geq p(2N)$. If $\left[p(1)+p(2)+\cdots+p(n)\right] / \sum_{i=1}^{2N} p(i) > \rho$, $0 < \rho < 1$ is a given parameter representing the energy percentage, the interference-plus-noise subspace can be constructed as $\bar{\mathbf{U}}_{i+n} = [\mathbf{e}_{n+1}, \mathbf{e}_{n+2}, \cdots, \mathbf{e}_{2N}]$. Thus, the new estimated interference-plus-noise subspace projection matrix can be obtained as $\tilde{\mathbf{P}}_{i+n} = \bar{\mathbf{U}}_{i+n} \bar{\mathbf{U}}_{i+n}^H$.

Next, we construct a projection constraint. By formulating the noncircular coefficient (13) at different direction θ,

$$\hat{\gamma}(\theta) = -\frac{\mathbf{a}(\theta)^H \mathbf{E}\mathbf{a}(\theta)^*}{\mathbf{a}(\theta)^H \mathbf{D}\mathbf{a}(\theta)} \cdot \frac{\mathbf{a}(\theta)^H \mathbf{a}(\theta)}{\mathbf{a}(\theta)^H (\mathbf{I}_N - \hat{\eta}\mathbf{R}_x^{-1})\mathbf{a}(\theta)} \tag{14}$$

where $\mathbf{a}(\theta)$ is the steering vector associated with a hypothetical direction θ based on the known array geometry structure, the ESV that comes from direction θ can be constructed as $\tilde{\mathbf{a}}_{\hat{\gamma}}(\theta) = \left[\mathbf{a}(\theta)^T, \hat{\gamma}(\theta)^* \mathbf{a}(\theta)^H\right]^T$. Fig. 1 depicts the relation of $\left\|\tilde{\mathbf{P}}_{i+n}^H \tilde{\mathbf{a}}_{\hat{\gamma}}(\theta)\right\|$ with θ, where $\|\cdot\|$ denotes the Euclidean norm. The example set up is the following. A uniform linear array (ULA) composed of $N = 4$ omnidirectional sensors with half a wavelength apart, one SOI and two interferences are all BPSK signals arriving from the directions 0°, 40°, -30°, with 10, 20, 20dB, and noncircularity rates $e^{j\frac{\pi}{6}}$, $e^{-j\frac{2\pi}{3}}$, $e^{j\frac{\pi}{3}}$ respectively. The SOI is assumed from $\bar{\theta}=0^\circ$. The number of snapshots is 300.

In Fig. 1, we can find that the term $\left\|\tilde{\mathbf{P}}_{i+n}^H \tilde{\mathbf{a}}_{\hat{\gamma}}(\theta)\right\|$ has the smallest values around the presumed direction $\bar{\theta}$, and a relative large value outside the SOI's angular section. Thus, we can constraint the projection to ensure the modified ESV $\hat{\tilde{\mathbf{a}}}_{\hat{\gamma}}$ does not converge to any interference

$$\left\|\tilde{\mathbf{P}}_{i+n}^H \hat{\tilde{\mathbf{a}}}_{\hat{\gamma}}\right\| \leq \left\|\tilde{\mathbf{P}}_{i+n}^H \bar{\tilde{\mathbf{a}}}_{\hat{\gamma}}\right\| \tag{15}$$

As there exists mismatch between the assumed ESV and the actual one, the constructed interference-plus-noise subspace projection matrix $\tilde{\mathbf{P}}_{i+n}$ is not so accurate, which may lead to the projection of the actual ESV $\left\|\tilde{\mathbf{P}}_{i+n}^H \tilde{\mathbf{a}}_\gamma\right\|$ is not the minimum value. Thus, we get a more reasonable constraint by relaxing the constraint (15) to

$$\left\|\tilde{\mathbf{P}}_{i+n}^H \hat{\tilde{\mathbf{a}}}_{\hat{\gamma}}\right\| \leq \max_{\theta \in \Theta} \left\|\tilde{\mathbf{P}}_{i+n}^H \tilde{\mathbf{a}}_{\hat{\gamma}}(\theta)\right\| \tag{16}$$

where $\Theta = \left[\theta_{\min}, \theta_{\max}\right]$ is a known angular sector, which the SOI locates in. This angular sector is assumed to be distinguishable from interfering signals, which can be obtained from low resolution direction finding methods. The constraint means that the modified ESV just has less correlation with $\tilde{\mathbf{P}}_{i+n}$ than the largest one in the SOI's angular section. The reason to change the constraint can be explain by Fig. 2. The experiment's conditions are the same as Fig. 1, except that the SOI's direction is assumed from 0° but actually comes from 4°, and the SV of both SOI and interferences have gain and phase errors, which are drawn from the random generators $N(1, 0.05)$ and $N(0, 0.05\pi)$.

The values of $\left\|\tilde{\mathbf{P}}_{i+n}^H \tilde{\mathbf{a}}_\gamma\right\|$ and $\left\|\tilde{\mathbf{P}}_{i+n}^H \bar{\tilde{\mathbf{a}}}_{\hat{\gamma}}\right\|$ are specially pointed out in Fig. 2, the new bound $\max_{\theta \in \Theta} \left\|\tilde{\mathbf{P}}_{i+n}^H \tilde{\mathbf{a}}_{\hat{\gamma}}(\theta)\right\|$ is the maximum value in the SOI's angular section, $\Theta = [-5, 5]$. We can find the projection of the actual ESV $\left\|\tilde{\mathbf{P}}_{i+n}^H \tilde{\mathbf{a}}_\gamma\right\|$ is larger than the presumed one $\left\|\tilde{\mathbf{P}}_{i+n}^H \bar{\tilde{\mathbf{a}}}_{\hat{\gamma}}\right\|$, but still under the new bound. Thus the constraint (15) is unreachable, but the revised one (16) is still satisfied.

The presumed ESV is corrected as $\hat{\tilde{\mathbf{a}}}_{\hat{\gamma}} = \bar{\tilde{\mathbf{a}}}_{\hat{\gamma}} + \tilde{\mathbf{e}}$, the mismatch vector $\tilde{\mathbf{e}}$ can be decomposed into two orthogonal components, $\tilde{\mathbf{e}} = \tilde{\mathbf{e}}_\perp + \tilde{\mathbf{e}}_\parallel$, where $\tilde{\mathbf{e}}_\parallel$ is parallel to $\bar{\tilde{\mathbf{a}}}_{\hat{\gamma}}$ and $\tilde{\mathbf{e}}_\perp$ is orthogonal to $\bar{\tilde{\mathbf{a}}}_{\hat{\gamma}}$. As any scaling of the SV does not impact the output SINR, we can ignore the $\tilde{\mathbf{e}}_\parallel$ component [14], and $\tilde{\mathbf{e}}_\perp$ can be found by maximizing the output power of the WL beamformer $\hat{P}_{WL}\left(\tilde{\mathbf{a}}_\gamma\right) = 1 / \left(\tilde{\mathbf{a}}_\gamma^H \hat{\mathbf{R}}_{\tilde{x}}^{-1} \tilde{\mathbf{a}}_\gamma\right)$ under the projection constraint:

$$\begin{aligned} &\min_{\mathbf{e}_\perp} \left(\bar{\tilde{\mathbf{a}}}_{\hat{\gamma}} + \tilde{\mathbf{e}}_\perp\right)^H \hat{\mathbf{R}}_{\tilde{x}}^{-1} \left(\bar{\tilde{\mathbf{a}}}_{\hat{\gamma}} + \tilde{\mathbf{e}}_\perp\right) \\ &\text{subject to } \; \bar{\tilde{\mathbf{a}}}_{\hat{\gamma}}^H \tilde{\mathbf{e}}_\perp = 0 \\ &\left\|\tilde{\mathbf{P}}_{i+n}^H \left(\bar{\tilde{\mathbf{a}}}_{\hat{\gamma}} + \tilde{\mathbf{e}}_\perp\right)\right\| \leq \max_{\theta \in \Theta} \left\|\tilde{\mathbf{P}}_{i+n}^H \tilde{\mathbf{a}}_{\hat{\gamma}}(\theta)\right\| \end{aligned} \tag{17}$$

The optimization problem (17) is a feasible quadratically constrained quadratic programming (QCQP) problem and can be easily solved with the help of CVX Toolbox [15].

Finally, the estimated ESV of the SOI is $\hat{\tilde{\mathbf{a}}}_{\hat{\gamma}} = \bar{\tilde{\mathbf{a}}}_{\hat{\gamma}} + \tilde{\mathbf{e}}_\perp$, and the WL weight vector of the proposed WL beamformer can be computed as

$$\tilde{\mathbf{w}}_{pro} = \left(\hat{\tilde{\mathbf{a}}}_{\hat{\gamma}}^H \hat{\mathbf{R}}_{\tilde{x}}^{-1} \hat{\tilde{\mathbf{a}}}_{\hat{\gamma}}\right)^{-1} \hat{\mathbf{R}}_{\tilde{x}}^{-1} \hat{\tilde{\mathbf{a}}}_{\hat{\gamma}} \tag{18}$$

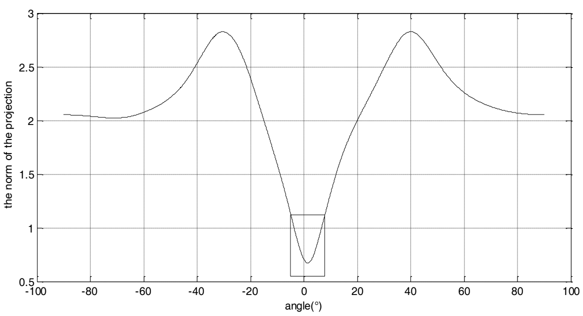

Fig. 1. The norm of the projection versus angle

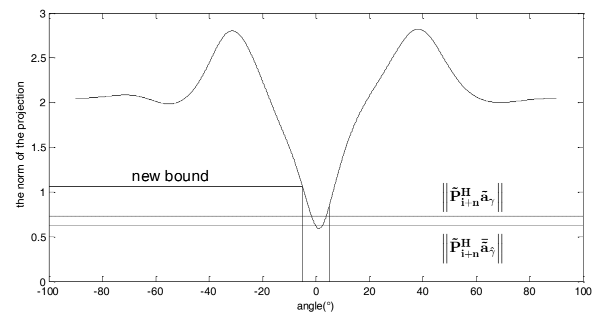

Fig. 2. The norm of the projection versus angle (with errors)

4. SIMULATION RESULTS

In our simulations, a ULA with $N=4$ omnidirectional sensors spaced half a wavelength is used. The additive noise is modeled as complex circularly symmetric Gaussian zero-mean spatially and temporally white process. The desired signal and two interferences are all BPSK. The desired signal is assumed to be from the presumed direction $\bar{\theta}_s = 0^\circ$ with noncircularity phase 60°. Two interferences are assumed to be from -30° and 40°, with noncircularity phase -120° and 150°, respectively. The interference-to-noise ratio (INR) is equal to 20 dB. The proposed beamformer (18) is compared to the robust WL-RCB [10], the reconstruct method [11], and the conventional robust beamformer [12], the optimal SINRs of Capon's MVDR and optimal WL MVDR are also shown in figures. SOI's angular sector is set to be $\Theta = [-5, 5]$ for the proposed method and [11], the value ε_γ=0.1, ε_a=0.3N is used in [10], ρ=0.9 is set in [12] and our method, two hundred Monte Carlo trials are simulated for each calculation.

In the first experiment, we consider signal look direction mismatch for both desired signal and interferences. The random direction-of-arrival (DOA) mismatch is uniformly distributed in $[-4^\circ, 4^\circ]$, which changes from run to run but remain fixed from snapshot to snapshot. Fig.3 shows the output signal-to-interference-plus-noise ratio (SINR) versus the input signal-to-noise ratio (SNR), with 300 snapshots. The results in Fig. 3 reveal that the proposed robust WL beamformer has a comparative performance with the reconstruct method, and much better performance than other tested algorithms.

In the second experiment, we consider both signal look direction mismatch for both desired signal and interferences, and calibration errors caused by gain and phase perturbations in each antenna. The random DOA mismatch is the same as the first experiment. The gain and phase errors are drawn from the uniformly distributed generator in $[-0.02, 0.02]$ and $[-5^\circ, 5^\circ]$. Fig. 4 shows the output SINR versus the input SNR, with 300 snapshots.

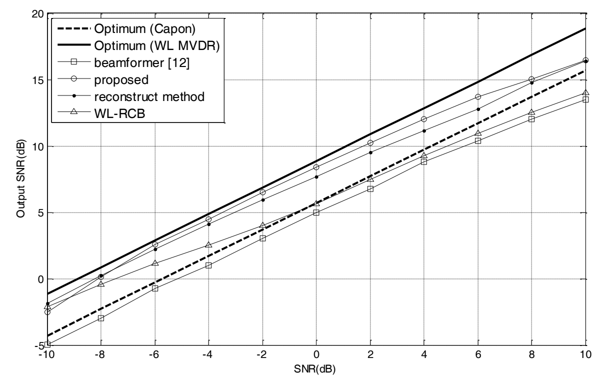

Fig. 3. Output SINR versus SNR

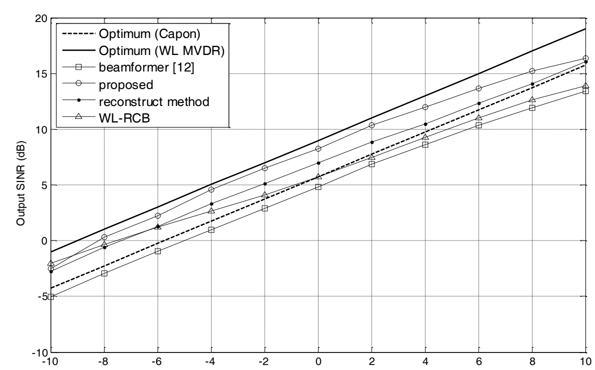

Fig. 4. Output SINR versus SNR

The results in Fig. 4 reveal that the proposed robust WL beamformer not only outperforms the conventional robust adaptive beamformer but also has a better performance than other robust WL beamforming algorithms. The results in Fig. 3 and Fig. 4 show the proposed algorithm can provide more robust against various mismatches.

5. CONCLUSION

In this paper, we have proposed a robust WL beamformer based on a projection constraint. The proposed method needs much less priori information and provides more robust than previous WL beamformers. The experiments show the proposed method achieves a better performance than other algorithms.

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